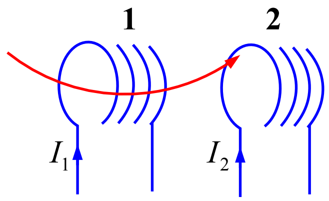
12—5 互感

一、互感现象及其规律



I_1 激发的磁场穿过 2 的磁通量

$\Phi_{21} \propto I_1$，$\Phi_{21} = M_{21} I_1$

I_2 激发的磁场穿过 1 的磁通量

$\Phi_{12} \propto I_2$，$\Phi_{12} = M_{12} I_2$

M_{21}：1 对 2 的互感系数，M_{12}：2 对 1 的互感系数

$M_{21} \equiv M_{12} = M$：互感系数，SI：H

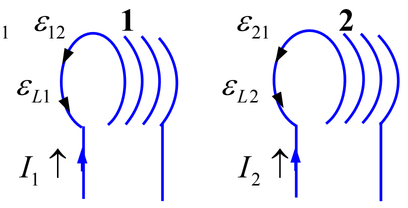
M：形状、大小、面积、匝数、磁介质、相对位置有关

$\Phi_{21} = MI_1$，$\Phi_{12} = MI_2$

推论：if，$I_1 = I_2$，$\Phi_{12} = \Phi_{21}$

$$\varepsilon_{21} = -\frac{d\Phi_{21}}{dt} = -M\frac{dI_1}{dt}$$

$$\varepsilon_{12} = -\frac{d\Phi_{12}}{dt} = -M\frac{dI_2}{dt}$$



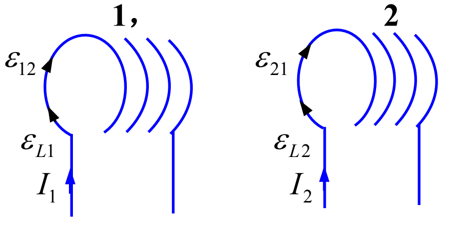
ε_{21}、ε_{12} 互感电动势

两线圈磁场相互加强时，电流的流入端称为两线圈的同名端

电流由同名端流入时，互感电动势正方向与自感电动势相同

两线圈磁场相互削弱时，电流的流入端称为两线圈的异名端

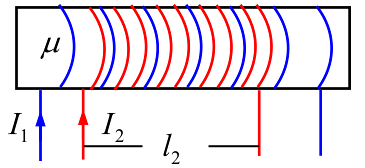
电流由异名端流入时，互感电动势正方向与自感电动势相反



二、M 的计算

$$M = \frac{\Phi_{21}}{I_1} = \frac{\Phi_{12}}{I_2}，\quad M = \varepsilon_{21}/(-\frac{dI_1}{dt}) = \varepsilon_{12}/(-\frac{dI_2}{dt})$$

例：两个共轴螺线管，$n_1\ l_1$，$n_2 l_2$，半径 R，$l_1, l_2 >> R$



（1）证明：$M_{21} = M_{12}$（2）M 与 L_1、L_2 的关系

解：（1）$B_1 = \mu n_1 I_1$，$B_2 = \mu n_2 I_2$

$$\Phi_{21} = N_2 B_1 S = n_2 l_2 \mu n_1 I_1 \pi R^2，\quad M_{21} = \frac{\Phi_{21}}{I_1} = \mu n_1 n_2 l_2 \pi R^2$$

$$\Phi_{12} = N_1' B_2 S = n_1 l_2 \mu n_2 I_2 \pi R^2，\quad M_{12} = \frac{\Phi_{12}}{I_2} = \mu n_1 n_2 l_2 \pi R^2$$

$$M_{21} = M_{12} = M = \mu n_1 n_2 l_2 \pi R^2$$

（2）$L_1 = \mu n_1^2 l_1 \pi R^2$，$L_2 = \mu n_2^2 l_2 \pi R^2$

$$\sqrt{L_1 L_2} = \mu n_1 n_2 \sqrt{l_1 l_2} \pi R^2$$

$$\frac{M}{\sqrt{L_1 L_2}} = \frac{l_2}{\sqrt{l_1 l_2}} = \sqrt{\frac{l_2}{l_1}}，\quad M = \sqrt{\frac{l_2}{l_1}} \sqrt{L_1 L_2}$$

一般，$M = k\sqrt{L_1 L_2}$，常数$0 \le k \le 1$：耦合系数

由两个线圈的相对位置决定

讨论：

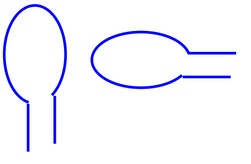
（1）无漏磁耦合，$k = 1$，$M = \sqrt{L_1 L_2}$

如果，$L_1 = L_2 = L$，$M = L$

（2）松耦合，两线圈相距很远

或垂直摆放

$k \approx 0$，$M \approx 0$



例：线圈串联

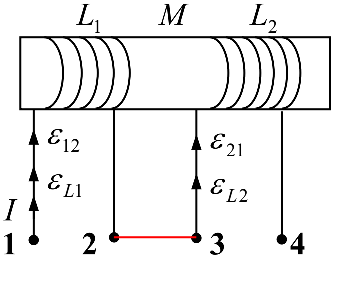
解：（1）顺串联（磁场相互加强）

$$\varepsilon_L = \varepsilon_{L1} + \varepsilon_{12} + \varepsilon_{L2} + \varepsilon_{21}$$

$$= -L_1 \frac{dI}{dt} - M\frac{dI}{dt} - L_2 \frac{dI}{dt} - M\frac{dI}{dt}$$

$$= -(L_1 + L_2 + 2M)\frac{dI}{dt}$$

$$L = \varepsilon_L / (-\frac{dI}{dt}) = L_1 + L_2 + 2M$$



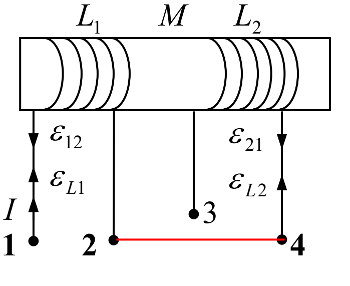
（2）反串联（磁场相互削弱）

$$\varepsilon_L = \varepsilon_{L1} - \varepsilon_{12} + \varepsilon_{L2} - \varepsilon_{21}$$

$$= -L_1 \frac{dI}{dt} + M\frac{dI}{dt} - L_2 \frac{dI}{dt} + M\frac{dI}{dt}$$

$$= -(L_1 + L_2 - 2M)\frac{dI}{dt}$$

$$L = \varepsilon_L / (-\frac{dI}{dt}) = L_1 + L_2 - 2M$$



讨论：（1）顺串联还是反串联看磁场是相互加强还是削弱

（2）顺串联 $L = L_1 + L_2 + 2k\sqrt{L_1 L_2}$

反串联 $L = L_1 + L_2 - 2k\sqrt{L_1 L_2}$

无漏磁耦合 $k = 1$

顺串联 $L = L_1 + L_2 + 2\sqrt{L_1 L_2}$

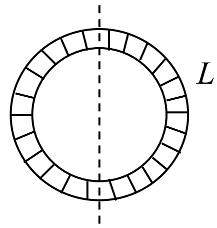
反串联 $L=L_1+L_2-2\sqrt{L_1L_2}$

$L_1=L_2$，顺串联 $L=4L_1$，反串联 $L=0$

松耦合 $k=0$， $L=L_1+L_2$

（3）螺绕环自感系数 L

半个螺绕环自感系数 L'



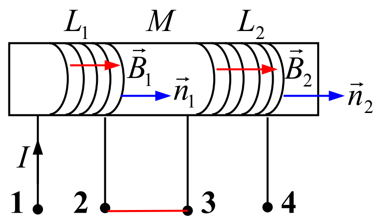
$$L'>\frac{L}{2},\quad L'=\frac{L}{2},\quad L'<\frac{L}{2}$$

$$L=L'+L'+2M=2(L'+M)$$

$$L'=\frac{L}{2}-M<\frac{L}{2}$$

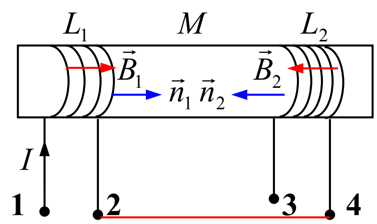
（4）用磁通量计算

（i）顺串联：



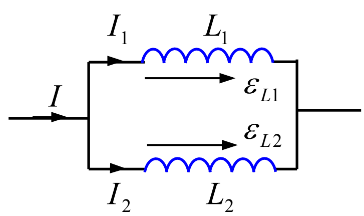
$$\Phi=\Phi_{11}+\Phi_{12}+\Phi_{21}+\Phi_{22}$$
$$=L_1I+MI+MI+L_2I$$
$$=(L_1+L_2+2M)I\text{，}$$
$$L=\Phi/I=L_1+L_2+2M$$

（ii）反串联：



$$\Phi=\Phi_{11}+\Phi_{12}+\Phi_{21}+\Phi_{22}$$
$$=L_1I-MI-MI+L_2I$$
$$=(L_1+L_2-2M)I\text{，}$$
$$L=\Phi/I=L_1+L_2-2M$$

例：线圈并联（L_1，L_2，M =0）



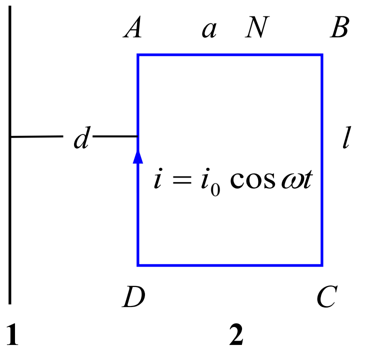
$$I=I_1+I_2\text{，}\quad \frac{dI}{dt}=\frac{dI_1}{dt}+\frac{dI_2}{dt}$$

$$\varepsilon_{L1}=-L_1\frac{dI_1}{dt}\text{，}\quad \varepsilon_{L2}=-L_2\frac{dI_2}{dt}$$

$$\varepsilon_L=-L\frac{dI}{dt}$$

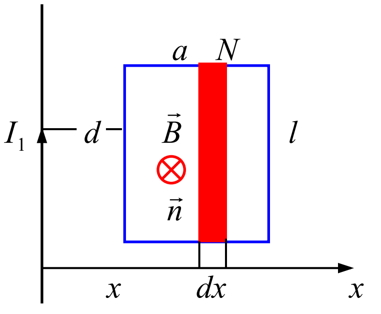
$$\frac{\varepsilon_L}{-L}=\frac{\varepsilon_{L1}}{-L_1}+\frac{\varepsilon_{L2}}{-L_2}\text{，}\quad \varepsilon_L=\varepsilon_{L1}=\varepsilon_{L2}\text{，}\quad \frac{1}{L}=\frac{1}{L_1}+\frac{1}{L_2}\text{，}\quad L=\frac{L_1L_2}{L_1+L_2}$$

例：



求：直导线中的感应电动势

解：$\varepsilon_{12} = -M\dfrac{dI_2}{dt} = Mi_0\omega\sin\omega t$



$$M = \frac{\Phi_{21}}{I_1} = \frac{\Phi_{12}}{I_2}$$

$$\Phi_{21} = N\int_S \vec{B}\cdot d\vec{S} = N\int_S B\cos\theta dS$$

$$= N\int_d^{d+a}\frac{\mu_0 I_1}{2\pi x}l dx = \frac{\mu_0 N I_1 l}{2\pi}\ln\frac{d+a}{d}$$

$$M = \frac{\Phi_{21}}{I_1} = \frac{\mu_0 N l}{2\pi}\ln\frac{d+a}{d}，\quad \varepsilon_{12} = \frac{\mu_0 N l}{2\pi}\ln\frac{d+a}{d}\cdot i_0\omega\sin\omega t$$