

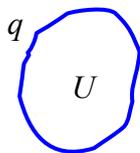
第3节 电容 电容器

一、孤立导体的电容

$$U \propto q,$$

$$\frac{q}{U} = C : \text{电容}$$

SI: C/V=F (法拉)

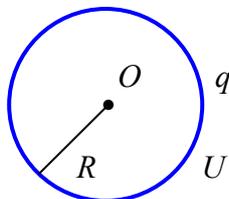


例: 导体球

解:
$$U = \frac{q}{4\pi\epsilon_0 R}$$

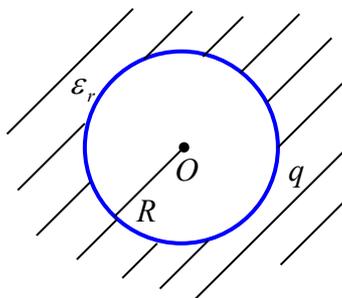
$$C_0 = \frac{q}{U} = 4\pi\epsilon_0 R$$

$$C_{\text{地}} = 4\pi\epsilon_0 R = 4\pi \times 8.85 \times 10^{-12} \times 6400 \times 10^3 \approx 7.0 \times 10^{-4} \text{ (F)}$$



$$U = \frac{q}{4\pi\epsilon_0\epsilon_r R}$$

$$C = \frac{q}{U} = 4\pi\epsilon_0\epsilon_r R = \epsilon_r C_0$$



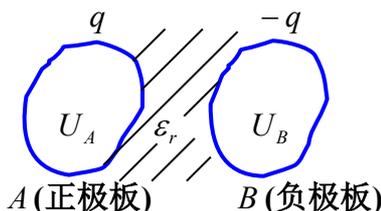
二、电容器及电容

$$U_{AB} = U_A - U_B$$

$$q \propto U_{AB}$$

$$\frac{q}{U_{AB}} = C$$

SI: F (法拉)



三、几种典型的电容器

1、平行板电容器

$$E = \frac{E_0}{\epsilon_r} = \frac{\sigma}{\epsilon_0\epsilon_r} = \frac{Q}{\epsilon_0\epsilon_r S}$$

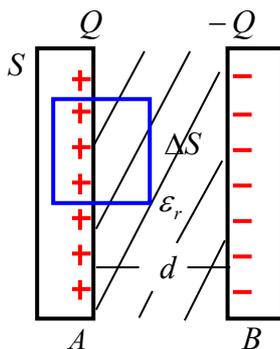
$$\oint_S \vec{D} \cdot d\vec{S} = D\Delta S = \sigma\Delta S,$$

$$D = \sigma = \epsilon_0\epsilon_r E,$$

$$E = \frac{\sigma}{\epsilon_0\epsilon_r} = \frac{Q}{\epsilon_0\epsilon_r S}$$

$$U_{AB} = Ed = \frac{Q}{\epsilon_0\epsilon_r S} d$$

$$C = \frac{Q}{U_{AB}} = \frac{\epsilon_0\epsilon_r S}{d} = \epsilon_r C_0, \quad C_0 = \frac{\epsilon_0 S}{d}$$



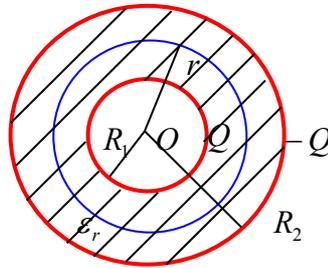
2、球形电容器

$$E = \frac{E_0}{\varepsilon_r} = \frac{Q}{4\pi\varepsilon_0\varepsilon_r r^2}$$

$$\oint_S \vec{D} \cdot d\vec{S} = D4\pi r^2 = Q,$$

$$D = \frac{Q}{4\pi r^2} = \varepsilon_0\varepsilon_r E,$$

$$E = \frac{Q}{4\pi\varepsilon_0\varepsilon_r r^2}$$



$$U = \int_{R_1}^{R_2} \vec{E} \cdot d\vec{l} = \int_{R_1}^{R_2} \frac{Q}{4\pi\varepsilon_0\varepsilon_r r^2} dr = \frac{Q}{4\pi\varepsilon_0\varepsilon_r} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{Q(R_2 - R_1)}{4\pi\varepsilon_0\varepsilon_r R_1 R_2}$$

$$C = \frac{Q}{U} = \frac{4\pi\varepsilon_0\varepsilon_r R_1 R_2}{R_2 - R_1} = \varepsilon_r C_0, \quad C_0 = \frac{4\pi\varepsilon_0 R_1 R_2}{R_2 - R_1}$$

讨论：(1) R_1 固定， $R_2 \rightarrow \infty$ ， $C \rightarrow 4\pi\varepsilon_0\varepsilon_r R_1$

(2) 令 $R_2 - R_1 = d$ ， $d \ll R_1, R_2$ ， $R_1 R_2 \approx R_1^2$

$$C \rightarrow \frac{4\pi\varepsilon_0\varepsilon_r R_1^2}{d} = \frac{\varepsilon_0\varepsilon_r S}{d}$$

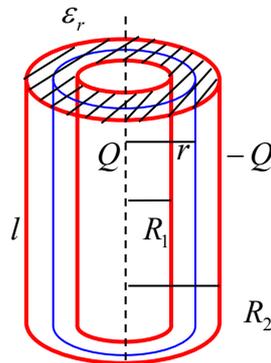
3、柱形电容器 ($l \gg R_1, R_2$)

$$E = \frac{E_0}{\varepsilon_r} = \frac{\lambda}{2\pi\varepsilon_0\varepsilon_r r} = \frac{Q}{2\pi\varepsilon_0\varepsilon_r r l}$$

$$\oint_S \vec{D} \cdot d\vec{S} = D2\pi r l = Q,$$

$$D = \frac{Q}{2\pi r l} = \varepsilon_0\varepsilon_r E,$$

$$E = \frac{Q}{2\pi\varepsilon_0\varepsilon_r r l},$$



$$U = \int_{R_1}^{R_2} \vec{E} \cdot d\vec{l} = \int_{R_1}^{R_2} \frac{Q}{2\pi\varepsilon_0\varepsilon_r r l} dr = \frac{Q}{2\pi\varepsilon_0\varepsilon_r l} \ln \frac{R_2}{R_1}$$

$$C = \frac{Q}{U} = \frac{2\pi\varepsilon_0\varepsilon_r l}{\ln(R_2/R_1)} = \varepsilon_r C_0, \quad C_0 = \frac{2\pi\varepsilon_0 l}{\ln(R_2/R_1)}$$

例：求电容

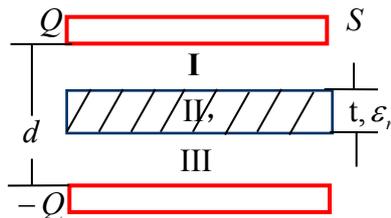
解： $E_1 = E_3 = E_0$

$$E_2 = \frac{E_0}{\varepsilon_r}, \quad E_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 S}$$

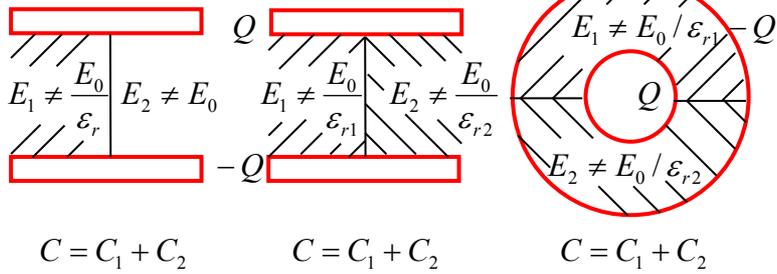
$$U = E_2 t + E_1 (d - t)$$

$$= \frac{E_0}{\varepsilon_r} t + E_0 (d - t) = \frac{Q}{\varepsilon_0 S} \left(\frac{t}{\varepsilon_r} + d - t \right)$$

$$C = \frac{Q}{U} = \frac{\varepsilon_0 S}{(t/\varepsilon_r) + d - t} = \frac{\varepsilon_0\varepsilon_r S}{t + (d - t)\varepsilon_r}$$

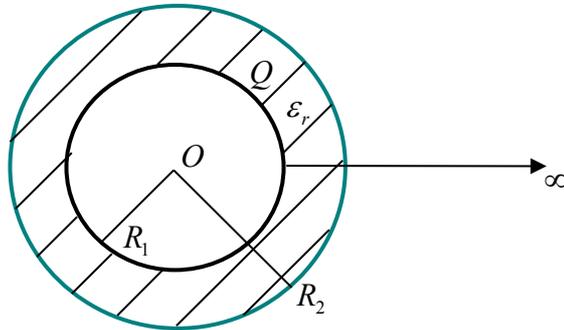


$E = E_0/\epsilon_r$ 不成立的情况:



例: 导体球, 外包一层电介质

求: 电容



解:
$$E = \begin{cases} 0 & r < R_1 \\ \frac{Q}{4\pi\epsilon_0\epsilon_r r^2} & R_1 < r < R_2 \\ \frac{Q}{4\pi\epsilon_0 r^2} & r > R_2 \end{cases}$$

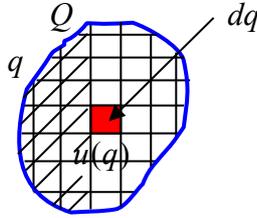
导体球的电势

$$\begin{aligned} U &= \int_{R_1}^{\infty} E \cos\theta dl \\ &= \int_{R_1}^{R_2} \frac{Q}{4\pi\epsilon_0\epsilon_r r^2} dr + \int_{R_2}^{\infty} \frac{Q}{4\pi\epsilon_0 r^2} dr \\ &= \frac{Q}{4\pi\epsilon_0\epsilon_r} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) + \frac{Q}{4\pi\epsilon_0} \frac{1}{R_2} \\ &= \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{\epsilon_r} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) + \frac{1}{R_2} \right] \\ C &= \frac{Q}{U} = \frac{4\pi\epsilon_0}{\frac{1}{\epsilon_r} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) + \frac{1}{R_2}} = \frac{4\pi\epsilon_0\epsilon_r R_1 R_2}{R_2 - R_1 + \epsilon_r R_1} \end{aligned}$$

第 4 节 电场的能量

一、带电体的静电能

已搬运的电荷 q 在 dq 所在
位置产生的电势 $u(q)$
搬运 dq 过程外力克服电场力
做功 $dA = u(q)dq$



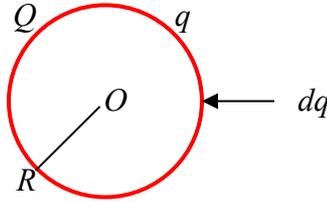
外力克服电场力做功的总和：静电能

$$W = \int_0^Q u(q) dq$$

例：孤立导体球的静电能

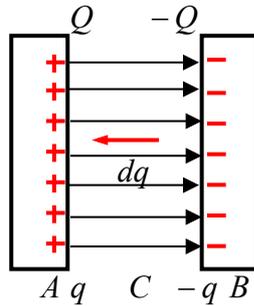
解： $u = \frac{q}{4\pi\epsilon_0 R}$, $u dq = \frac{q}{4\pi\epsilon_0 R} dq$

$$W = \int_0^Q u(q) dq = \int_0^Q \frac{q}{4\pi\epsilon_0 R} dq = \frac{Q^2}{8\pi\epsilon_0 R}$$



二、电容器的能量

已搬运的电荷 q
电势差 $u_{AB} = q/C$
搬运 dq 过程外力克服电场力
做功 $dA = u_{AB} dq$
外力克服电场力做功的总和：
电容器的能量



$$W = \int_0^Q u_{AB} dq = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}, \quad Q, V_{AB}, Q = CV_{AB}$$

$$W = \frac{Q^2}{2C} = \frac{1}{2} QV_{AB} = \frac{1}{2} CV_{AB}^2 \quad \text{—— 电容器储能公式}$$

对任意的电容器成立

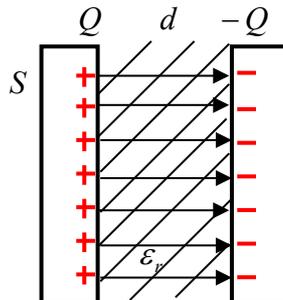
孤立导体球： $C = 4\pi\epsilon_0 R$, $W = \frac{Q^2}{2C} = \frac{Q^2}{8\pi\epsilon_0 R}$

带电系统的能量就是它的电场的能量

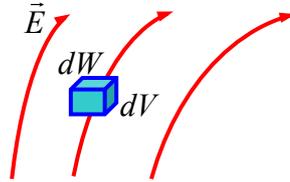
三、电场的能量

$$C = \frac{\epsilon_0 \epsilon_r S}{d}, \quad V_{AB} = Ed$$

$$\begin{aligned} W &= \frac{Q^2}{2C} = \frac{1}{2} CV_{AB}^2 \\ &= \frac{1}{2} \frac{\epsilon_0 \epsilon_r S}{d} (Ed)^2, \\ &= \frac{1}{2} \epsilon_0 \epsilon_r E^2 Sd = \frac{1}{2} \epsilon_0 \epsilon_r E^2 V \end{aligned}$$



能量分布在电场中
 电场是能量的携带者
 电场能量密度



$$w = \frac{dW}{dV}$$

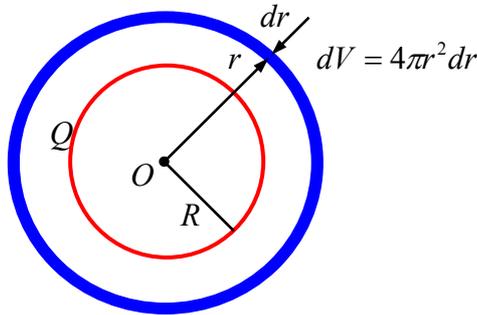
$$w = \frac{dW}{dV} = \frac{1}{2} \varepsilon_0 \varepsilon_r E^2 = \frac{1}{2} DE = \frac{1}{2} \vec{D} \cdot \vec{E}, \quad w \propto E^2$$

均匀电场, $W = wV$

非均匀电场, $dW = wdV$

$$W = \int dW = \int_V wdV = \int_V \frac{1}{2} \varepsilon_0 \varepsilon_r E^2 dV$$

例: 孤立导体球电场的能量



解:
$$E = \begin{cases} 0 & r < R \\ \frac{Q}{4\pi\varepsilon_0 r^2} & r > R \end{cases}$$

$$w = \frac{1}{2} \varepsilon_0 \varepsilon_r E^2 = \begin{cases} 0 & r < R \\ \frac{Q^2}{32\pi^2 \varepsilon_0 r^4} & r > R \end{cases}$$

$$\begin{aligned} W &= \int_V wdV = \int_{r < R} wdV + \int_{r > R} wdV \\ &= \int_R^\infty \frac{Q^2}{32\pi^2 \varepsilon_0 r^4} 4\pi r^2 dr = \frac{Q^2}{8\pi\varepsilon_0 R} \end{aligned}$$

静电学, 静电能 \Leftrightarrow 电容器的能量 \Leftrightarrow 电场能量
 计算带电系统静电能或电场能量的方法:

(1) 电容器, $W = \frac{Q^2}{2C}$

(2) 已知 \vec{E} , $W = \int_V wdV = \int_V \frac{1}{2} \varepsilon_0 \varepsilon_r E^2 dV$

(3) 搬运方法, $W = \int_0^Q u(q) dq$

$$w = \frac{1}{2} \varepsilon_0 \varepsilon_r E^2 > 0, \quad W = \int_V wdV > 0$$