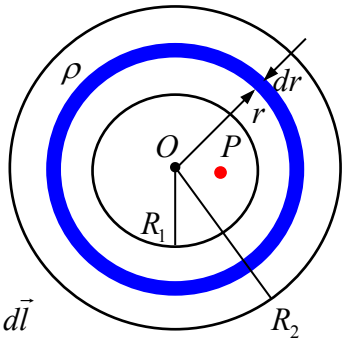
静电场习题课

例：均匀带电球壳

求：腔内电势



解：方法 I：$U=\int_P^{\infty}\vec{E}\cdot d\vec{l}$

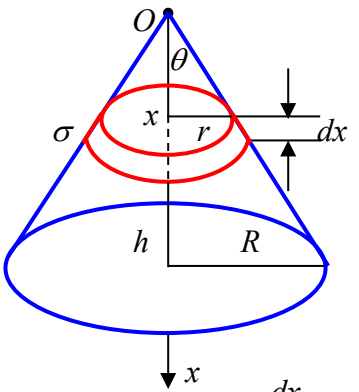
方法 II：$dV=4\pi r^2dr$，$dq=\rho dV=\rho 4\pi r^2dr$

$$dU=\frac{dq}{4\pi\varepsilon_0 r}=\frac{\rho 4\pi r^2dr}{4\pi\varepsilon_0 r}=\frac{\rho}{\varepsilon_0}rdr$$

$$U=\int dU=\int_{R_1}^{R_2}\frac{\rho}{\varepsilon_0}rdr=\frac{\rho}{2\varepsilon_0}(R_2^2-R_1^2)$$

例：圆锥侧面均匀带电

证明：O 的电势与圆锥高度无关

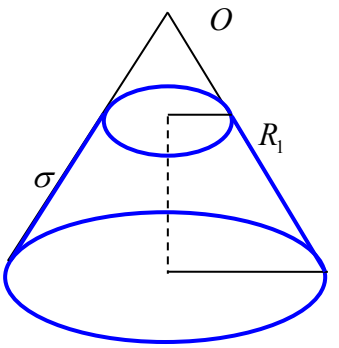


证：$dS=2\pi r\dfrac{dx}{\cos\theta}$，$dq=\sigma dS=\sigma 2\pi r\dfrac{dx}{\cos\theta}$

$$dU=\frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{x^2+r^2}}=\frac{1}{4\pi\varepsilon_0}\frac{\sigma 2\pi r dx/\cos\theta}{\sqrt{x^2+r^2}}=\frac{\sigma}{2\varepsilon_0}tg\theta dx$$

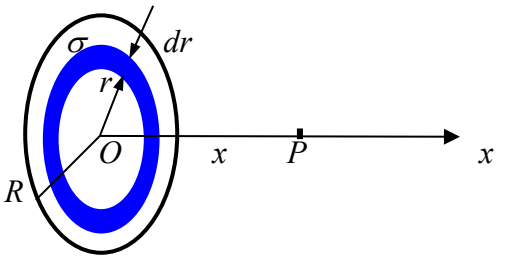
$$U=\int dU=\int_0^h\frac{\sigma}{2\varepsilon_0}tg\theta dx=\frac{\sigma}{2\varepsilon_0}tg\theta\cdot h=\frac{\sigma R}{2\varepsilon_0}$$

***，



R_2，$U_O=\dfrac{\sigma}{2\varepsilon_0}(R_2-R_1)$

例：均匀带电圆盘轴线上U



解：$dS = 2\pi r dr$， $dq = \sigma dS = \sigma 2\pi r dr$

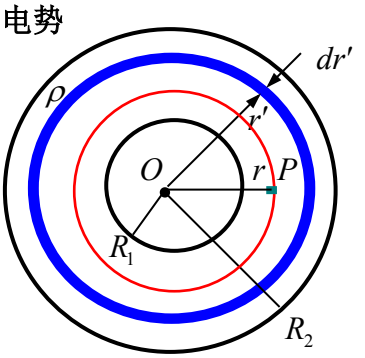
$$dU = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{x^2+r^2}} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma 2\pi r dr}{\sqrt{x^2+r^2}} = \frac{\sigma}{2\varepsilon_0}\frac{r dr}{\sqrt{x^2+r^2}}$$

$$U = \int dU = \int_0^R \frac{\sigma}{2\varepsilon_0}\frac{r dr}{\sqrt{x^2+r^2}}$$

$$= \frac{\sigma}{2\varepsilon_0}\sqrt{x^2+r^2}\Big|_0^R = \frac{\sigma}{2\varepsilon_0}(\sqrt{x^2+R^2} - |x|)$$

例：均匀带电球壳

求：壳中半径r处的电势



解：$U = U_1 + U_2$

$$U_1 = \frac{q_1}{4\pi\varepsilon_0 r} = \frac{1}{4\pi\varepsilon_0 r}\rho\frac{4}{3}\pi(r^3 - R_1^3) = \frac{\rho}{3\varepsilon_0}(r^2 - \frac{R_1^3}{r})$$

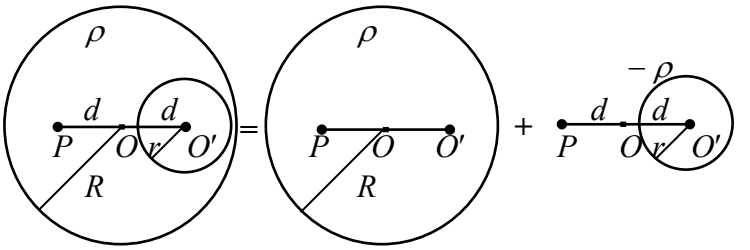
$dV = 4\pi r'^2 dr'$， $dq = \rho dV = \rho 4\pi r'^2 dr'$

$$dU_2 = \frac{dq}{4\pi\varepsilon_0 r'} = \frac{\rho 4\pi r'^2 dr'}{4\pi\varepsilon_0 r'} = \frac{\rho}{\varepsilon_0} r' dr'$$

$$U_2 = \int dU_2 = \int_r^{R_2}\frac{\rho}{\varepsilon_0} r' dr' = \frac{\rho}{2\varepsilon_0}(R_2^2 - r^2)$$

$$U = U_1 + U_2 = \frac{\rho}{6\varepsilon_0}(3R_2^2 - r^2 - \frac{2R_1^3}{r})$$

例：均匀带电球体（R，ρ），挖掉半径为r的小球体



求：（1）O'处的场强

（2）P处的场强，$O'OP$三点共线，$\overline{OP}=d$

解：（1）$\vec{E}_{O'}=\vec{E}_1+\vec{E}_2$

$$E_1=\frac{\rho}{3\varepsilon_0}d\text{，方向}O\to O'$$

$$E_2=0$$

$$E_{O'}=\frac{\rho}{3\varepsilon_0}d\text{，方向}O\to O'$$

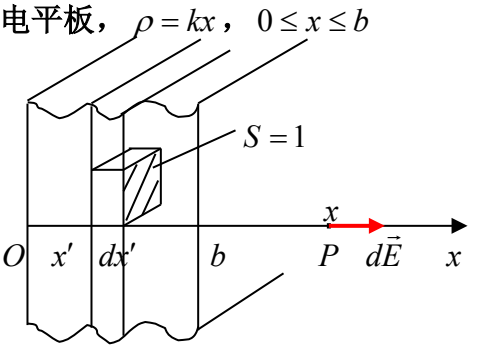
（2）$\vec{E}_P=\vec{E}_1+\vec{E}_2$

$$E_1=\frac{\rho}{3\varepsilon_0}d\text{，方向}O\to P$$

$$E_2=\frac{1}{4\pi\varepsilon_0(2d)^2}\rho\frac{4}{3}\pi r^3=\frac{\rho}{3\varepsilon_0}\frac{r^3}{4d^2}\text{，方向}P\to O$$

$$E_P=E_1-E_2=\frac{\rho}{3\varepsilon_0}(d-\frac{r^3}{4d^2})\text{，方向}O\to P$$

例：无限大带电平板，$\rho=kx$，$0\le x\le b$



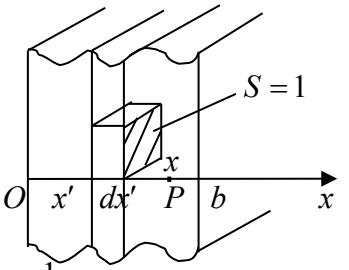
求：（1）板外的电场（2）板内的电场（3）$\vec{E}=0$的位置

解：方法 I：

（1）$dE=\dfrac{\sigma}{2\varepsilon_0}$，$\sigma=\rho\cdot 1\cdot dx'=kx'dx'$，

$$dE=\frac{1}{2\varepsilon_0}kx'dx'$$

$$E=\int dE=\int_0^b\frac{1}{2\varepsilon_0}kx'dx'=\frac{k}{4\varepsilon_0}b^2$$



（2） $dE=\dfrac{1}{2\varepsilon_0}kx'dx'$

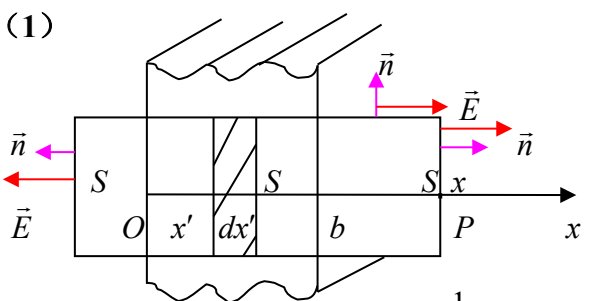
$$E=\int_0^x\frac{1}{2\varepsilon_0}kx'dx'-\int_x^b\frac{1}{2\varepsilon_0}kx'dx'=\frac{k}{4\varepsilon_0}x^2-\frac{k}{4\varepsilon_0}(b^2-x^2)$$

$$=\frac{k}{4\varepsilon_0}(2x^2-b^2)$$

（3） $E=0$， $2x^2-b^2=0$， $x=(\sqrt{2}/2)b$

方法 II：高斯定理

（1）



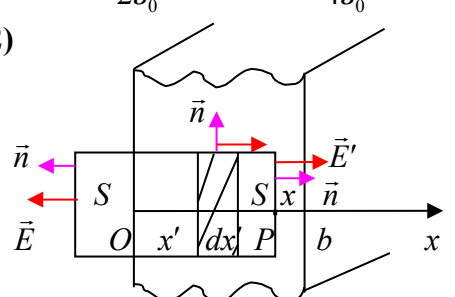
$$\Phi=\oint_S\vec{E}\cdot d\vec{S}=ES+ES=2ES=\frac{1}{\varepsilon_0}\sum_{内}q_i$$

$dV=Sdx'$， $dq=\rho dV=kx'Sdx'$

$$\sum_{内}q_i=\int dq=\int_0^b kx'Sdx'=\frac{1}{2}kSb^2$$

$$2ES=\frac{1}{2\varepsilon_0}kSb^2,\quad E=\frac{k}{4\varepsilon_0}b^2$$

(2)



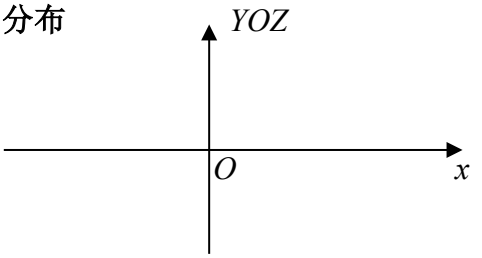
$$\Phi=\oint_S\vec{E}\cdot d\vec{S}=E'S+ES=\frac{1}{\varepsilon_0}\sum_{内}q_i$$

$$dq=\rho dV=kx'Sdx',\quad \sum_{内}q_i=\int dq=\int_0^x kx'Sdx'=\frac{1}{2}kSx^2$$

$$E'S+ES=\frac{1}{2\varepsilon_0}kSx^2,\quad E'=\frac{k}{2\varepsilon_0}x^2-\frac{k}{4\varepsilon_0}b^2=\frac{k}{4\varepsilon_0}(2x^2-b^2)$$

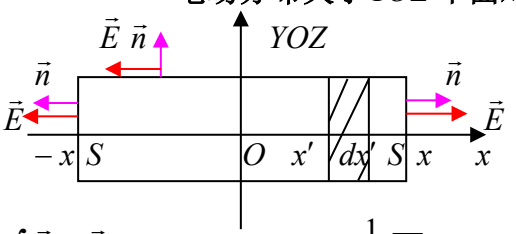
例：电荷体密度按 $\rho=\rho_0\cos x$ 分布于空间之中

求：电场的分布



解：$\cos(-x)=\cos x$，电荷分布关于 YOZ 平面对称

电场分布关于 YOZ 平面对称



$$\Phi=\oint_S\vec{E}\cdot d\vec{S}=ES+ES=2ES=\frac{1}{\varepsilon_0}\sum_{内}q_i$$

$$dV=Sdx'，\quad dq=\rho dV=\rho_0\cos x'Sdx'$$

$$\sum_{内}q_i=\int dq=\int_{-x}^{x}\rho_0\cos x'Sdx'=2\rho_0S\sin x$$

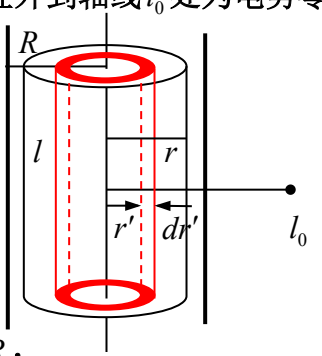
$$2ES=\frac{1}{\varepsilon_0}2\rho_0S\sin x，\quad E=\frac{\rho_0}{\varepsilon_0}\sin x$$

$E>0$，沿 x 轴正向，$E<0$，沿 x 轴负向

例：无限长带电圆柱体（R、$\rho=Ar$，A 为常数）

求：（1）电场

（2）取圆柱外到轴线 l_0 处为电势零点，求电势



解：（1）$r<R$，

$$\Phi=\oint_S\vec{E}\cdot d\vec{S}=E2\pi rl=\frac{1}{\varepsilon_0}\sum_{内}q_i$$

$$dV=2\pi r'dr'l，\quad dq=\rho dV=Ar'\ 2\pi r'dr'l$$

$$\sum_{内}q_i=\int dq=\int_0^r 2\pi Alr'^2dr'=\frac{2}{3}\pi Alr^3$$

$$E2\pi rl=\frac{2}{3\varepsilon_0}\pi Alr^3，\quad E=\frac{A}{3\varepsilon_0}r^2$$