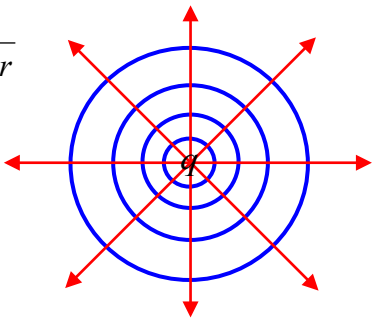
第 6 节　场强与电势的关系

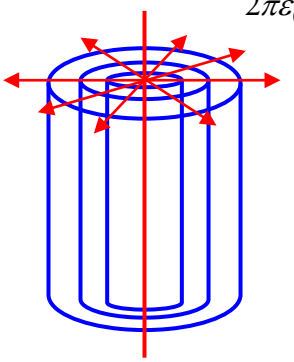
$\vec{E}$、U，$U=\int_P^R \vec{E}\cdot d\vec{l}$

一、等势面

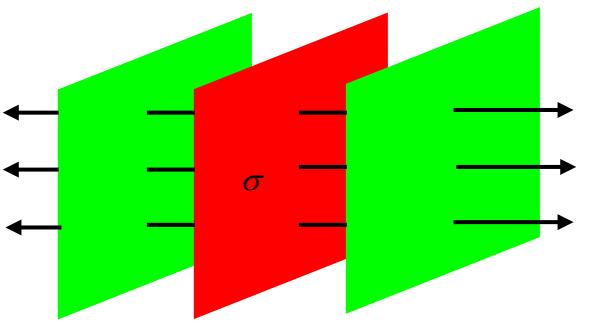
1、点电荷， $U=\dfrac{q}{4\pi\varepsilon_0 r}$



2、无限长均匀带电直线， $U=-\dfrac{\lambda}{2\pi\varepsilon_0}\ln\dfrac{r}{r_0}$



3、无限大均匀带电平面， $U=-\dfrac{\sigma}{2\varepsilon_0}r$



约定：相邻的两个等势面的电势差都相等

性质：

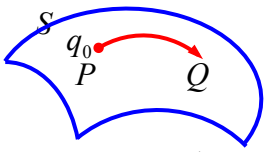
（1）沿等势面移动电荷，电场力作功为零

（2）等势面与电力线处处正交，

电力线的方向为电势降落的方向

证明：（1）$U_P = U_Q$

$A = q_0 U_{PQ} = 0$

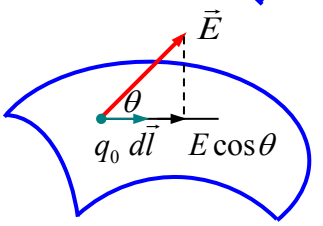


（2）反证法：

设等势面上某点处

$\vec{E}$ 与等势面不正交

即 $\theta \neq \dfrac{\pi}{2}$

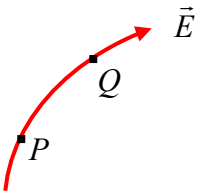


$E\cos\theta \neq 0$， $dA = q_0 \vec{E} \cdot d\vec{l} = q_0 E\cos\theta dl \neq 0$，矛盾

$U_P - U_Q = \int_P^Q \vec{E} \cdot d\vec{l}$

$= \int_P^Q E\cos\theta dl = \int_P^Q E dl > 0$

$U_P > U_Q$



二、场强与电势的微分关系

$dU > 0$， $U_{II} > U_I$

等势面的正法线方向：

沿电势增加的方向

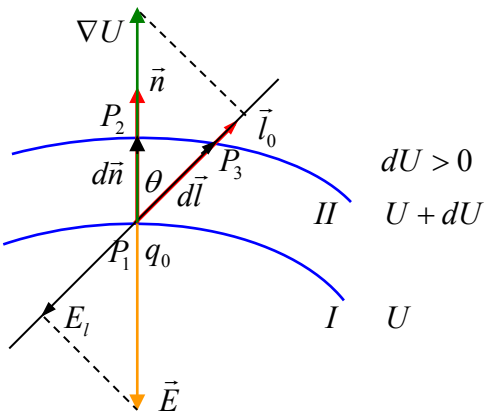
$\overline{P_1P_3} > \overline{P_1P_2}$

$dn = dl\cos\theta$

$\dfrac{dU}{dn}$、$\dfrac{dU}{dl}$（方向导数）

$\dfrac{dU}{dl} = \dfrac{dU}{dn/\cos\theta} = \dfrac{dU}{dn}\cos\theta$，

$|\cos\theta| \leq 1$， $\left|\dfrac{dU}{dl}\right| \leq \dfrac{dU}{dn}$



定义：电势梯度 ∇U 的方向：电势增加率最大的方向

大小：$|\nabla U| = \dfrac{dU}{dn}$

$$\boxed{\nabla U = \frac{dU}{dn}\vec{n}}$$

$dA = q_0 \vec{E} \cdot d\vec{n} = q_0 U_{P_1P_2} \Rightarrow q_0 E(-1)dn = q_0[U - (U + dU)]$

$E(-1)dn = -dU$， $E = \dfrac{dU}{dn}$

$$\boxed{\vec{E} = -\nabla U = -\frac{dU}{dn}\vec{n}}$$

$\vec{E} \cdot \vec{l}_0 = -\dfrac{dU}{dn}\vec{n} \cdot \vec{l}_0$

$$\boxed{E_l = -\frac{dU}{dn}\cos\theta = -\frac{dU}{dl} = -\frac{\partial U}{\partial l}}$$

$Oxyz$：$\vec{l}_0 = \vec{i}, \vec{j}, \vec{k}$，

$$\boxed{E_x = -\frac{\partial U}{\partial x}，\quad E_y = -\frac{\partial U}{\partial y}，\quad E_z = -\frac{\partial U}{\partial z}}$$ **SI：V/m**

例：均匀带电圆环（q，R）轴线上电势 $U = \dfrac{q}{4\pi\varepsilon_0\sqrt{x^2+R^2}}$

求：轴线上 $\vec{E}$

解： $E_x = -\dfrac{\partial U}{\partial x} = -\dfrac{q}{4\pi\varepsilon_0}(-\dfrac{1}{2})(x^2+R^2)^{-3/2}\cdot 2x = \dfrac{1}{4\pi\varepsilon_0}\dfrac{qx}{(x^2+R^2)^{3/2}}$

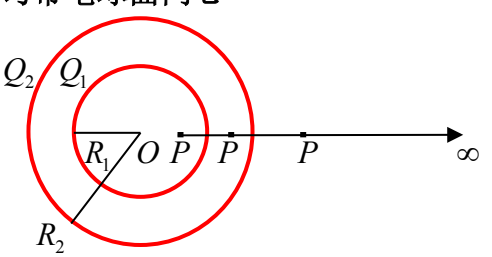
由于对称性， E_y **=0，** E_z **=0**

例：电势 $U = A\ln(x^2+y^2)$

求： $\vec{E}$

解： $E_x = -\dfrac{\partial U}{\partial x} = -A\dfrac{2x}{x^2+y^2}$，$E_y = -\dfrac{\partial U}{\partial y} = -A\dfrac{2y}{x^2+y^2}$，$E_z = -\dfrac{\partial U}{\partial z}$ **=0**

例：两个均匀带电球面同心



解：
$$E = \begin{cases} 0 & r < R_1 \\ \dfrac{Q_1}{4\pi\varepsilon_0 r^2} & R_1 < r < R_2 \\ \dfrac{Q_1+Q_2}{4\pi\varepsilon_0 r^2} & r > R_2 \end{cases}$$

1、 $r < R_1$

$$U = \int_P^\infty \vec{E}\cdot d\vec{l} = \int_P^\infty E\cos\theta dl$$
$$= \int_P^{R_1} E\cos\theta dl + \int_{R_1}^{R_2} E\cos\theta dl + \int_{R_2}^\infty E\cos\theta dl$$
$$= \int_{R_1}^{R_2}\frac{Q_1}{4\pi\varepsilon_0 r^2}dr + \int_{R_2}^\infty \frac{Q_1+Q_2}{4\pi\varepsilon_0 r^2}dr$$
$$= \frac{Q_1}{4\pi\varepsilon_0}(\frac{1}{R_1}-\frac{1}{R_2}) + \frac{Q_1+Q_2}{4\pi\varepsilon_0}\frac{1}{R_2} = \frac{Q_1}{4\pi\varepsilon_0 R_1} + \frac{Q_2}{4\pi\varepsilon_0 R_2}$$

2、 $R_1 < r < R_2$

$$U = \int_P^\infty \vec{E}\cdot d\vec{l} = \int_P^\infty E\cos\theta dl = \int_P^{R_2} E\cos\theta dl + \int_{R_2}^\infty E\cos\theta dl$$

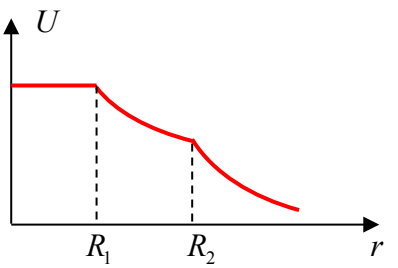
$$=\int_r^{R_2}\frac{Q_1}{4\pi\varepsilon_0 r^2}dr+\int_{R_2}^{\infty}\frac{Q_1+Q_2}{4\pi\varepsilon_0 r^2}dr$$

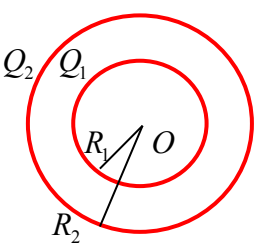
$$=\frac{Q_1}{4\pi\varepsilon_0}(\frac{1}{r}-\frac{1}{R_2})+\frac{Q_1+Q_2}{4\pi\varepsilon_0}\frac{1}{R_2}=\frac{Q_1}{4\pi\varepsilon_0 r}+\frac{Q_2}{4\pi\varepsilon_0 R_2}$$

3、 $r>R_2$

$$U=\int_P^{\infty}\vec{E}\cdot d\vec{l}=\int_P^{\infty}E\cos\theta dl$$

$$=\int_r^{\infty}\frac{Q_1+Q_2}{4\pi\varepsilon_0 r^2}dr=\frac{Q_1+Q_2}{4\pi\varepsilon_0}\frac{1}{r}=\frac{Q_1}{4\pi\varepsilon_0 r}+\frac{Q_2}{4\pi\varepsilon_0 r}$$





1、 $r<R_1$ $\qquad U=\frac{Q_1}{4\pi\varepsilon_0 R_1}+\frac{Q_2}{4\pi\varepsilon_0 R_2}$

2、 $R_1<r<R_2$ $\qquad U=\frac{Q_1}{4\pi\varepsilon_0 r}+\frac{Q_2}{4\pi\varepsilon_0 R_2}$

3、 $r>R_2$ $\qquad U=\frac{Q_1}{4\pi\varepsilon_0 r}+\frac{Q_2}{4\pi\varepsilon_0 r}$

静电场小结

库仑定律
场强迭加原理

⇒

高斯定理：$\oint_S\vec{E}\cdot d\vec{S}=\frac{1}{\varepsilon_0}\sum_{内}q_i$

环路定理：$\oint_L\vec{E}\cdot d\vec{l}=0$

静电场是有源场，电荷是静电场的场源

静电场是保守场，是引入电势的先决条件

$\vec{E}$ 满足矢量迭加原理，U 满足标量迭加原理

$U=\int_P^R\vec{E}\cdot d\vec{l}$，$\vec{E}=-\nabla U=-\frac{dU}{dn}\vec{n}$