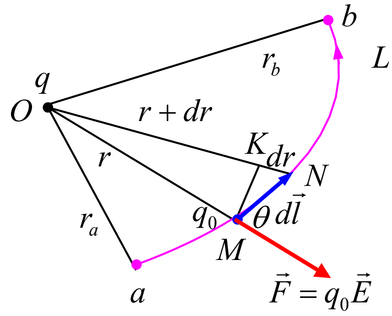
第 4 节　静电力的功　电势能

$\vec{E}$，q_0，$\vec{F}=q_0\vec{E}$

一、静电力的功　环路定理

1、点电荷的电场



$dA=\vec{F}\cdot d\vec{l}$

$=q_0\vec{E}\cdot d\vec{l}$，

$A=\int dA=\int_{a(L)}^{b}\vec{F}\cdot d\vec{l}$

$=q_0\int_{a(L)}^{b}\vec{E}\cdot d\vec{l}$，

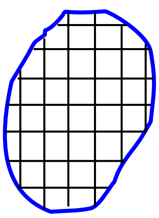
$=q_0\int_{a(L)}^{b}E\cos\theta dl$，$\overline{OM}=\overline{OK}=r$，$\overline{KN}=dr$

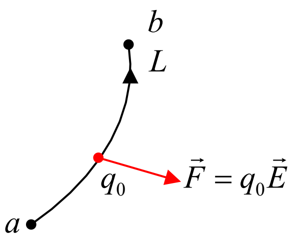
$\angle KNM\approx\theta$，$dr=dl\cos\theta$，$E=\dfrac{q}{4\pi\varepsilon_0 r^2}$

$A=\int_{r_a}^{r_b}\dfrac{qq_0}{4\pi\varepsilon_0 r^2}dr=\dfrac{qq_0}{4\pi\varepsilon_0}(\dfrac{1}{r_a}-\dfrac{1}{r_b})$

点电荷的电场对 q_0 作功与路径无关

2、任意带电体的电场





$A=\int_{a(L)}^{b}\vec{F}\cdot d\vec{l}=q_0\int_{a(L)}^{b}\vec{E}\cdot d\vec{l}$，$\vec{E}=\vec{E}_1+\vec{E}_2+\cdots+\vec{E}_n$

$A=q_0\int_{a(L)}^{b}(\vec{E}_1+\vec{E}_2+\cdots+\vec{E}_n)\cdot d\vec{l}$

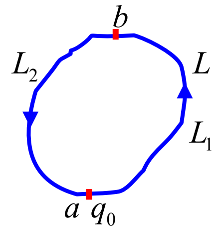
$=q_0\int_{a(L)}^{b}\vec{E}_1\cdot d\vec{l}+q_0\int_{a(L)}^{b}\vec{E}_2\cdot d\vec{l}+\cdots+q_0\int_{a(L)}^{b}\vec{E}_n\cdot d\vec{l}$

任意的静电场对 q_0 作功与路径无关

静电场：保守场

静电力：保守力

3、静电场的环路定理



$A=q_0\oint_L\vec{E}\cdot d\vec{l}$

$=q_0\int_{a(L1)}^{b}\vec{E}\cdot d\vec{l}+q_0\int_{b(L2)}^{a}\vec{E}\cdot d\vec{l}$，

$=q_0\int_{a(L1)}^{b}\vec{E}\cdot d\vec{l}-q_0\int_{a(-L2)}^{b}\vec{E}\cdot d\vec{l}$，

=0

$\boxed{\oint_L\vec{E}\cdot d\vec{l}=0}$ —— 静电场的环路定理

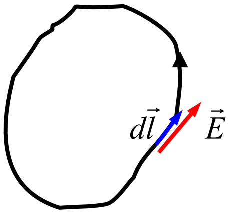
⇕

静电力作功与路径无关

例：静电场的电力线不能是闭合曲线

证：反证法，设静电场的某条电力线是闭合曲线

$$\oint_L \vec{E}\cdot d\vec{l} = \oint_L E\cos\theta dl > 0$$



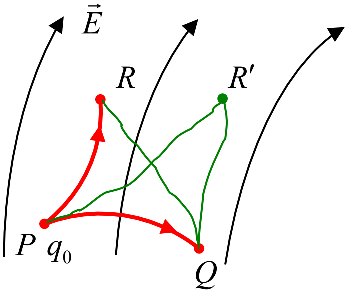
静电场的环路定理只适用于静电场

二、电势能

R：参考点

$$q_0\int_P^R \vec{E}\cdot d\vec{l}$$

定义：电势能 $W_P = q_0\int_P^R \vec{E}\cdot d\vec{l}$



注意：

（1）W_P 与 A 的区别

（2）A 与 ΔW 的关系

$$A = q_0\int_P^Q \vec{E}\cdot d\vec{l} = q_0\int_P^R \vec{E}\cdot d\vec{l} + q_0\int_R^Q \vec{E}\cdot d\vec{l}$$

$$= q_0\int_P^R \vec{E}\cdot d\vec{l} - q_0\int_Q^R \vec{E}\cdot d\vec{l} = W_P - W_Q = -(W_Q - W_P)$$

$W_Q - W_P = \Delta W$：电势能增量，$\boxed{A = -\Delta W}$

$A>0$，$\Delta W<0$，$W\downarrow$；$A<0$，$\Delta W>0$，$W\uparrow$

（3）参考点：$W_R = q_0\int_R^R \vec{E}\cdot d\vec{l} = 0$

理论上：“∞”，$W_P = q_0\int_P^\infty \vec{E}\cdot d\vec{l}$

工程上：“大地”

R'：参考点，$W_P' = q_0\int_P^{R'} \vec{E}\cdot d\vec{l}$，$W_P \neq W_P'$

电势能的数值只具有相对意义

q_0 在静电场中任意两点上电势能的差值与参考点的选择无关

第 5 节　电势和电势差

一、定义　$W_P = q_0\int_P^R \vec{E}\cdot d\vec{l}$，$W_P \propto q_0$，$W_P/q_0 = \int_P^R \vec{E}\cdot d\vec{l}$

电势：$U_P = W_P/q_0 = \int_P^R \vec{E}\cdot d\vec{l}$

注意：（1）标量，SI：J/C=V（伏）

（2）$W_P = q_0 U_P$

（3）$U_P - U_Q = U_{PQ}$：电压，SI：V

$$U_{PQ} = U_P - U_Q = \int_P^R \vec{E}\cdot d\vec{l} - \int_Q^R \vec{E}\cdot d\vec{l}$$

$$= \int_P^R \vec{E}\cdot d\vec{l} + \int_R^Q \vec{E}\cdot d\vec{l} = \int_P^Q \vec{E}\cdot d\vec{l}$$

$$\boxed{U_{PQ} = \int_P^Q \vec{E}\cdot d\vec{l}}$$

（4）q_0： $P \to Q$， $A = q_0\int_P^Q \vec{E}\cdot d\vec{l} = q_0 U_{PQ}$

（5）电势与参考点的选择有关

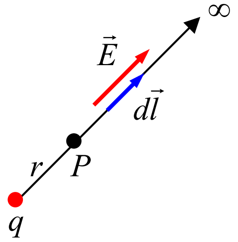
电压与参考点的选择无关

$U_R = \int_R^R \vec{E}\cdot d\vec{l} = 0$

参考点=电势能零点=电势零点

二、电势的计算

1、点电荷



$U_P = \int_P^R \vec{E}\cdot d\vec{l}$

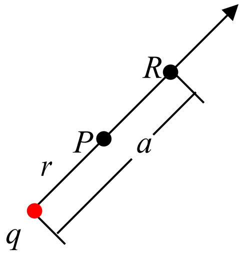
∞为参考点

$U_P = \int_P^\infty \vec{E}\cdot d\vec{l}$

$= \int_P^\infty E\cos\theta dl$， $\cos\theta = 1$， $dl = dr$， $E = \dfrac{q}{4\pi\varepsilon_0 r^2}$

$= \int_r^\infty \dfrac{q}{4\pi\varepsilon_0 r^2}dr = \dfrac{q}{4\pi\varepsilon_0 r}$

R：参考点

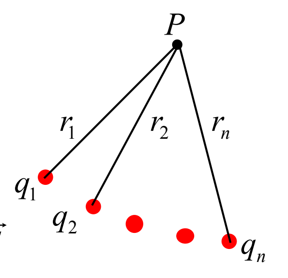


$U_P = \int_P^R \vec{E}\cdot d\vec{l}$

$= \int_r^a \dfrac{q}{4\pi\varepsilon_0 r^2}dr$，

$= \dfrac{q}{4\pi\varepsilon_0}(\dfrac{1}{r} - \dfrac{1}{a})$。

2、点电荷系



∞为参考点

$U_P = \int_P^\infty \vec{E}\cdot d\vec{l}$

$\vec{E} = \vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n$

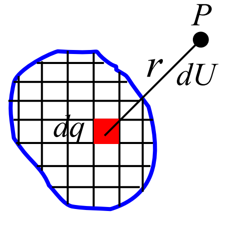
$U_P = \int_P^\infty (\vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n)\cdot d\vec{l}$

$= \int_P^\infty \vec{E}_1\cdot d\vec{l} + \int_P^\infty \vec{E}_2\cdot d\vec{l} + \cdots + \int_P^\infty \vec{E}_n\cdot d\vec{l}$

$= \dfrac{q_1}{4\pi\varepsilon_0 r_1} + \dfrac{q_2}{4\pi\varepsilon_0 r_2} + \cdots + \dfrac{q_n}{4\pi\varepsilon_0 r_n}$

$U_P = U_1 + U_2 + \cdots + U_n$ ———— 电势迭加原理

3、连续电荷分布的电势



$dU = \dfrac{dq}{4\pi\varepsilon_0 r}$

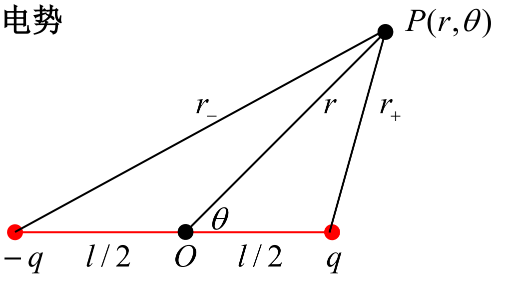
$U = \int dU = \int \dfrac{dq}{4\pi\varepsilon_0 r}$

计算电势的两种方法：

（1）已知$\vec{E}$，$U_P=\int_P^R \vec{E}\cdot d\vec{l}$

（2）$U=\begin{cases}\sum U_i & \text{点电荷系}\\ \int\frac{dq}{4\pi\varepsilon_0 r} & \text{电荷连续分布}\end{cases}$

例：电偶极子的电势



解：$U=U_++U_-$，∞：**参考点**

$$U=\frac{q}{4\pi\varepsilon_0 r_+}+\frac{-q}{4\pi\varepsilon_0 r_-}=\frac{q}{4\pi\varepsilon_0}\frac{r_--r_+}{r_-r_+}$$

$$r_+^2=r^2+(l/2)^2-2r(l/2)\cos\theta$$

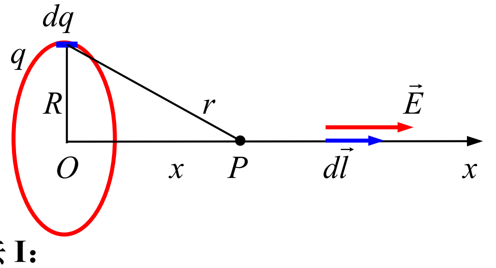
$$r_-^2=r^2+(l/2)^2+2r(l/2)\cos\theta$$

$$r_-^2-r_+^2=(r_--r_+)(r_-+r_+)=2rl\cos\theta$$

$r>>l$，$r_-+r_+\approx 2r$，$r_--r_+\approx l\cos\theta$，$r_-r_+\approx r^2$

$$U\approx\frac{q}{4\pi\varepsilon_0}\frac{l\cos\theta}{r^2}=\frac{1}{4\pi\varepsilon_0}\frac{P\cos\theta}{r^2}=\frac{1}{4\pi\varepsilon_0}\frac{P\cos\theta r}{r^3}=\frac{1}{4\pi\varepsilon_0}\frac{\vec{P}\cdot\vec{r}}{r^3}$$

例：均匀带电细圆环轴线上的电势



解：方法 I:

$$dU=\frac{dq}{4\pi\varepsilon_0 r}，\quad U=\int dU=\int\frac{dq}{4\pi\varepsilon_0 r}=\frac{q}{4\pi\varepsilon_0 r}=\frac{q}{4\pi\varepsilon_0\sqrt{x^2+R^2}}$$

方法 II:

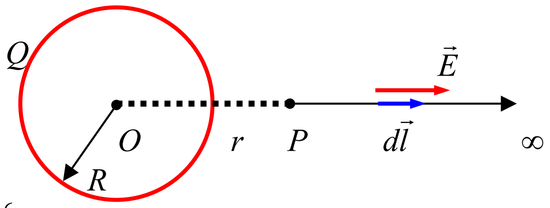
$$U_P=\int_P^\infty \vec{E}\cdot d\vec{l}=\int_P^\infty E\cos\theta dl$$

$\cos\theta=1$，$dl=dx$，$E=\frac{1}{4\pi\varepsilon_0}\frac{qx}{(x^2+R^2)^{3/2}}$

$$U=\int_x^\infty\frac{1}{4\pi\varepsilon_0}\frac{qx}{(x^2+R^2)^{3/2}}dx=\frac{q}{4\pi\varepsilon_0}\left(-\frac{1}{\sqrt{x^2+R^2}}\right)\Bigg|_x^\infty$$

$$=\frac{q}{4\pi\varepsilon_0}\frac{1}{\sqrt{x^2+R^2}}$$

例：均匀带电球面

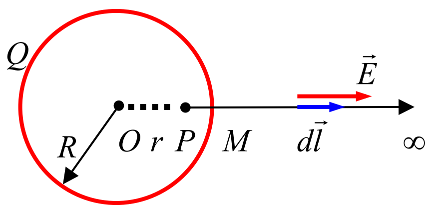


解： $E=\begin{cases}0 & r<R\\ \dfrac{Q}{4\pi\varepsilon_0 r^2} & r>R\end{cases}$

（1）$r>R$，$U_P=\int_P^{\infty}\vec{E}\cdot d\vec{l}=\int_P^{\infty}E\cos\theta\cdot dl$

$$=\int_r^{\infty}\frac{Q}{4\pi\varepsilon_0 r^2}dr=\frac{Q}{4\pi\varepsilon_0 r}$$

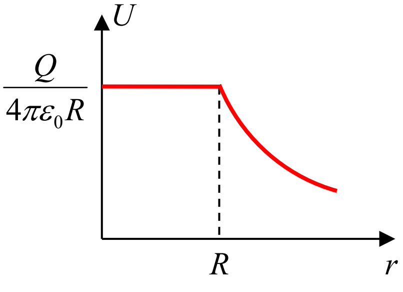
（2）$r<R$



$$U_P=\int_P^{\infty}\vec{E}\cdot d\vec{l}=\int_P^{\infty}E\cos\theta\cdot dl=\int_P^{M}E\cos\theta\cdot dl+\int_M^{\infty}E\cos\theta\cdot dl$$

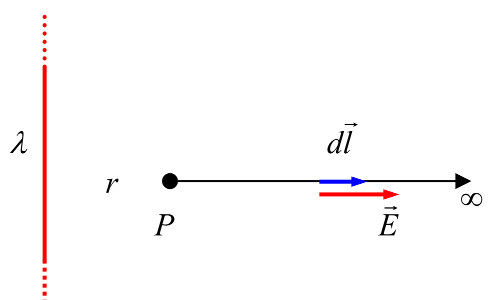
$$=\int_R^{\infty}\frac{Q}{4\pi\varepsilon_0 r^2}dr=\frac{Q}{4\pi\varepsilon_0 R},$$

$$U=\begin{cases}\dfrac{Q}{4\pi\varepsilon_0 R} & r<R\\ \dfrac{Q}{4\pi\varepsilon_0 r} & r>R\end{cases}$$



电势是连续的

例：无限长均匀带电直线



解： $E=\dfrac{\lambda}{2\pi\varepsilon_0 r}$，$U_P=\int_P^{\infty}\vec{E}\cdot d\vec{l}=\int_P^{\infty}E\cos\theta\cdot dl$

$$=\int_r^{\infty}\frac{\lambda}{2\pi\varepsilon_0 r}dr=\frac{\lambda}{2\pi\varepsilon_0}\ln r\Big|_r^{\infty}$$，无意义