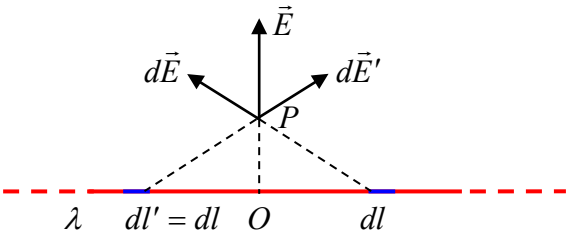
场强计算与高斯定理习题课

例：无限长均匀带电直线 $E=\dfrac{\lambda}{2\pi\varepsilon_0 r}$

解：



$$\Phi=\oint_S \vec{E}\cdot d\vec{S}$$

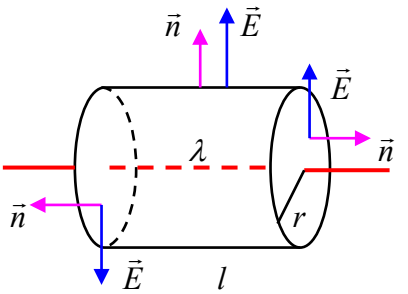
$$=\oint_S E\cos\theta dS\,,$$

$$=\int_{侧} E\cos\theta dS\,,$$

$$+\int_{左} E\cos\theta dS\,,$$

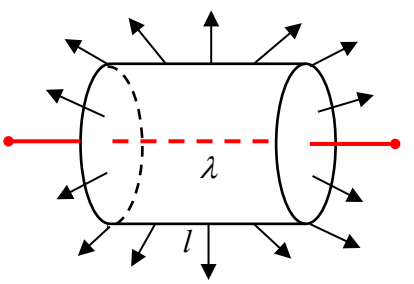
$$+\int_{右} E\cos\theta dS\,,$$

$$=E\int_{侧} dS=E\cdot 2\pi rl=\frac{\lambda l}{\varepsilon_0}\,,\quad E=\frac{\lambda}{2\pi\varepsilon_0 r}$$

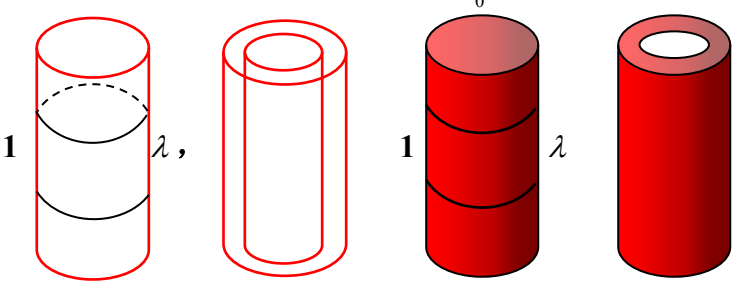


问题 1、高斯面只包围了部分电荷，求出的场强是这一部分电荷的场强还是整根均匀带电直线的场强？

问题 2、对于一段有限长均匀带电直线段，能否用该方法求其场强？



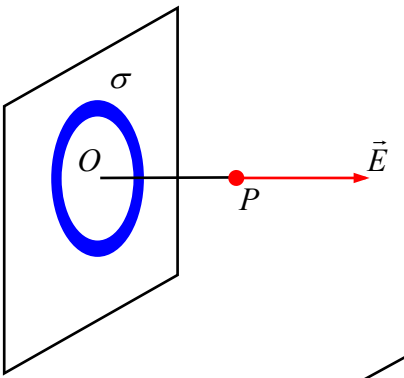
$$\Phi=\oint_S \vec{E}\cdot d\vec{S}=\frac{\lambda l}{\varepsilon_0}$$

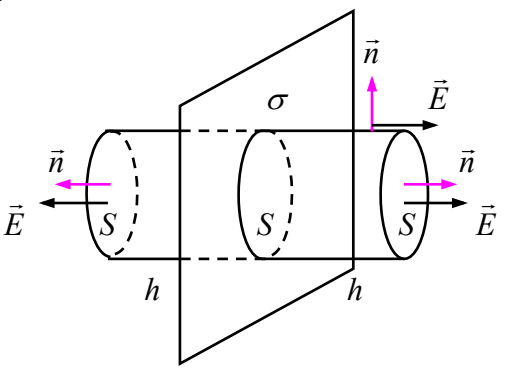


轴对称电场

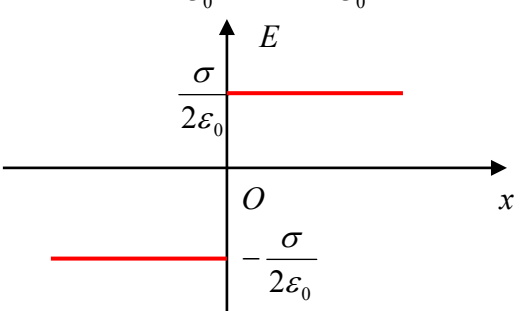
例：无限大均匀带电平面 $E=\dfrac{\sigma}{2\varepsilon_0}$

解：

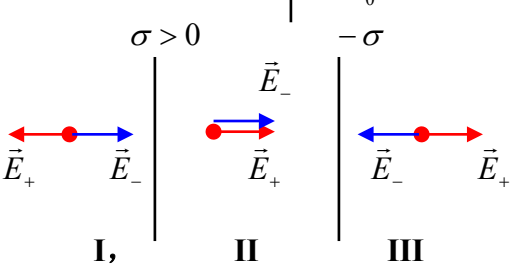




$$\Phi=\oint_S \vec{E}\cdot d\vec{S}$$
$$=\oint_S E\cos\theta dS,$$
$$=\int_{侧} E\cos\theta dS,$$
$$+\int_{左} E\cos\theta dS,$$
$$+\int_{右} E\cos\theta dS$$
$$=0+ES+ES=2ES=\frac{\sigma S}{\varepsilon_0},\quad E=\frac{\sigma}{2\varepsilon_0}$$



例：



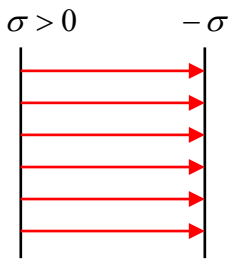
求：$\vec{E}$

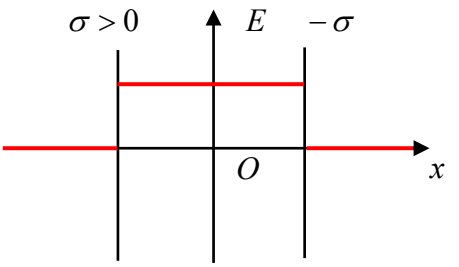
解：$\vec{E}=\vec{E}_+ + \vec{E}_-$

I、$\vec{E}=0$

II、$E=E_+ + E_- = \dfrac{\sigma}{2\varepsilon_0}+\dfrac{\sigma}{2\varepsilon_0}=\dfrac{\sigma}{\varepsilon_0}$

III、$\vec{E}=0$





关于高斯定理：

1、$\Phi=\oint_S \vec{E}\cdot d\vec{S}=\dfrac{1}{\varepsilon_0}\sum_{内} q_i$

Φ 仅与 $\sum_{内} q_i$ 有关，$\vec{E}$ 与所有电荷及其分布有关

2、如果 Φ 已知，$\sum_{内} q_i=\varepsilon_0\oint_S \vec{E}\cdot d\vec{S}=\varepsilon_0\Phi$

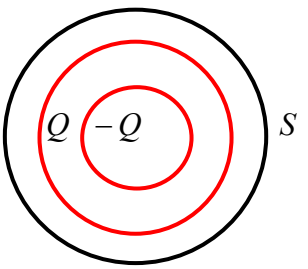
但仅由 Φ 和高斯定理不能完全确定高斯面内电荷分布

如 Φ **=0**，$\sum_{内} q_i=0$

判断下面几种说法的正确性：

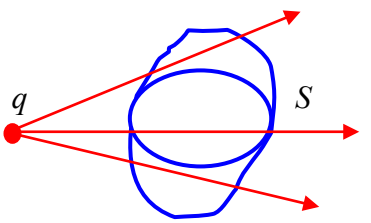
（1）如果高斯面上 $\vec{E}$ 处处为零，则高斯面内必无电荷

$\Phi=\oint_S \vec{E}\cdot d\vec{S}=0$，$\sum_{内} q_i=0$



（2）如果高斯面内无电荷，则高斯面上 $\vec{E}$ 处处为零

$\sum_{内} q_i=0$，$\Phi=\oint_S \vec{E}\cdot d\vec{S}=\dfrac{1}{\varepsilon_0}\sum_{内} q_i$ **=0**，$\not\Rightarrow \vec{E}=0$



（3）如果高斯面上 $\vec{E}$ 处处不为零，则高斯面内必有电荷

（4）如果高斯面内有电荷，则高斯面上 $\vec{E}$ 处处不为零

由高斯定理求电场强度的思路：

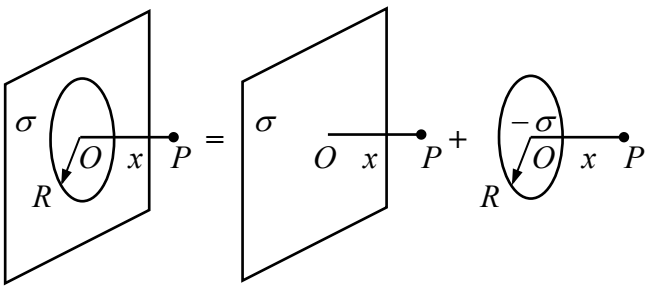
电荷分布的对称性 $\Rightarrow$ 电场分布的对称性

$\Rightarrow$ 适当的选取高斯面（$\vec{E}\perp\vec{n}$，$\vec{E}//\vec{n}$）

$\Rightarrow$ 将 E 从积分号内提出，化积分方程为代数方程求 E

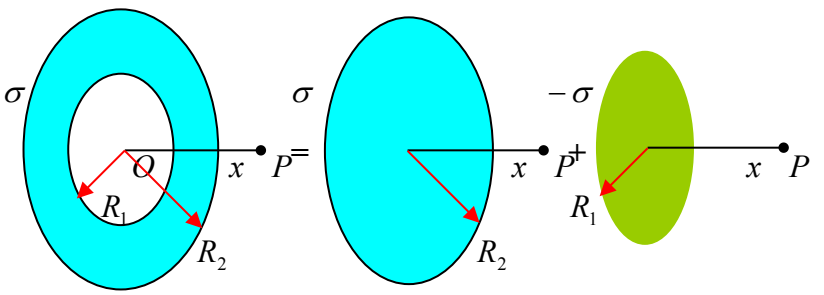
补偿法求电场强度

例：求圆孔轴线上的 $\vec{E}$



解：$E=\dfrac{\sigma}{2\varepsilon_0}+\dfrac{-\sigma}{2\varepsilon_0}(1-\dfrac{x}{\sqrt{x^2+R^2}})=\dfrac{\sigma}{2\varepsilon_0}\dfrac{x}{\sqrt{x^2+R^2}}$

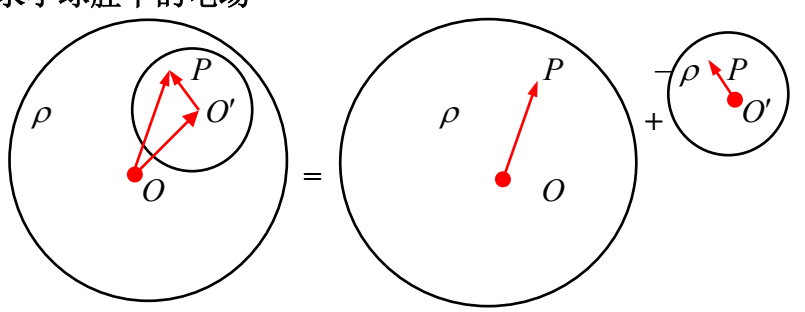
例：求轴线上的 $\vec{E}$



解：$E=\dfrac{\sigma}{2\varepsilon_0}(1-\dfrac{x}{\sqrt{x^2+R_2{}^2}})+\dfrac{-\sigma}{2\varepsilon_0}(1-\dfrac{x}{\sqrt{x^2+R_1{}^2}})$

$$=\frac{\sigma}{2\varepsilon_0}(\frac{x}{\sqrt{x^2+R_1{}^2}}-\frac{x}{\sqrt{x^2+R_2{}^2}})$$

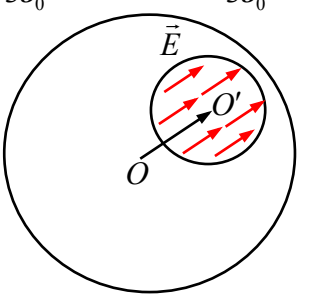
例：求小球腔中的电场



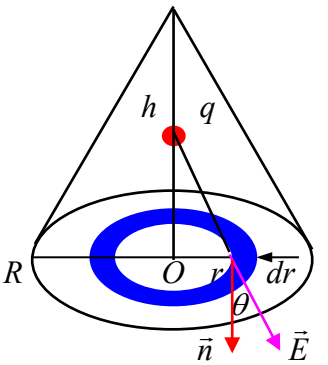
$$\vec{E}_P=\frac{\rho}{3\varepsilon_0}\overrightarrow{OP}+\frac{-\rho}{3\varepsilon_0}\overrightarrow{O'P}=\frac{\rho}{3\varepsilon_0}(\overrightarrow{OP}-\overrightarrow{O'P})=\frac{\rho}{3\varepsilon_0}\overrightarrow{OO'}$$

小球腔内是均匀电场

$E=\dfrac{\rho}{3\varepsilon_0}\overline{OO'}$，方向 $\overrightarrow{OO'}$



例：求通过圆锥侧面的电通量



解：$\Phi_{侧}=\int_{侧}\vec{E}\cdot d\vec{S}$

$$\Phi=\oint_S\vec{E}\cdot d\vec{S}$$

$$=\int_{侧}\vec{E}\cdot d\vec{S}+\int_{底}\vec{E}\cdot d\vec{S}=\frac{q}{\varepsilon_0}$$

$$\Phi_{侧}=\frac{q}{\varepsilon_0}-\Phi_{底}$$

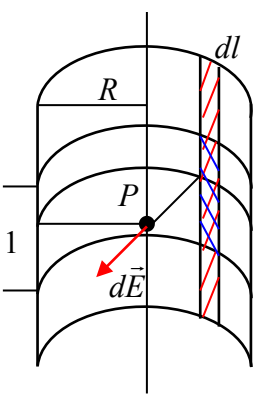
$$\Phi_{底}=\int_{底}\vec{E}\cdot d\vec{S}=\int_{底}E\cos\theta\cdot dS$$

$$dS=2\pi rdr，\quad E=\frac{q}{4\pi\varepsilon_0[r^2+(h/2)^2]}，\quad \cos\theta=\frac{h/2}{\sqrt{r^2+(h/2)^2}}$$

$$\Phi_{底}=\frac{qh}{4\varepsilon_0}\int_0^R\frac{rdr}{[r^2+(h/2)^2]^{3/2}}=\frac{q}{2\varepsilon_0}-\frac{q}{4\varepsilon_0}\frac{h}{\sqrt{R^2+(h/2)^2}}$$

$$\Phi_{侧}=\frac{q}{\varepsilon_0}-\Phi_{底}=\frac{q}{2\varepsilon_0}+\frac{q}{4\varepsilon_0}\frac{h}{\sqrt{R^2+(h/2)^2}}$$

例：无限长均匀带电半圆柱面
沿轴向单位长度带电 λ

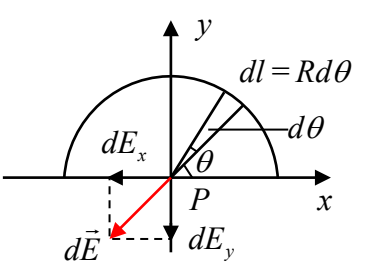


求：轴线上 $\vec{E}$

解：$\sigma=\dfrac{\lambda}{\pi R}$，$dE=\dfrac{\lambda'}{2\pi\varepsilon_0 R}$

$$\lambda'=\sigma dl=\frac{\lambda}{\pi R}dl$$

$$dE=\frac{1}{2\pi\varepsilon_0 R}\frac{\lambda}{\pi R}dl=\frac{\lambda}{2\pi^2\varepsilon_0 R^2}dl$$



$$dE_x=-dE\cos\theta$$

$$dE_y=-dE\sin\theta$$

$$E_x=\int dE_x=\int-dE\cos\theta$$

$$=\int_0^\pi-\frac{\lambda}{2\pi^2\varepsilon_0 R^2}Rd\theta\cos\theta，$$

$$=0$$

$$E_y=\int dE_y=\int-dE\sin\theta$$

$$=\int_0^\pi-\frac{\lambda}{2\pi^2\varepsilon_0 R^2}Rd\theta\sin\theta=\frac{\lambda}{2\pi^2\varepsilon_0 R}\cos\theta\Big|_0^\pi=-\frac{\lambda}{\pi^2\varepsilon_0 R}$$

$$\vec{E}=E_x\vec{i}+E_y\vec{j}=-\frac{\lambda}{\pi^2\varepsilon_0 R}\vec{j}$$