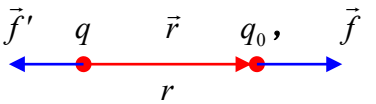
静　电　场

第 1 节　库仑定律

$f \propto \dfrac{qq_0}{r^2}$，$f = k\dfrac{qq_0}{r^2}$



q 对 q_0：$\vec{f} = k\dfrac{qq_0}{r^2}\cdot\dfrac{\vec{r}}{r}$

q_0 对 q：$\vec{f}' = -\vec{f}$，$k \approx 9\times10^9 Nm^2/C^2$

$k = \dfrac{1}{4\pi\varepsilon_0}$，$\varepsilon_0$：真空介电常数，$\varepsilon_0 \approx 8.85\times10^{-12} C^2N^{-1}m^{-2}$

$$\boxed{\vec{f} = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\cdot\frac{\vec{r}}{r}}$$

第 2 节　电场和电场强度

$q_1 \Leftrightarrow$ 电场 $\Leftrightarrow q_2$

一、电场强度

试验电荷 q_0：（1）电量很小，（2）体积很小

1、q_0，不同位置，$\vec{f}/q_0$ 一般不同

2、$q_0, q_0', q_0'', \cdots$，同一位置，$\vec{f}, \vec{f}', \vec{f}'', \cdots$

$\vec{f}/q_0 = \vec{f}'/q_0' = \vec{f}''/q_0'' = \cdots$

$\vec{f}/q_0$ 与试验电荷无关仅与电场中位置有关

定义：电场强度 $\vec{E} = \vec{f}/q_0$，SI：N/C

（1）$\vec{E} \not\propto \vec{f}$，$\vec{E} \not\propto \dfrac{1}{q_0}$

（2）$\vec{E}$ 与 q_0 的正、负无关

（3）点电荷 q_0：$\vec{f} = q_0\vec{E}$

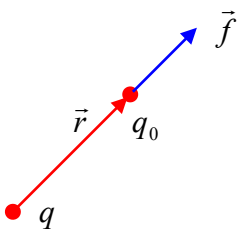
二、电场强度的计算

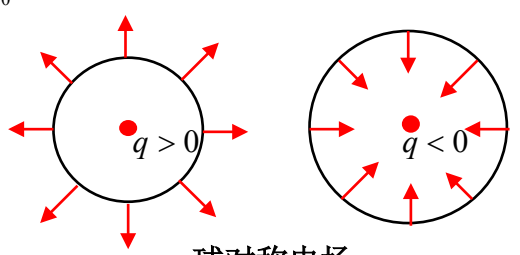
1、点电荷的电场

$\vec{f} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{qq_0}{r^2}\cdot\dfrac{\vec{r}}{r}$

$\vec{E} = \vec{f}/q_0 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}\cdot\dfrac{\vec{r}}{r}$，

$E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}$





球对称电场

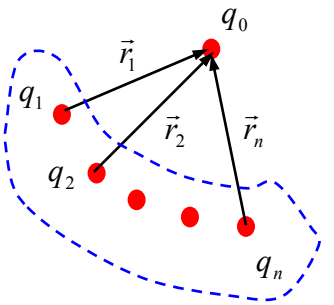
2、点电荷系

$$\vec{f}=\vec{f}_1+\vec{f}_2+\cdots+\vec{f}_n$$

$$=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_0}{r_1^2}\cdot\frac{\vec{r}_1}{r_1}+$$

$$+\frac{1}{4\pi\varepsilon_0}\frac{q_2q_0}{r_2^2}\cdot\frac{\vec{r}_2}{r_2}+$$

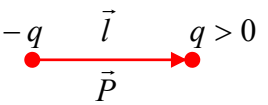
$$+\cdots+\frac{1}{4\pi\varepsilon_0}\frac{q_nq_0}{r_n^2}\cdot\frac{\vec{r}_n}{r_n}$$



$$\vec{E}=\frac{\vec{f}}{q_0}=\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2}\cdot\frac{\vec{r}_1}{r_1}+\frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2^2}\cdot\frac{\vec{r}_2}{r_2}+\cdots+\frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_n^2}\cdot\frac{\vec{r}_n}{r_n}$$

$\vec{E}=\vec{E}_1+\vec{E}_2+\cdots+\vec{E}_n$ ——— 场强迭加原理

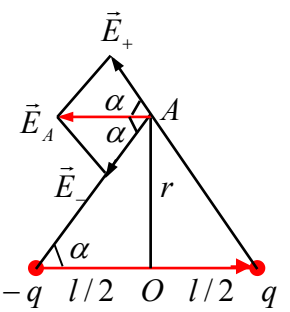
例：电偶极子



电偶极矩 $\vec{P}=q\vec{l}$，

$P=ql$，$-q$ 指向 q

（1）中垂线上某点 A



$$\vec{E}_A=\vec{E}_++\vec{E}_-$$

$$E_+=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2+(\frac{l}{2})^2},$$

$$E_-=E_+,$$

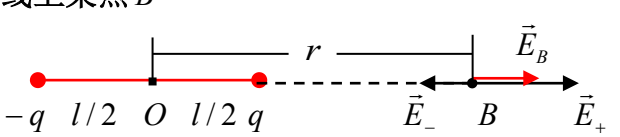
$$E_A=E_+\cos\alpha+E_-\cos\alpha=2E_+\cos\alpha$$

$$=2\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2+(l/2)^2}\frac{l/2}{\sqrt{r^2+(l/2)^2}}=\frac{1}{4\pi\varepsilon_0}\frac{P}{[r^2+(l/2)^2]^{3/2}}$$

$$\vec{E}_A=-\frac{1}{4\pi\varepsilon_0}\frac{\vec{P}}{[r^2+(l/2)^2]^{3/2}}$$

$r>>l$，$\vec{E}_A\approx-\frac{1}{4\pi\varepsilon_0}\frac{\vec{P}}{r^3}$

（2）延长线上某点 B



$$\vec{E}_B=\vec{E}_++\vec{E}_-$$

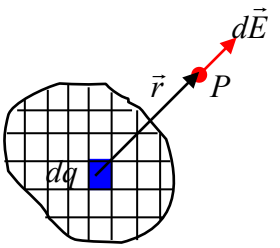
$$E_+=\frac{1}{4\pi\varepsilon_0}\frac{q}{(r-\frac{l}{2})^2},\quad E_-=\frac{1}{4\pi\varepsilon_0}\frac{q}{(r+\frac{l}{2})^2}$$

$$E_B=E_+-E_-=\frac{1}{4\pi\varepsilon_0}\frac{2rP}{(r-\frac{l}{2})^2(r+\frac{l}{2})^2}$$

$$\vec{E}_B = \frac{1}{4\pi\varepsilon_0}\frac{2r\vec{P}}{(r-\frac{l}{2})^2(r+\frac{l}{2})^2}$$

$r >> l$， $\vec{E}_B \approx \frac{1}{4\pi\varepsilon_0}\frac{2\vec{P}}{r^3}$

3、连续电荷分布的电场



$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\cdot\frac{\vec{r}}{r}$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

$$\vec{E} = \int d\vec{E} = \int \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\cdot\frac{\vec{r}}{r}$$

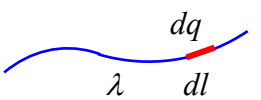
$Oxyz$： $d\vec{E} = dE_x\vec{i} + dE_y\vec{j} + dE_z\vec{k}$

$E_x = \int dE_x$， $E_y = \int dE_y$， $E_z = \int dE_z$

$\vec{E} = E_x\vec{i} + E_y\vec{j} + E_z\vec{k}$，一般情况下， $E \neq \int dE$

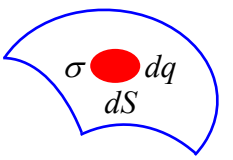
（1）电荷线密度 $\lambda = \frac{dq}{dl}$

$dq = \lambda dl$



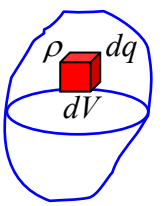
（2）电荷面密度 $\sigma = \frac{dq}{dS}$

$dq = \sigma dS$



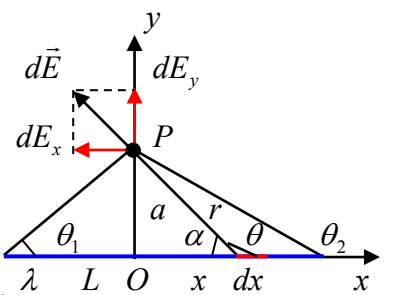
（3）电荷体密度 $\rho = \frac{dq}{dV}$

$dq = \rho dV$



例：均匀带电直线段

解： $dq = \lambda dx$



$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{r^2},$$

$dE_x = dE\cos\theta$

$dE_y = dE\sin\theta$

$$E_x = \int dE_x = \int dE\cos\theta = \int \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{r^2}\cos\theta$$

$a/x = tg\alpha = tg(\pi-\theta) = -tg\theta$， $x = -actg\theta$， $dx = a\csc^2\theta d\theta$

$a/r = \sin\alpha = \sin(\pi-\theta) = \sin\theta$， $r = a\csc\theta$

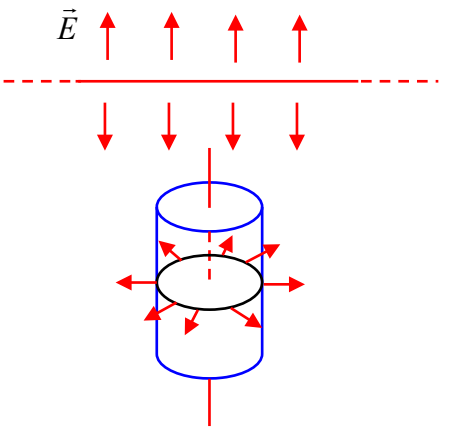
$$E_x=\int\frac{1}{4\pi\varepsilon_0}\frac{\lambda a\csc^2\theta d\theta}{a^2\csc^2\theta}\cos\theta=\frac{\lambda}{4\pi\varepsilon_0 a}\int_{\theta_1}^{\theta_2}\cos\theta d\theta$$

$$=\frac{\lambda}{4\pi\varepsilon_0 a}(\sin\theta_2-\sin\theta_1)$$

$$E_y=\int dE_y=\int dE\sin\theta=\frac{\lambda}{4\pi\varepsilon_0 a}\int_{\theta_1}^{\theta_2}\sin\theta d\theta$$

$$=\frac{\lambda}{4\pi\varepsilon_0 a}(\cos\theta_1-\cos\theta_2)$$

$$\vec{E}=E_x\vec{i}+E_y\vec{j}$$



讨论：λ 不变，$L\to\infty$

$\theta_1=0$，$\theta_2=\pi$

$E_x=0$

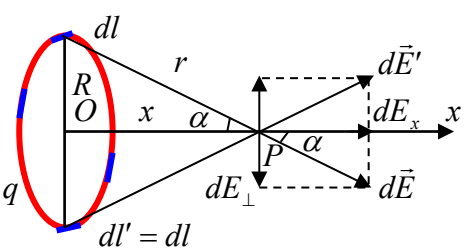
$$E_y=\frac{\lambda}{2\pi\varepsilon_0 a}$$

$$E=\frac{\lambda}{2\pi\varepsilon_0 a}$$

轴对称电场

例：均匀带电细圆环

解：$dq=\lambda dl$，（$\lambda=\dfrac{q}{2\pi R}$）



$$dE=\frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}=\frac{1}{4\pi\varepsilon_0}\frac{\lambda dl}{r^2},$$

$$dE_x=dE\cos\alpha$$

$$dE_\perp=dE\sin\alpha$$

由于对称性，$\vec{E}_\perp=0$

$$E=E_x=\int dE_x=\int dE\cos\alpha=\int\frac{1}{4\pi\varepsilon_0}\frac{\lambda dl}{r^2}\cos\alpha$$

$$=\frac{\lambda\cos\alpha}{4\pi\varepsilon_0 r^2}\int dl=\frac{\lambda\cos\alpha}{4\pi\varepsilon_0 r^2}2\pi R=\frac{q\cos\alpha}{4\pi\varepsilon_0 r^2},\quad \cos\alpha=\frac{x}{r},\quad r=\sqrt{x^2+R^2}$$

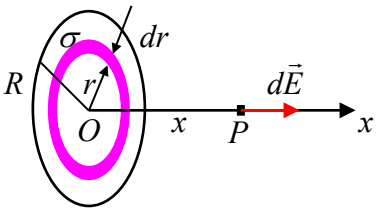
$$E=\frac{1}{4\pi\varepsilon_0}\frac{qx}{(x^2+R^2)^{3/2}}$$

讨论：（1）$x=0$，圆心处，$E=0$

（2）$x>>R$，$E\approx\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{x^2}$

例：均匀带电圆盘

求：轴线上 $\vec{E}$



解：$dS = 2\pi r dr$

$dq = \sigma dS = \sigma 2\pi r dr$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{x dq}{(x^2+r^2)^{3/2}}$$

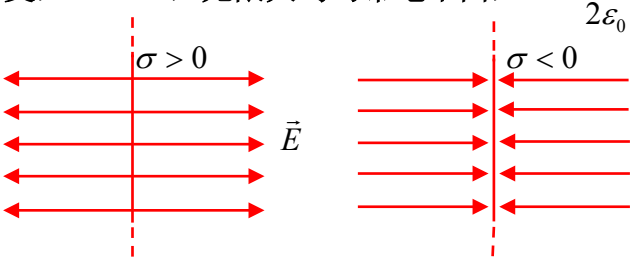
$$= \frac{1}{4\pi\varepsilon_0}\frac{x\sigma 2\pi r dr}{(x^2+r^2)^{3/2}} = \frac{\sigma x}{2\varepsilon_0}\frac{r dr}{(x^2+r^2)^{3/2}}$$

$$E = \int dE = \int_0^R \frac{\sigma x}{2\varepsilon_0}\frac{r dr}{(x^2+r^2)^{3/2}}$$

$$= \frac{\sigma x}{2\varepsilon_0}\int_0^R \frac{\frac{1}{2}d(x^2+r^2)}{(x^2+r^2)^{3/2}} = \frac{\sigma x}{2\varepsilon_0}\left(-\frac{1}{\sqrt{x^2+r^2}}\right)\Bigg|_0^R$$

$$= \frac{\sigma x}{2\varepsilon_0}\left(-\frac{1}{\sqrt{x^2+R^2}}+\frac{1}{x}\right) = \frac{\sigma}{2\varepsilon_0}\left(1-\frac{x}{\sqrt{x^2+R^2}}\right)$$

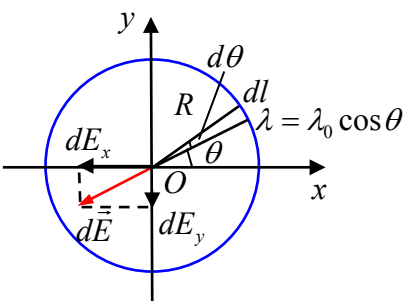
讨论：σ 不变，$R\to\infty$，无限大均匀带电平面，$E = \dfrac{\sigma}{2\varepsilon_0}$



例：细圆环（R）

$\lambda = \lambda_0\cos\theta$

求：圆心处 $\vec{E}$



解：$dl = R d\theta$

$dq = \lambda dl = \lambda_0\cos\theta R d\theta$

$$dE = \frac{dq}{4\pi\varepsilon_0 R^2} = \frac{\lambda_0\cos\theta d\theta}{4\pi\varepsilon_0 R}$$

$dE_x = -dE\cos\theta$，$dE_y = -dE\sin\theta$

$$E_x = \int dE_x = -\int dE\cos\theta = -\int_0^{2\pi}\frac{\lambda_0}{4\pi\varepsilon_0 R}\cos^2\theta d\theta$$

$$= -\frac{\lambda_0}{4\pi\varepsilon_0 R}\int_0^{2\pi}\frac{1+\cos 2\theta}{2}d\theta = -\frac{\lambda_0}{4\varepsilon_0 R}$$

$$E_y = \int dE_y = -\int dE\sin\theta = -\int_0^{2\pi}\frac{\lambda_0}{4\pi\varepsilon_0 R}\cos\theta\sin\theta d\theta = \mathbf{0}$$

$$\vec{E} = -\frac{\lambda_0}{4\varepsilon_0 R}\vec{i}$$