Chapter 10B

FIR Digital Filter Design

Part B

Computer-Aided Design of FIR Digital Filters

1. Design of Equiripple Linear-Phase FIR Filters

- The linear-phase FIR filter obtained by minimizing the peak absolute value of

$$\varepsilon = \max_{\omega \in R} |E(\omega)|$$

which is usually called the equiripple FIR filter

- After ε is minimized, the weighted error function $|E(\omega)|$ exhibits an equiripple behavior in the frequency range R

- The general form of frequency response of a causal linear-phase FIR filter of length $2M+1$:

$$H(e^{j\omega}) = e^{-jN\omega/2} e^{j\beta} \breve{H}(\omega)$$

where the amplitude response $\breve{H}(\omega)$ is a real function of ω

- Weighted error function is given by

$$E(\omega) = W(\omega)\left[\breve{H}(\omega) - D(\omega)\right]$$

where $D(\omega)$ is the desired amplitude response and $W(\omega)$ is a positive weighting function

Parks-McClellan Algorithm

- Based on iteratively adjusting the coefficients of $\breve{H}(\omega)$ until the peak absolute value of $E(\omega)$ is minimized
- If peak absolute value of $E(\omega)$ in a band $\omega_a \le \omega \le \omega_b$ is ε_0, then the absolute error satisfies

$$\left|\breve{H}(\omega) - D(\omega)\right| \le \frac{\varepsilon_0}{|W(\omega)|}, \quad \omega_a \le \omega \le \omega_b$$

- For filter design,

$$D(\omega) = \begin{cases} 1, & \text{in the passband} \\ 0, & \text{in the stopband} \end{cases}$$

- $\breve{H}(\omega)$ is required to satisfy the above desired response with a ripple of $\pm\delta_p$ in the passband and a ripple of δ_s in the stopband

1. Design of Equiripple Linear-Phase FIR Filters

- Thus, weighting function can be chosen either as

$$W(\omega) = \begin{cases} 1, & \text{in the passband} \\ \delta_p / \delta_s, & \text{in the stopband} \end{cases}$$

or

$$W(\omega) = \begin{cases} \delta_s / \delta_p, & \text{in the passband} \\ 1, & \text{in the stopband} \end{cases}$$