Chapter 9

IIR Digital Filter Design

Part A

Preliminary Considerations

1. Preliminary Considerations

- **Digital Filter Specifications**
- **Selection of the Filter Type**
- **Basic Approaches to Digital Filter Design**
- **Estimation of the Filter Order**
- **Scaling the Digital Filter**

1.1 Digital Filter Specifications

Objective :

Determination of a realizable transfer function $G(z)$ approximating a given frequency response specification is an important step in the development of a digital filter

- If an IIR filter is desired, $G(z)$ should be a stable rational function
- Digital filter design is the process of deriving the transfer function $G(z)$

1.1 Digital Filter Specifications

- Usually, either the magnitude and/or the phase (delay) response is specified for the design of digital filter for most applications.
- In most practical applications, the problem of interest is the development of a realizable approximation to a given magnitude response specification.

1.1 Digital Filter Specifications

Digital Filter Design Steps

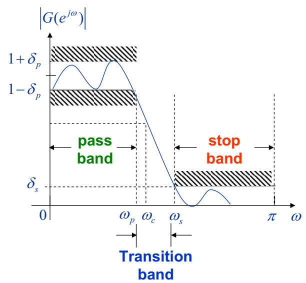
A choice between IIR and FIR digital filter has to be made

① Derivation of a realizable transfer function $G(z)$

② Realization of $G(z)$ using a suitable filter structure

1.1 Digital Filter Specifications

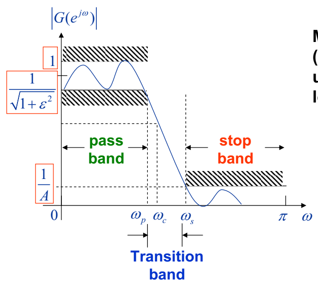
Digital Filter Specifications



- Passband:
  $1-\delta_p \le |G(e^{j\omega})| \le 1+\delta_p, \quad |\omega| \le \omega_p$
- Stopband:
  $|G(e^{j\omega})| \le \delta_s, \quad \omega_s \le \omega \le \pi$
- Peak Passband ripple:
  $\alpha_p = -20\log_{10}(1-\delta_p)$ dB
- Minimum Stopband attenuation
  $\alpha_s = -20\log_{10}(\delta_s)$ dB

1.1 Digital Filter Specifications

Normalized Specifications



Maximum value of the magnitude (gain) is assumed to be unity or the minimum value of the loss function is 0 dB

$$\alpha_{\max} = 20\log_{10}\left(\sqrt{1+\varepsilon^2}\right) \text{ dB}$$

$$\alpha_{\max} \cong -20\log_{10}(1-2\delta_p)$$

$$\cong 2\delta_p \text{ dB}$$

1.1 Digital Filter Specifications

- Frequency specifications are normalized using the sampling rate:

$$\omega_p = \frac{\Omega_p}{F_T} = \frac{2\pi F_p}{F_T} = 2\pi F_p T$$

$$\omega_s = \frac{\Omega_s}{F_T} = \frac{2\pi F_s}{F_T} = 2\pi F_s T$$

- $\omega = \pi$ corresponds to half the sampling rate, $F_T/2$

Q: What is the condition for non-overlapping?

1.2 Selection of the Filter Type

- **FIR filters**:

✓ Linear phase response

✓ Stability with quantized coefficients

✗ Higher order required than using IIR filters

1.2 Selection of the Filter Type

- **IIR filters:**

✓ Better attenuation properties

✓ Closed form approximation formulas

✗ Nonlinear phase response

✗ Instability with finite wordlength computation

✓ Lower order

N_{FIR}/N_{IIR} is typically of the order of tens (or more)

1.3 Basic Approaches to Digital Filter Design

IIR Filter Design

An analog filter transfer function $H_a(s)$ is transformed into the desired digital filter transfer function $G(z)$

- Analog approximation techniques are highly advanced
- They usually yield closed-form solutions

1.3 Basic Approaches to Digital Filter Design

IIR Filter Design

- Extensive tables are available for analog filter design or the methods are easy to program
- Digital filters often replace (or simulate) analog filters

$$H_a(s) = \frac{P_a(s)}{D_a(s)} \Longrightarrow G(z) = \frac{P(z)}{D(z)}$$

1.3 Basic Approaches to Digital Filter Design

IIR Filter Design

- The basic idea behind the conversion of an analog prototype transfer function $H_a(s)$ to a digital filter transfer function $G(z)$ is to apply a mapping from the s-domain to the z-domain so that the *essential properties* of the analog frequency response are preserved.

1.3 Basic Approaches to Digital Filter Design

IIR Filter Design

- Requirements for the transform are:
  - The imaginary axis ($j\Omega$) of the s-plane is mapped onto the unit circle in the z-plane
  - Stable $H_a(s)$ must be transformed into a stable $G(z)$

1.3 Basic Approaches to Digital Filter Design

FIR Filter Design

- No analog prototype filters are available
- FIR filter design is based on a direct approximation of the specified magnitude response
- A linear phase response is usually required

1.3 Basic Approaches to Digital Filter Design

FIR Filter Design

- FIR transfer function:

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

- The corresponding frequency response:

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n) e^{-j\omega n}$$

- Linear phase requirement:

$$h(n) = \pm h(N-1-n)$$

1.4 Estimation of the Filter Order

- **IIR Design --** Filter order is solved from the approximation formulas
- **FIR Design --** Several formulas proposed for estimating the minimum length of the impulse response

Kaiser: $$N \cong \frac{-20\log_{10}\left(\sqrt{\delta_p \delta_s}\right) - 13}{14.6(\omega_s - \omega_p)/2\pi}$$

1.4 Estimation of the Filter Order

- N is inversely proportional to the normalized transition width and does not depend on the location of the transition band
- N depends also on the product of δ_p and δ_s

1.5 Scaling the Digital Filter

- $G(z)$ has to be scaled in magnitude so that the maximum gain in the passband is unity
- Notice that the scaling coefficient K does not affect the shape of the magnitude response, i.e., it does not affect the locations of poles and zeros in the z-plane

1.5 Scaling the Digital Filter

- **Lowpass filter:** Unity gain at zero frequency $\omega = 0$ (or z=1)

$$KG(e^{j\omega})\big|_{\omega=0} = KG(z)\big|_{z=1} \Rightarrow K = 1/G(1)$$

- **Highpass filter:** Unity gain at $\omega = \pi$ (or z=−1)

$$KG(e^{j\omega})\big|_{\omega=\pi} = KG(z)\big|_{z=-1} \Rightarrow K = 1/G(-1)$$