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Cooperative Output Regulation for Linear Multiagent Systems via Distributed Fixed-Time Event-Triggered Control

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***Abstract*— In this article, we consider the cooperative output regulation for linear multiagent systems (MASs) via the distributed event-triggered strategy in fixed time. A novel fixed-time event-triggered control protocol is proposed using a dynamic compensator method. It is shown that based on the designed control scheme, the cooperative output regulation problem is addressed in fixed time and the agents in the communication network are subject to intermittent communication with their neighbors. Simultaneously, with the proposed event-triggering mechanism, Zeno behavior can be ruled out by choosing the appropriate parameters. Different from the existing strategies, both the compensator and control law are designed with intermittent communication in fixed time, where the convergence time is independent of any initial conditions. Moreover, for the case that the states are not available, the output regulation problem can further be addressed by the distributed observer-based output feedback controller with the fixed-time event-triggered compensator and event-triggered mechanism. Finally, a simulation example is provided to illustrate the effectiveness of the theoretical results.**

***Index Terms*— Cooperative control, event-triggered protocol, fixed time, multiagent systems (MASs), output regulation.**

I. Introduction

THE investigation of cooperative control of multiagent systems (MASs) has become one of the hot spots in recent years, due to its wide application in modern engineering, such as mechanical arm operation and satellites alignment. Research on cooperative control problems usually includes consensus, formation, and flocking. In addition, cooperative output regulation has also received many scholars' universal attention, which aims to enable agents to track given reference input and restrain external disturbances. The cooperative output regulation problems can include some cooperative control problems, such as leader-following consensus [1]–[3] and synchronization. Some important results of cooperative output regulation of MASs have been reported in [4] and [5].

It is worth noting that in the above studies, the interactive information model of MAS depends on continuous communication between adjacent agents. However, it is not difficult to know that continuous communication would bring about tremendous resource waste. Moreover, each agent usually has limited onboard computing resources and communication bandwidth, which further limits the application of MASs in practical engineering. Due to this, more works have been focused on intermittent communication control approaches. One traditional intermittent communication control strategy is developed based on periodic sampling and may be called time-triggered sampling, where information exchange occurs when a sampling period elapses. However, once the sampling period is small, it will lead to communication resource wastage. For this reason, the intermittent communication control based on event-triggered is proposed, where a control task related to communication executes only when a triggering condition is met. In this regard, the event-triggered method can save processing and communication resources and ensure the control performance of the system.

Recently, event-triggered control strategies have been used to solve various cooperative control problems. The core goal of the event-triggered cooperative control problem is to design an event-based distributed control protocol, including a triggering function that can exclude Zeno behavior. In the past decade, much research has been done on the event-triggered consensus problems. For instance, the consensus problems were investigated for general linear MASs [6]–[8], first-order MASs [9], second-order MASs [10]–[12], and high-order nonlinear MASs [13]–[15] via event-triggered control technology. Later, the synchronization problems for homogeneous linear MASs were solved with event-triggered control strategy in [16] and [17]. More recently, the cooperative output regulation problems have been attracting a lot of attention, and many researchers are working on the event-triggered mechanisms. The cooperative output regulation problems have been studied for linear MASs under fixed and switching topologies in [18] and [19], respectively. Based on these, the results were extended to nonlinear MASs in [20]. The cooperative robust practical output regulation problem was solved with

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event-triggered control strategy based on output information, and the practical problem allows certain degree error of tracking.

However, the distributed control protocols with event-triggered mechanism proposed in the above literature [6]–[20] only realize the asymptotic stability, where the error vector for cooperative control in such MASs will converge to the zero vector with an exponential decay rate, at best. In the analysis of cooperative control problems, an important performance index for a proposed cooperative control protocol is the convergence rate. In many practical engineering applications, especially for some systems with high control precision, such as missile or satellite system control and flying control, the system is required to achieve fast tracking of the target in the communication process. Then, the finite-time consensus problem has been promoted to achieve high-speed convergence [21]–[23].

Nevertheless, the convergence time in [21]–[23] depends on the initial states of the agents, which prohibits their applications if the initial states are sufficiently large or unavailable in advance. As a consequence, a new concept of fixed-time stability was proposed in [24]. It is assumed that the settling time is uniformly bounded and regardless of the initial conditions. In [25]–[27], the fixed-time consensus problems for the second- and high-order MASs were addressed. However, in [25]–[27], continuous communication between adjacent agents was required. In this case, the fixed-time control problems were studied based on the event-triggered technology, which can reduce communication and computing resources. Event-triggered fixed-time consensus problems for linear and nonlinear MASs with integrator dynamics were investigated in [28]–[30]. However, in [29] and [30], the adjacent agents' states are still needed to be monitored continuously. Then, the fixed-time consensus of a class of general linear MASs with the proposed event-triggered control mechanism was studied in [31]. However, the system dynamics are homogeneous. In practice, state information of an agent is not easy to acquire, while only the output information is available. Then, in [32], combined with the event-triggered control scheme, the problem about the fixed-time output consensus of high-order linear MASs has been paid more attention. However, few results have been reported in the study on the fixed-time output consensus via event-triggered control.

In addition, as we know, the cooperative output regulation problems can include some cooperative control problems, such as leader-following consensus. The cooperative output regulation problem was considered for heterogeneous linear MASs by a fully distributed event-triggered control law in [33]. In [34] and [35], the cooperative output regulation problem for switched linear MASs was concerned by the event-triggered scheme, and the event-triggered condition was developed to exclude the Zeno behavior. Beyond that, works on the event-triggered cooperative output regulation problems are still few, especially the fixed time event-triggered cooperative output regulation problem.

Motivated by these observations, the main contributions of this article are threefold.

First, to address the fixed-time cooperative output regulation problem, this article proposes a novel distributed fixed-time event-triggered compensator and fixed-time control protocol to achieve high-speed convergence in contrast to existing works, such as [18]–[20], where the error vector for cooperative output regulation in such MASs will converge to the zero vector with an exponential decay rate, at best.

Second, compared with the event-triggering mechanisms in [25]–[27], [29], and [30], the event-triggering mechanism proposed in this article does not require continuous communication between adjacent agents. We note that the event-triggering mechanisms, which can avoid continuous communication, have also been designed to solve consensus problems [31], [32]. However, these works achieve distributed asymptotic consensus. Meanwhile, different from [18]–[20] and [33]–[35], the triggering function (9) in this article introduced the fixed-time convergent function to govern the triggering threshold, so as to realize the convergence of the estimation error within fixed time, and the cooperative output regulation problem is addressed in fixed time without regard to any initial conditions.

Third, the event-triggered control scheme proposed in this article can not only track given reference but also restrain disturbances. In other words, the event-triggered control scheme is more general than [25]–[32], and it can also be adopted to solve the leader-following consensus and synchronization problems.

II. Problem Formulation and Preliminaries

A. Notations

Let R^N be the N dimensional Euclidean space. I_N represents the $N \times N$ identity matrix. $1_N = (1, \ldots, 1)^T \in R^N$. The superscript T represents the transpose. $\text{diag}\{\cdot\}$ represents a diagonal matrix. For any $b \in R$, $\text{sig}(b)^k = \text{sign}(b)|b|^k$ for a positive k. $\|\cdot\|$ denotes the Euclidean norm.

B. Graph Theory

In this section, the communications among the agents can be denoted by a directed graph $\bar{G}$. $A = [a_{ij}] \in R^{N\times N}$ refers to the adjacency matrix. The Laplacian matrix is $L = [l_{ij}] \in R^{N\times N}$, and the pinning matrix is $B = \text{diag}\{b_1, \ldots, b_N\}$ with $b_i > 0, i = 1, \ldots, N$ if the follower i can receive the information from the leader; otherwise, $b_i = 0$. Moreover, define the matrix L_1 as $L_1 = L + B$.

C. Problem Formulation

In this article, we consider the heterogeneous linear MASs described by the dynamics

$$\begin{cases} \dot{x}_i = A_i x_i + B_i u_i + F_i \upsilon \\ y_{mi} = C_{mi} x_i + D_{mi} u_i + F_{mi} \upsilon \\ \tilde{e}_i = C_i x_i + D_i u_i + E_i \upsilon \end{cases} \tag{1}$$

where $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^u$, $y_{mi} \in \mathbb{R}^{p_{mi}}$, and $\tilde{e}_i \in \mathbb{R}^{p_i}$ represent the state, control input, measurement output, and regulated output of agent i, respectively. The constant matrices A_i, B_i, F_i, C_{mi}, D_{mi}, F_{mi}, C_i, D_i, and E_i have compatible dimensions. $\upsilon \in \mathbb{R}^q$ denotes the exogenous signal representing the disturbance to be rejected or the reference input to be tracked and is generated by the exosystem as follows:

$$\dot{\upsilon} = S_0 \upsilon \tag{2}$$

where the constant matrix $S_0 \in \mathbb{R}^{q \times q}$.

The cooperative output regulation problem to be solved has been clarified as follows.

Definition 1: Consider the MASs (1) and (2) under a directed graph $\bar{G}$, and the problem is called to be the linear cooperative output regulation if we can design a fixed-time event-triggered distributed controller such that the following conditions hold.

1) When υ is bounded, the trajectory of the closed-loop system is bounded.
2) For any initial condition, the regulated output $\tilde{e}_i = 0$ holds within fixed time T, $i = 1, \dots, N$.

The following assumptions and useful lemmas are needed before proceeding.

Assumption 1: The graph $\bar{G}$ contains one directed spanning tree with the leader as its root.

Assumption 2: All eigenvalues of matrix S_0 have nonnegative real parts.

Assumption 3: The pairs (A_i, B_i), $i = 1, \dots, N$, are stabilizable.

Assumption 4: The pairs (C_{mi}, A_i), $i = 1, \dots, N$, are detectable.

Assumption 5: The following equations have a solution pair (X_i, U_i):

$$\begin{aligned} X_i S_0 &= A_i X_i + B_i U_i + F_i \\ 0 &= C_i X_i + D_i U_i + E_i, \quad i = 1, \dots, N. \end{aligned} \tag{3}$$

Lemma 1 [25]: If $x_1, x_2, \dots, x_N \geq 0$, $0 < q < 1$, then $(\sum_{i=1}^{N} x_i)^q \leq \sum_{i=1}^{N} x_i^q \leq N^{1-q} (\sum_{i=1}^{N} x_i)^q$.

Lemma 2 [25]: If $x_1, x_2, \dots, x_N \geq 0$, $q > 1$, then $N^{1-q} (\sum_{i=1}^{N} x_i)^q \leq \sum_{i=1}^{N} x_i^q \leq (\sum_{i=1}^{N} x_i)^q$.

Lemma 3 [24]: For system $\dot{x} = f(x)$ with $f(0) = 0$, if there exists a positive radially unbounded continuous function $V(x)$ satisfying $\dot{V}(x) \leq -\gamma V^{\alpha} - \delta V^{\beta}$ with $\gamma, \delta > 0$, $0 < \alpha < 1$ and $\beta > 1$, then the fixed-time stability can be achieved and the settling time T satisfies $T \leq T_{\max} := (1/(\gamma(1-\alpha))) + (1/(\delta(\beta - 1)))$.

Lemma 4 [37]: Given any vector $x \in R^n$, a symmetric positive-definite matrix $P \in R^{n \times n}$, and a symmetric matrix $Q \in R^{n \times n}$, then $\lambda_{\min}(P^{-1}Q) x^T P x \leq x^T Q x \leq \lambda_{\max}(P^{-1}Q) x^T P x$ holds.

III. Main Results

In this section, a distributed fixed-time event-triggered compensator and the corresponding distributed controller will be designed to address the output regulation problem for the MASs (1) and (2).

A distributed fixed-time event-triggered compensator is presented for each agent

$$\begin{aligned} \dot{\eta}_i = {} & S_0 \eta_i + \mu_1 G \hat{\omega}_i + \mu_2 G \operatorname{sig}(P(\hat{\omega}_i + e_i))^{\frac{2\alpha}{\alpha+1}} \\ & + \mu_3 G \operatorname{sig}(P(\hat{\omega}_i + e_i))^{\frac{\alpha+\beta}{\alpha+1}} \end{aligned} \tag{4}$$

$$\hat{\omega}_i = \sum_{j=1}^{N} a_{ij} (\hat{\eta}_i - \hat{\eta}_j) + a_{i0} (\hat{\eta}_i - \upsilon) \tag{5}$$

where $\eta_i \in \mathbb{R}^q$ is the compensator state. Coupling gains $\mu_1, \mu_2, \mu_3 > 0$. G and P are gain matrices to be designed. $0 < \alpha < (1/3)$ and $\beta > 1$. $\hat{\eta}_i$ denotes the open-loop estimate of η_i during $t \in [t_k^i, t_{k+1}^i)$. t_k^i is the triggering instant of η_i. The open-loop estimate of $\hat{\eta}_i$ is designed as

$$\begin{cases} \dot{\hat{\eta}}_i(t) = S_0 \hat{\eta}_i(t), & t \in [t_k^i, t_{k+1}^i) \\ \hat{\eta}_i(t) = \eta_i(t), & t = t_k^i. \end{cases} \tag{6}$$

Remark 1: It should be pointed out that not each follower agent can obtain the leader's information directly, while only the follower agent who is connected to the leader can acquire the leader's information. Therefore, the dynamic compensator (4) in this article is designed for each agent to track the exosystem state via intermittent communication. The designed dynamic compensator (4) in this article is independent of η_j, but depends on open-loop estimates $\hat{\eta}_j$. Therefore, each agent does not need to be constantly aware of the compensator states of its neighbors so as to save processing and communication resources.

Remark 2: In [33]–[35], the dynamic compensators with event-triggered mechanism were designed to solve the consensus and output regulation problems. However, different from [33]–[35] where the tracking errors between the designed compensator states and leaders states converge to zero asymptotically, the designed compensator state in this article can estimate the exosystem state in fixed time, which can be deduced by the proof of Theorem 1.

e_i in (4) is the measurement error and is defined as

$$e_i = \hat{\eta}_i - \eta_i, \quad t \in [t_k^i, t_{k+1}^i). \tag{7}$$

The triggering time instant is given by

$$t_k^i = \inf\{t > t_{k-1}^i \,|\, f_{\eta_i} \geq 0\} \tag{8}$$

where f_{η_i} is the triggering function of η_i and is given by

$$\begin{aligned} f_{\eta_i} = {} & c_1 \|e_i\|^2 + c_2 \|e_i\|^{\frac{4\alpha}{\alpha+1}} + c_3 \|e_i\|^{\frac{2(\alpha+\beta)}{\alpha+1}} + \imath_{11} \|\hat{\omega}_i\|^2 \\ & + \imath_9 \|\hat{\omega}_i\|^{\frac{4\alpha}{\alpha+1}} + \imath_{13} \|\hat{\omega}_i\|^{\frac{2(\alpha+\beta)}{\alpha+1}} - \upsilon \theta_i |\phi_i|^{2\alpha} \end{aligned} \tag{9a}$$

$$\dot{\phi}_i = -\gamma_i \operatorname{sig}(\phi_i)^{\alpha} - \sigma_i \operatorname{sig}(\phi_i)^{\beta} \tag{9b}$$

where $0 < \alpha < (1/3)$, $\beta > 1$, $\gamma_i > 0$, $\sigma_i > 0$, $c_1 = k_1 \|\mu_1 (L_1 \otimes P^2)\|^2 + (\omega k_5/2) \|L_1 \otimes P S_0 + I_N \otimes P S_0 - \mu_1 L_1 \otimes PG\|^2 + (qN)^{-1} \|(L_1 + I_N) \otimes P\|^2 + 2(\imath_3 + 2)(\|(L_1 + I_N) \otimes P\|^2 + 2\|L_1 \otimes P\|^2)$, $\imath_3 = (\omega k_4/2) + (\omega k_5/2) + ((\omega(\mu_2^2 k_6 + \mu_3^2 k_7) \|I_N \otimes PG\|^2)/2)$, $c_2 = \|(L_1 + I_N) \otimes P\|^{(4\alpha/(\alpha+1))} + (qN)^{((1-3\alpha)/(\alpha+1))} 2^{(4\alpha/(\alpha+1))} (\imath_4 + 2) \|L_1 \otimes P\|^{(4\alpha/(\alpha+1))} + (qN)^{((1-3\alpha)/(\alpha+1))} 2^{(2\alpha/(\alpha+1))} (\imath_4 + 2) \|((L_1 + I_N) \otimes P) e\|^{(4\alpha/(\alpha+1))}$, $\imath_4 = k_2 \|\mu_2 I_N \otimes P^2\|^2 + (\omega/2k_6)$, $c_3 = (qN)^{1-((2(\alpha+\beta))/(\alpha+1))} \|(L_1 + I_N) \otimes P\|^{((2(\alpha+\beta))/(\alpha+1))} + (qN)^{((\beta-1)/(\alpha+1))} 2^{((\alpha+\beta)/(\alpha+1))} (\imath_5 + 2) \|(L_1 + I_N) \otimes P \|^{((2(\alpha+\beta))/(\alpha+1))} + (qN)^{((2(\beta-1))/(\alpha+1))} 2^{((2(\alpha+\beta))/(\alpha+1))} (\imath_5 + 2) \|L_1 \otimes P\|^{((2(\alpha+\beta))/(\alpha+1))}$, $\imath_5 = k_3 \|\mu_3 I_N \otimes P^2\|^2 + (\omega/2k_7)$, $\imath_9 = (qN)^{((1-3\alpha)/(\alpha+1))} 2^{(4\alpha/(\alpha+1))} (\imath_4 + 2) \|P\|^{(4\alpha/(\alpha+1))}$, $\imath_{11} = 4(\imath_3 + 2)\|P\|^2$, and $\imath_{13} = (qN)^{((2(\beta-1))/(\alpha+1))} 2^{((2(\alpha+\beta))/(\alpha+1))} (\imath_5 + 2) \|P\|^{((2(\alpha+\beta))/(\alpha+1))}$, here under the conditions of

Theorem 1 required to determine the parameters k_1, k_2, and k_3.

Remark 3: First, compared with [8]–[10] and [20], in this article, a novel triggering function is proposed. In the triggering function, the measurement error contains its compensator state and its open-loop estimate of compensator state at the latest triggered instant, and the relative estimate contains the latest broadcast open-loop estimates of compensator states from its neighbors and its open-loop estimate of compensator state at the latest triggered instant. Therefore, the communication between adjacent agents in the proposed event-triggered mechanism (9) is not continuous but intermittent, which can reduce the communication loss between agents.

Remark 4: Note that the designed dynamic compensator (4) in this article is independent of η_j but depends upon open-loop estimates $\hat{\eta}_j$. The triggering condition (8) is checked first, and then, the transmission of compensator state η_i is performed accordingly. When the triggering condition (8) holds, the compensator state η_i will be transmitted to its neighbors at the latest triggered instant. Therefore, interactions between agents and their neighbors can be implemented by intermittent communication.

Remark 5: Different from [18]–[20] and [33]–[35], the triggering function (9) in this article introduced the fixed-time convergent function ϕ_i to govern the triggering threshold, so as to realize the convergence of the estimation error within fixed time.

Theorem 1: Consider the MASs described by (1) and (2), Assumptions 1 and 2 are satisfied. Then, under the distributed fixed-time event-triggered compensator (4) and event-triggering mechanism (9), $\lim_{t\to T_1}\tilde{\eta}_i = 0$ can be achieved in fixed time $T_1 = ((\alpha+1)/(\vartheta_{\min}(1-\alpha))) + ((\alpha+1)/(\vartheta_{\min}3^{((1-\beta)/(\alpha+1))}(\beta-1)))$ and the Zeno behavior can be ruled out, if $G = -P$, where $P > 0$ satisfies the following algebraic Riccati equation (ARE):

$$PS_0 + S_0^T P - P^2 + I = 0. \tag{10}$$

Moreover, $\Lambda = PS_0 + S_0^T P - \mu_1\lambda_{\min}^{L_1} P^2 < 0$ with $\lambda_{\min}^{L_1} = \lambda_{\min}(L_1 + L_1^T)$ and $\iota_1 = \lambda_{\max}(\Lambda) + (\omega k_4/2)\|L_1\otimes PS_0 - \mu_1 L_1\otimes PG\|^2 + (1/k_1) + (1/k_2) + (1/k_3) < 0$.

Proof: Let $\tilde{\eta}_i = \eta_i - \upsilon$, $\tilde{\eta} = [\tilde{\eta}_1^T,\ldots,\tilde{\eta}_N^T]^T$, $\eta = [\eta_1^T,\ldots,\eta_1^T]^T$, and $\bar{\upsilon} = 1_N\otimes\upsilon$. Then, we have

$$\tilde{\eta} = \eta - \bar{\upsilon}. \tag{11}$$

Let $\hat{\omega} = [\hat{\omega}_1^T,\ldots,\hat{\omega}_N^T]^T$, $\hat{\eta} = [\hat{\eta}_1^T,\ldots,\hat{\eta}_N^T]^T$, and $e = [e_1^T,\ldots,e_N^T]^T$. Then, $\hat{\eta} - \bar{\upsilon} = e + \tilde{\eta}$. From the definition of $\hat{\omega}_i$ in (5), the compact of $\hat{\omega}$ can be written as

$$\begin{aligned}\hat{\omega} &= (L_1\otimes I_q)(\hat{\eta} - \bar{\upsilon})\\ &= (L_1\otimes I_q)(e+\tilde{\eta}).\end{aligned} \tag{12}$$

From (4) and (12), we can obtain

$$\begin{aligned}\dot{\eta} = (I_N\otimes S_0)\eta &+ \mu_1(L_1\otimes G)(e+\tilde{\eta})\\ &+ \mu_2(I_N\otimes G)\mathrm{sig}((L_1\otimes P)(e+\tilde{\eta}) + (I_N\otimes P)e)^{\frac{2\alpha}{\alpha+1}}\\ &+ \mu_3(I_N\otimes G)\mathrm{sig}((L_1\otimes P)(e+\tilde{\eta}) + (I_N\otimes P)e)^{\frac{\alpha+\beta}{\alpha+1}}.\end{aligned} \tag{13}$$

By (11) and (13), it follows that:

$$\begin{aligned}\dot{\tilde{\eta}} = ((I_N\otimes S_0) &+ \mu_1(L_1\otimes G))\tilde{\eta} + \mu_1(L_1\otimes G)e\\ &+ \mu_2(I_N\otimes G)\mathrm{sig}(\psi)^{\frac{2\alpha}{\alpha+1}} + \mu_3(I_N\otimes G)\mathrm{sig}(\psi)^{\frac{\alpha+\beta}{\alpha+1}}\end{aligned} \tag{14}$$

where $\psi = (L_1\otimes P)(e+\tilde{\eta}) + (I_N\otimes P)e$.

Construct the following valid Lyapunov function candidate:

$$V = \tilde{\eta}^T(I_N\otimes P)\tilde{\eta} + \frac{\omega}{2}(\mathrm{sig}(\psi))^T\mathrm{sig}(\psi) + \sum_{i=1}^{N}\frac{\upsilon}{1+\alpha}\mathrm{sig}(\phi_i)^{\alpha}\phi_i \tag{15}$$

where constants $\omega > 0$, $\upsilon > 0$, $0 < \alpha < (1/3)$, and ϕ_i has been defined in (9).

Let $V_1 = \tilde{\eta}^T(I_N\otimes P)\tilde{\eta}$, $V_2 = (\omega/2)(\mathrm{sig}(\psi))^T\mathrm{sig}(\psi)$, and $V_3 = \sum_{i=1}^{N}(\upsilon/(1+\alpha))\mathrm{sig}(\phi_i)^{\alpha}\phi_i$. Then, we have $\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3$. First, the derivative of V_1 along equations (14) is defined as

$$\begin{aligned}\dot{V}_1 = 2\tilde{\eta}^T&[(I_N\otimes PS_0) + \mu_1(L_1\otimes PG)]\tilde{\eta}\\ &+ 2\mu_1\tilde{\eta}^T(L_1\otimes PG)e + 2\mu_2\tilde{\eta}^T(I_N\otimes PG)\mathrm{sig}(\psi)^{\frac{2\alpha}{\alpha+1}}\\ &+ 2\mu_3\tilde{\eta}^T(I_N\otimes PG)\mathrm{sig}(\psi)^{\frac{\alpha+\beta}{\alpha+1}}.\end{aligned} \tag{16}$$

Note that

$$\begin{aligned}&2\tilde{\eta}^T[(I_N\otimes PS_0) + \mu_1(L_1\otimes PG)]\tilde{\eta}\\ &\quad= \tilde{\eta}^T\left[\left(I_N\otimes\left(PS_0 + S_0^T P\right)\right) - \mu_1\left(\left(L_1 + L_1^T\right)\otimes P^2\right)\right]\tilde{\eta}\\ &\quad\le \tilde{\eta}^T\left[I_N\otimes\left(PS_0 + S_0^T P - \mu_1\lambda_{\min}^{L_1}P^2\right)\right]\tilde{\eta}\end{aligned} \tag{17}$$

where Lemma 4 has been used.

Substituting (17) and (10a) into (16) and by Young's inequality, one has

$$\begin{aligned}\dot{V}_1 \le \lambda_{\max}^{\Lambda}\|\tilde{\eta}\|^2 &+ \frac{\|\tilde{\eta}\|^2}{k_1} + k_1\left\|\mu_1\left(L_1\otimes P^2\right)\right\|^2\|e\|^2\\ &+ \frac{\|\tilde{\eta}\|^2}{k_2} + k_2\left\|\mu_2\left(I_N\otimes P^2\right)\right\|^2\left(\mathrm{sig}(\psi)^{\frac{2\alpha}{\alpha+1}}\right)^T\mathrm{sig}(\psi)^{\frac{2\alpha}{\alpha+1}}\\ &+ \frac{\|\tilde{\eta}\|^2}{k_3} + k_3\left\|\mu_3\left(I_N\otimes P^2\right)\right\|^2\left(\mathrm{sig}(\psi)^{\frac{\alpha+\beta}{\alpha+1}}\right)^T\mathrm{sig}(\psi)^{\frac{\alpha+\beta}{\alpha+1}}\end{aligned} \tag{18}$$

where $k_1, k_2, k_3 > 0$.

Since $e_i = \hat{\eta}_i - \eta_i$, and based on (4) and (6), one gets

$$\begin{aligned}\dot{e} = (I_N\otimes S_0)e &- \mu_1(L_1\otimes G)(e+\tilde{\eta}) - \mu_2(I_N\otimes G)\\ &\times\mathrm{sig}(\psi)^{\frac{2\alpha}{\alpha+1}} - \mu_3(I_N\otimes G)\mathrm{sig}(\psi)^{\frac{\alpha+\beta}{\alpha+1}}.\end{aligned} \tag{19}$$

Second, the derivative of V_2 along (14) and (19) is defined as

$$\begin{aligned}\dot{V}_2 &= \omega(\mathrm{sig}(\psi))^T\left((L_1\otimes P)\left(\dot{e} + \dot{\tilde{\eta}}\right) + (I_N\otimes P)\dot{e}\right)\\ &= \omega(\mathrm{sig}(\psi))^T\Big((L_1\otimes PS_0 - \mu_1 L_1\otimes PG)\tilde{\eta}\\ &\qquad + (L_1\otimes PS_0 + I_N\otimes PS_0 - \mu_1 L_1\otimes PG)e\\ &\qquad - (\mu_2 I_N\otimes PG)\mathrm{sig}(\psi)^{\frac{2\alpha}{\alpha+1}}\\ &\qquad - (\mu_3 I_N\otimes PG)\mathrm{sig}(\psi)^{\frac{\alpha+\beta}{\alpha+1}}\Big)\\ &\le \left(\frac{\omega}{2k_4} + \frac{\omega}{2k_5}\right)(\mathrm{sig}(\psi))^T\mathrm{sig}(\psi)\\ &\quad + \frac{\omega k_4}{2}\|L_1\otimes PS_0 - \mu_1 L_1\otimes PG\|^2\|\tilde{\eta}\|^2\end{aligned}$$

$$+\frac{\omega k_5}{2}\|L_1\otimes PS_0+I_N\otimes PS_0-\mu_1L_1\otimes PG\|^2\|e\|^2$$
$$+\frac{\omega\left(\mu_2^2k_6+\mu_3^2k_7\right)\|I_N\otimes PG\|^2}{2}(\mathrm{sig}(\psi))^T\mathrm{sig}(\psi)$$
$$+\frac{\omega}{2k_6}\left(\mathrm{sig}(\psi)^{\frac{2\alpha}{\alpha+1}}\right)^T\mathrm{sig}(\psi)^{\frac{2\alpha}{\alpha+1}}$$
$$+\frac{\omega}{2k_7}\left(\mathrm{sig}(\psi)^{\frac{\alpha+\beta}{\alpha+1}}\right)^T\mathrm{sig}(\psi)^{\frac{\alpha+\beta}{\alpha+1}} \quad (20)$$

where $k_4, k_5, k_6, k_7 > 0$.

Moreover, the derivative of V_3 is

$$\dot{V}_3=\sum_{i=1}^{N}\upsilon\,\mathrm{sig}(\phi_i)^\alpha\dot{\phi}_i=\sum_{i=1}^{N}\left(-\upsilon\gamma_i|\phi_i|^{2\alpha}-\upsilon\sigma_i|\phi_i|^{\alpha+\beta}\right). \quad (21)$$

Combined with (18), (20), and (21), we can obtain

$$\dot{V}\le\iota_1\|\tilde{\eta}\|^2+\iota_2\|e\|^2+(\iota_3+2-2)(\mathrm{sig}(\psi))^T\mathrm{sig}(\psi)$$
$$+(\iota_4+2-2)\left(\mathrm{sig}(\psi)^{\frac{2\alpha}{\alpha+1}}\right)^T\mathrm{sig}(\psi)^{\frac{2\alpha}{\alpha+1}}$$
$$+(\iota_5+2-2)\left(\mathrm{sig}(\psi)^{\frac{\alpha+\beta}{\alpha+1}}\right)^T\mathrm{sig}(\psi)^{\frac{\alpha+\beta}{\alpha+1}}$$
$$-\sum_{i=1}^{N}\left(\upsilon\gamma_i|\phi_i|^{2\alpha}+\upsilon\sigma_i|\phi_i|^{\alpha+\beta}\right) \quad (22)$$

where $\iota_2 = k_1\|\mu_1(L_1\otimes P^2)\|^2 + (\omega k_5/2)\|L_1\otimes PS_0 + I_N\otimes PS_0-\mu_1L_1\otimes PG\|^2$, $\iota_3 = (\omega k_4/2)+(\omega k_5/2)+((\omega(\mu_2^2k_6+\mu_3^2k_7)\|I_N\otimes PG\|^2)/2)$, $\iota_4 = k_2\|\mu_2 I_N\otimes P^2\|^2+(\omega/2k_6)$, and $\iota_5 = k_3\|\mu_3 I_N\otimes P^2\|^2+(\omega/2k_7)$.

By Lemma 1, note that

$$\left(\mathrm{sig}(\psi)^{\frac{2\alpha}{\alpha+1}}\right)^T\mathrm{sig}(\psi)^{\frac{2\alpha}{\alpha+1}}$$
$$=\|(L_1\otimes P)(e+\tilde{\eta})+(I_N\otimes P)e\|_{\frac{4\alpha}{\alpha+1}}^{\frac{4\alpha}{\alpha+1}}$$
$$\le(qN)^{\frac{1-3\alpha}{\alpha+1}}\|(L_1\otimes P)(e+\tilde{\eta})+(I_N\otimes P)e\|^{\frac{4\alpha}{\alpha+1}}$$
$$\le(qN)^{\frac{1-3\alpha}{\alpha+1}}\left(2\|(L_1\otimes P)\tilde{\eta}\|^2+2\|((L_1+I_N)\otimes P)e\|^2\right)^{\frac{2\alpha}{\alpha+1}}$$
$$\le(qN)^{\frac{1-3\alpha}{\alpha+1}}2^{\frac{2\alpha}{\alpha+1}}\left(\|(L_1\otimes P)\tilde{\eta}\|^{\frac{4\alpha}{\alpha+1}}+\|((L_1+I_N)\otimes P)e\|^{\frac{4\alpha}{\alpha+1}}\right) \quad (23)$$

$$-\left(\mathrm{sig}(\psi)^{\frac{2\alpha}{\alpha+1}}\right)^T\mathrm{sig}(\psi)^{\frac{2\alpha}{\alpha+1}}$$
$$=-\sum_{i=1}^{qN}|\psi_i|^{\frac{4\alpha}{\alpha+1}}\le-\left(\sum_{i=1}^{qN}|\psi_i|\right)^{\frac{4\alpha}{\alpha+1}}=-\|\psi\|^{\frac{4\alpha}{\alpha+1}}$$
$$=-\|(L_1\otimes P)\tilde{\eta}+((L_1+I_N)\otimes P)e\|^{\frac{4\alpha}{\alpha+1}}$$
$$\le-\|(L_1\otimes P)\tilde{\eta}\|^{\frac{4\alpha}{\alpha+1}}+\iota_6\|e\|^{\frac{4\alpha}{\alpha+1}} \quad (24)$$

where $\iota_6=\|(L_1+I_N)\otimes P\|^{(4\alpha/(\alpha+1))}$.

Similarly, by Lemma 2, one gets

$$(\mathrm{sig}(\psi))^T\mathrm{sig}(\psi)$$
$$=\|(L_1\otimes P)(e+\tilde{\eta})+(I_N\otimes P)e\|_2^2$$
$$\le\|(L_1\otimes P)(e+\tilde{\eta})+(I_N\otimes P)e\|^2$$
$$\le 2\|(L_1\otimes P)\tilde{\eta}\|^2+2\|((L_1+I_N)\otimes P)e\|^2 \quad (25)$$

$$\left(\mathrm{sig}(\psi)^{\frac{\alpha+\beta}{\alpha+1}}\right)^T\mathrm{sig}(\psi)^{\frac{\alpha+\beta}{\alpha+1}}$$
$$=\|(L_1\otimes P)(e+\tilde{\eta})+(I_N\otimes P)e\|_{\frac{2(\alpha+\beta)}{\alpha+1}}^{\frac{2(\alpha+\beta)}{\alpha+1}}$$
$$\le\left(2\|(L_1\otimes P)\tilde{\eta}\|^2+2\|((L_1+I_N)\otimes P)e\|^2\right)^{\frac{\alpha+\beta}{\alpha+1}}$$
$$\le(qN)^{\frac{\beta-1}{\alpha+1}}2^{\frac{\alpha+\beta}{\alpha+1}}\left(\|(L_1\otimes P)\tilde{\eta}\|^{\frac{2(\alpha+\beta)}{\alpha+1}}+\|((L_1+I_N)\otimes P)e\|^{\frac{2(\alpha+\beta)}{\alpha+1}}\right) \quad (26)$$

$$-(\mathrm{sig}(\psi))^T\mathrm{sig}(\psi)-\left(\mathrm{sig}(\psi)^{\frac{\alpha+\beta}{\alpha+1}}\right)^T\mathrm{sig}(\psi)^{\frac{\alpha+\beta}{\alpha+1}}$$
$$=-\sum_{i=1}^{qN}|\psi_i|^2-\sum_{i=1}^{qN}|\psi_i|^{\frac{2(\alpha+\beta)}{\alpha+1}}$$
$$\le-(qN)^{-1}\left(\sum_{i=1}^{qN}|\psi_i|\right)^2-(qN)^{1-\frac{2(\alpha+\beta)}{\alpha+1}}\left(\sum_{i=1}^{qN}|\psi_i|\right)^{\frac{2(\alpha+\beta)}{\alpha+1}}$$
$$=-(qN)^{-1}\|\psi\|^2-(qN)^{1-\frac{2(\alpha+\beta)}{\alpha+1}}\|\psi\|^{\frac{2(\alpha+\beta)}{\alpha+1}}$$
$$=-(qN)^{-1}\|(L_1\otimes P)\tilde{\eta}+((L_1+I_N)\otimes P)e\|^2$$
$$-(qN)^{1-\frac{2(\alpha+\beta)}{\alpha+1}}\|(L_1\otimes P)\tilde{\eta}+((L_1+I_N)\otimes P)e\|^{\frac{2(\alpha+\beta)}{\alpha+1}}$$
$$\le-(qN)^{-1}\|(L_1\otimes P)\tilde{\eta}\|^2+\iota_7\|e\|^2$$
$$-(qN)^{1-\frac{2(\alpha+\beta)}{\alpha+1}}\|(L_1\otimes P)\tilde{\eta}\|^{\frac{2(\alpha+\beta)}{\alpha+1}}+\iota_8\|e\|^{\frac{2(\alpha+\beta)}{\alpha+1}} \quad (27)$$

where $\iota_7=(qN)^{-1}\|(L_1+I_N)\otimes P\|^2$ and $\iota_8 = (qN)^{1-((2(\alpha+\beta))/(\alpha+1))}\|(L_1+I_N)\otimes P\|^{((2(\alpha+\beta))/(\alpha+1))}$.

Based on (12) and Lemma 1, we have

$$\|(L_1\otimes P)\tilde{\eta}\|^{\frac{4\alpha}{\alpha+1}}$$
$$=\left(\|(I_N\otimes P)\hat{\omega}-(L_1\otimes P)e\|^2\right)^{\frac{2\alpha}{\alpha+1}}$$
$$\le\left(2\|(I_N\otimes P)\hat{\omega}\|^2+2\|(L_1\otimes P)e\|^2\right)^{\frac{2\alpha}{\alpha+1}}$$
$$\le\left(2\|(I_N\otimes P)\hat{\omega}\|^2\right)^{\frac{2\alpha}{\alpha+1}}+\left(2\|(L_1\otimes P)e\|^2\right)^{\frac{2\alpha}{\alpha+1}}$$
$$\le 2^{\frac{2\alpha}{\alpha+1}}\|P\|^{\frac{4\alpha}{\alpha+1}}\|\hat{\omega}\|^{\frac{4\alpha}{\alpha+1}}+2^{\frac{2\alpha}{\alpha+1}}\|L_1\otimes P\|^{\frac{4\alpha}{\alpha+1}}\|e\|^{\frac{4\alpha}{\alpha+1}}. \quad (28)$$

Similarly, by Lemma 2, one gets

$$\|(L_1\otimes P)\tilde{\eta}\|^2$$
$$\le 2\|(I_N\otimes P)\hat{\omega}\|^2+2\|(L_1\otimes P)e\|^2$$
$$=2\|P\|^2\|\hat{\omega}\|^2+2\|L_1\otimes P\|^2\|e\|^2 \quad (29)$$

$$\|(L_1\otimes P)\tilde{\eta}\|^{\frac{2(\alpha+\beta)}{\alpha+1}}$$
$$\le\left(2\|(I_N\otimes P)\hat{\omega}\|^2+2\|(L_1\otimes P)e\|^2\right)^{\frac{\alpha+\beta}{\alpha+1}}$$
$$\le(qN)^{\frac{\beta-1}{\alpha+1}}\left(2\|(I_N\otimes P)\hat{\omega}\|^2\right)^{\frac{\alpha+\beta}{\alpha+1}}$$
$$+(qN)^{\frac{\beta-1}{\alpha+1}}\left(2\|(L_1\otimes P)e\|^2\right)^{\frac{\alpha+\beta}{\alpha+1}}$$
$$\le(qN)^{\frac{\beta-1}{\alpha+1}}2^{\frac{\alpha+\beta}{\alpha+1}}\|P\|^{\frac{2(\alpha+\beta)}{\alpha+1}}\|\hat{\omega}\|^{\frac{2(\alpha+\beta)}{\alpha+1}}$$
$$+(qN)^{\frac{\beta-1}{\alpha+1}}2^{\frac{\alpha+\beta}{\alpha+1}}\|L_1\otimes P\|^{\frac{2(\alpha+\beta)}{\alpha+1}}\|e\|^{\frac{2(\alpha+\beta)}{\alpha+1}}. \quad (30)$$

Substituting (28) into (23), (29) into (25), and (30) into (26) yields

$$(\iota_4+2)\left(\mathrm{sig}(\psi)^{\frac{2\alpha}{\alpha+1}}\right)^T\mathrm{sig}(\psi)^{\frac{2\alpha}{\alpha+1}}\le\iota_9\|\hat{\omega}\|^{\frac{4\alpha}{\alpha+1}}+\iota_{10}\|e\|^{\frac{4\alpha}{\alpha+1}} \quad (31)$$

where $\iota_9 = (qN)^{((1-3\alpha)/(\alpha+1))}2^{(4\alpha/(\alpha+1))}(\iota_4+2)\|P\|^{(4\alpha/(\alpha+1))}$ and $\iota_{10} = (qN)^{((1-3\alpha)/(\alpha+1))}2^{(4\alpha/(\alpha+1))}(\iota_4+2)\|L_1\otimes$

$P\|^{(4\alpha/(\alpha+1))}+(qN)^{((1-3\alpha)/(\alpha+1))}2^{(2\alpha/(\alpha+1))}(\iota_4+2)\|((L_1+I_N)\otimes P)e\|^{(4\alpha/(\alpha+1))}$

$$\begin{aligned}&(\iota_3+2)(\operatorname{sig}(\psi))^T\operatorname{sig}(\psi)\\&\quad+(\iota_5+2)\Big(\operatorname{sig}(\psi)^{\frac{\alpha+\beta}{\alpha+1}}\Big)^T\operatorname{sig}(\psi)^{\frac{\alpha+\beta}{\alpha+1}}\\&\quad\le \iota_{11}\|\hat{\omega}\|^2+\iota_{12}\|e\|^2+\iota_{13}\|\hat{\omega}\|^{\frac{2(\alpha+\beta)}{\alpha+1}}+\iota_{14}\|e\|^{\frac{2(\alpha+\beta)}{\alpha+1}}\end{aligned}\tag{32}$$

where $\iota_{11}=4(\iota_3+2)\|P\|^2$, $\iota_{12}=2(\iota_3+2)(\|(L_1+I_N)\otimes P\|^2+2\|L_1\otimes P\|^2)$, $\iota_{13}=(qN)^{((2(\beta-1))/(\alpha+1))}2^{((2(\alpha+\beta))/(\alpha+1))}(\iota_5+2)\|P\|^{((2(\alpha+\beta))/(\alpha+1))}$, and $\iota_{14}=(qN)^{((\beta-1)/(\alpha+1))}2^{((\alpha+\beta)/(\alpha+1))}(\iota_5+2)\|(L_1+I_N)\otimes P\|^{((2(\alpha+\beta))/(\alpha+1))}+(qN)^{((2(\beta-1))/(\alpha+1))}2^{((2(\alpha+\beta))/(\alpha+1))}(\iota_5+2)\|L_1\otimes P\|^{((2(\alpha+\beta))/(\alpha+1))}$.

By substituting (24), (27), (31), and (32) into (22), it follows that:

$$\begin{aligned}&\dot{V}\\&\le \iota_1\|\tilde{\eta}\|^2+(\iota_2+\iota_7+\iota_{12})\|e\|^2+\iota_9\|\hat{\omega}\|^{\frac{4\alpha}{\alpha+1}}\\&\quad+(\iota_6+\iota_{10})\|e\|^{\frac{4\alpha}{\alpha+1}}+\iota_{11}\|\hat{\omega}\|^2+\iota_{13}\|\hat{\omega}\|^{\frac{2(\alpha+\beta)}{\alpha+1}}\\&\quad+(\iota_8+\iota_{14})\|e\|^{\frac{2(\alpha+\beta)}{\alpha+1}}-\|(L_1\otimes P)\tilde{\eta}\|^{\frac{4\alpha}{\alpha+1}}\\&\quad-(qN)^{-1}\|(L_1\otimes P)\tilde{\eta}\|^2-(qN)^{1-\frac{2(\alpha+\beta)}{\alpha+1}}\|(L_1\otimes P)\tilde{\eta}\|^{\frac{2(\alpha+\beta)}{\alpha+1}}\\&\quad-\Big(\operatorname{sig}(\psi)^{\frac{2\alpha}{\alpha+1}}\Big)^T\operatorname{sig}(\psi)^{\frac{2\alpha}{\alpha+1}}-(\operatorname{sig}(\psi))^T\operatorname{sig}(\psi)\\&\quad-\Big(\operatorname{sig}(\psi)^{\frac{\alpha+\beta}{\alpha+1}}\Big)^T\operatorname{sig}(\psi)^{\frac{\alpha+\beta}{\alpha+1}}-\sum_{i=1}^{N}\upsilon\gamma_i|\phi_i|^{2\alpha}\\&\quad-\sum_{i=1}^{N}\upsilon\sigma_i|\phi_i|^{\alpha+\beta}.\end{aligned}\tag{33}$$

Due to the fact that $\iota_1<0$ and the triggering function $f_{\eta_i}\le 0$, one has

$$\begin{aligned}\dot{V}\le&-\|(L_1\otimes P)\tilde{\eta}\|^{\frac{4\alpha}{\alpha+1}}-(qN)^{1-\frac{2(\alpha+\beta)}{\alpha+1}}\|(L_1\otimes P)\tilde{\eta}\|^{\frac{2(\alpha+\beta)}{\alpha+1}}\\&-(qN)^{-1}\|(L_1\otimes P)\tilde{\eta}\|^2-\Big(\operatorname{sig}(\psi)^{\frac{2\alpha}{\alpha+1}}\Big)^T\operatorname{sig}(\psi)^{\frac{2\alpha}{\alpha+1}}\\&-(\operatorname{sig}(\psi))^T\operatorname{sig}(\psi)-\Big(\operatorname{sig}(\psi)^{\frac{\alpha+\beta}{\alpha+1}}\Big)^T\operatorname{sig}(\psi)^{\frac{\alpha+\beta}{\alpha+1}}\\&-\sum_{i=1}^{N}\upsilon(\gamma_i-\theta_i)|\phi_i|^{2\alpha}-\sum_{i=1}^{N}\upsilon\sigma_i|\phi_i|^{\alpha+\beta}.\end{aligned}\tag{34}$$

In (34), note that

$$\begin{aligned}&-\|(L_1\otimes P)\tilde{\eta}\|^{\frac{4\alpha}{\alpha+1}}-(qN)^{1-\frac{2(\alpha+\beta)}{\alpha+1}}\|(L_1\otimes P)\tilde{\eta}\|^{\frac{2(\alpha+\beta)}{\alpha+1}}\\&\quad=-\left(\frac{\tilde{\eta}^T\left({L_1}^TL_1\otimes P^2\right)\tilde{\eta}}{\tilde{\eta}^T(I_N\otimes P)\tilde{\eta}}\tilde{\eta}^T(I_N\otimes P)\tilde{\eta}\right)^{\frac{2\alpha}{\alpha+1}}\\&\qquad-(qN)^{1-\frac{2(\alpha+\beta)}{\alpha+1}}\left(\frac{\tilde{\eta}^T\left({L_1}^TL_1\otimes P^2\right)\tilde{\eta}}{\tilde{\eta}^T(I_N\otimes P)\tilde{\eta}}\tilde{\eta}^T(I_N\otimes P)\tilde{\eta}\right)^{\frac{\alpha+\beta}{\alpha+1}}\\&\quad\le-\vartheta_1V_1^{\frac{2\alpha}{\alpha+1}}-\vartheta_2V_1^{\frac{\alpha+\beta}{\alpha+1}}\end{aligned}\tag{35}$$

where $\vartheta_1=(((\lambda_{\min}({L_1}^TL_1\otimes P^2))/(\lambda_{\max}(I_N\otimes P))))^{(2\alpha/(\alpha+1))}$ and $\vartheta_2=(qN)^{1-((2(\alpha+\beta))/(\alpha+1))}((\lambda_{\min}({L_1}^TL_1\otimes P^2))/(\lambda_{\max}(I_N\otimes P)))^{((\alpha+\beta)/(\alpha+1))}$. What is more

$$\begin{aligned}&-\Big(\operatorname{sig}(\psi)^{\frac{2\alpha}{\alpha+1}}\Big)^T\operatorname{sig}(\psi)^{\frac{2\alpha}{\alpha+1}}-\Big(\operatorname{sig}(\psi)^{\frac{\alpha+\beta}{\alpha+1}}\Big)^T\operatorname{sig}(\psi)^{\frac{\alpha+\beta}{\alpha+1}}\\&\quad=-\sum_{i=1}^{N}\sum_{j=1}^{q}\left(\left|\psi_{ij}\right|^2\right)^{\frac{2\alpha}{\alpha+1}}-\sum_{i=1}^{N}\sum_{j=1}^{q}\left(\left|\psi_{ij}\right|^2\right)^{\frac{\alpha+\beta}{\alpha+1}}\\&\quad\le-\left(\frac{2}{\omega}\right)^{\frac{2\alpha}{\alpha+1}}\left(\sum_{i=1}^{N}\sum_{j=1}^{q}\frac{\omega}{2}\left|\psi_{ij}\right|^2\right)^{\frac{2\alpha}{\alpha+1}}\\&\qquad-(qN)^{1-\frac{\alpha+\beta}{\alpha+1}}\left(\frac{2}{\omega}\right)^{\frac{\alpha+\beta}{\alpha+1}}\left(\sum_{i=1}^{N}\sum_{j=1}^{q}\frac{\omega}{2}\left|\psi_{ij}\right|^2\right)^{\frac{\alpha+\beta}{\alpha+1}}\\&\quad=-\vartheta_3V_2^{\frac{2\alpha}{\alpha+1}}-\vartheta_4V_2^{\frac{\alpha+\beta}{\alpha+1}}\end{aligned}\tag{36}$$

where $\vartheta_3=(2/\omega)^{(2\alpha/(\alpha+1))}$ and $\vartheta_4=(qN)^{1-((\alpha+\beta)/(\alpha+1))}(2/\omega)^{((\alpha+\beta)/(\alpha+1))}$. Furthermore,

$$\begin{aligned}&-\sum_{i=1}^{N}\upsilon(\gamma_i-\theta_i)|\phi_i|^{2\alpha}-\sum_{i=1}^{N}\upsilon\sigma_i|\phi_i|^{\alpha+\beta}\\&\quad\le-\upsilon\gamma_{\theta\min}\sum_{i=1}^{N}\left(|\phi_i|^{\alpha+1}\right)^{\frac{2\alpha}{\alpha+1}}-\upsilon\sigma_{\min}\sum_{i=1}^{N}\left(|\phi_i|^{\alpha+1}\right)^{\frac{\alpha+\beta}{\alpha+1}}\\&\quad\le-\gamma_{\theta\min}\frac{(\alpha+1)^{\frac{2\alpha}{\alpha+1}}}{\upsilon^{\frac{\alpha-1}{\alpha+1}}}\sum_{i=1}^{N}\left(\frac{\upsilon}{\alpha+1}|\phi_i|^{\alpha+1}\right)^{\frac{2\alpha}{\alpha+1}}\\&\qquad-\sigma_{\min}N^{\frac{1-\beta}{\alpha+1}}\frac{(\alpha+1)^{\frac{\alpha+\beta}{\alpha+1}}}{\upsilon^{\frac{\beta-1}{\alpha+1}}}\sum_{i=1}^{N}\left(\frac{\upsilon}{\alpha+1}|\phi_i|^{\alpha+1}\right)^{\frac{\alpha+\beta}{\alpha+1}}\\&\quad=-\vartheta_5V_3^{\frac{2\alpha}{\alpha+1}}-\vartheta_6V_3^{\frac{\alpha+\beta}{\alpha+1}}\end{aligned}\tag{37}$$

where $\vartheta_5=\gamma_{\theta\min}(((\alpha+1)^{(2\alpha/(\alpha+1))})/(\upsilon^{((\alpha-1)/(\alpha+1))}))$, $\gamma_{\theta\min}=\min\{\gamma_i-\theta_i\}>0$, $\vartheta_6=\sigma_{\min}N^{((1-\beta)/(\alpha+1))}(((\alpha+1)^{((\alpha+\beta)/(\alpha+1))})/(\upsilon^{((\beta-1)/(\alpha+1))}))$, and $\sigma_{\min}=\min\{\sigma_i\}>0$.

According to (35)–(37), we have

$$\begin{aligned}\dot{V}\le&-\vartheta_1V_1^{\frac{2\alpha}{\alpha+1}}-\vartheta_2V_1^{\frac{\alpha+\beta}{\alpha+1}}-\vartheta_3V_2^{\frac{2\alpha}{\alpha+1}}-\vartheta_4V_2^{\frac{\alpha+\beta}{\alpha+1}}\\&-\vartheta_5V_3^{\frac{2\alpha}{\alpha+1}}-\vartheta_6V_3^{\frac{\alpha+\beta}{\alpha+1}}.\end{aligned}\tag{38}$$

Since $0<(2\alpha/(\alpha+1))<1$ and $((\alpha+\beta)/(\alpha+1))>1$, then, we obtain

$$\begin{aligned}\dot{V}\le&-\vartheta_{\min}\left(V_1^{\frac{2\alpha}{\alpha+1}}+V_2^{\frac{2\alpha}{\alpha+1}}+V_3^{\frac{2\alpha}{\alpha+1}}\right)\\&-\vartheta_{\min}\left(V_1^{\frac{\alpha+\beta}{\alpha+1}}+V_2^{\frac{\alpha+\beta}{\alpha+1}}+V_3^{\frac{\alpha+\beta}{\alpha+1}}\right)\\\le&-\vartheta_{\min}(V_1+V_2+V_3)^{\frac{2\alpha}{\alpha+1}}\\&-\vartheta_{\min}3^{1-\frac{\alpha+\beta}{\alpha+1}}(V_1+V_2+V_3)^{\frac{\alpha+\beta}{\alpha+1}}\\=&-\vartheta_{\min}V^{\frac{2\alpha}{\alpha+1}}-\vartheta_{\min}3^{1-\frac{\alpha+\beta}{\alpha+1}}V^{\frac{\alpha+\beta}{\alpha+1}}\end{aligned}\tag{39}$$

where $\vartheta_{\min}=\min\{\vartheta_1,\vartheta_2,\vartheta_3,\vartheta_4,\vartheta_5,\vartheta_6\}$. We can know $\lim_{t\to T_1}\tilde{\eta}_i=0$ in fixed time by Lemma 3, where $T_1=((\alpha+1)/(\vartheta_{\min}(1-\alpha)))+((\alpha+1)/(\vartheta_{\min}3^{((1-\beta)/(\alpha+1))}(\beta-1)))$. Next, we prove that the Zeno behavior can be ruled out on each interval.

The general solution to (4) for $[t_k^i,t_{k+1}^i)$ is

$$\begin{aligned}\eta_i=&\,e^{S_0\left(t-t_k^i\right)}\eta_i\left(t_k^i\right)+\int_{t_k^i}^{t}e^{S_0(t-s)}\\&\times\Big(\mu_2G\operatorname{sig}(P(\hat{\omega}_i+e_i))^{\frac{2\alpha}{\alpha+1}}\end{aligned}$$

$$+ \mu_3 G \mathrm{sig}(P(\hat{\omega}_i + e_i))^{\frac{\alpha+\beta}{\alpha+1}} + \mu_1 G \hat{\omega}_i\Big) ds. \quad (40)$$

Since $e_i(t_k^i) = 0$, and by (7), we get

$$e_i = -\int_{t_k^i}^{t} e^{S_0(t-s)} \Big(\mu_1 G \hat{\omega}_i + \mu_2 G \mathrm{sig}(P(\hat{\omega}_i + e_i))^{\frac{2\alpha}{\alpha+1}} + \mu_3 G \mathrm{sig}(P(\hat{\omega}_i + e_i))^{\frac{\alpha+\beta}{\alpha+1}}\Big) ds. \quad (41)$$

e_i can be bounded by $\|e_i\|$ as follows:

$$\|e_i\| \le \left\| \int_{t_k^i}^{t} e^{S_0(t-s)} \Big(\mu_1 G \hat{\omega}_i + \mu_2 G \mathrm{sig}(P(\hat{\omega}_i + e_i))^{\frac{2\alpha}{\alpha+1}} + \mu_3 G \mathrm{sig}(P(\hat{\omega}_i + e_i))^{\frac{\alpha+\beta}{\alpha+1}}\Big) ds \right\|. \quad (42)$$

Since $\tilde{\eta}_i$ and $e_i, i = 1, \ldots, N$ are bounded on $[0, \infty)$, we know from (12) that $\hat{\omega}, i = 1, \ldots, N$ are bounded on $[0, \infty)$. Besides this, $e^{S_0(t-t_k^i)}$ is bounded on any interval $[t_k^i, t_{k+1}^i)$. Therefore, there exists a positive number ϖ that guarantees $\|e^{S_0(t-s)}(\mu_1 G \hat{\omega}_i + \mu_2 G \mathrm{sig}(P(\hat{\omega}_i + e_i))^{(2\alpha/(\alpha+1))} + \mu_3 G \cdot \mathrm{sig}(P(\hat{\omega}_i + e_i))^{((\alpha+\beta)/(\alpha+1))})\| \le \varpi$. Then, we have $\|e_i\| \le \varpi(t - t_k^i)$, for $[t_k^i, t_{k+1}^i)$. Thus, from (9a), a lower bound of interevent time $t_{k+1}^i - t_k^i$ can be obtained by the solution of τ_i

$$c_1\|\varpi\|^2\tau_i^2 + c_2\|\varpi\|^{\frac{4\alpha}{\alpha+1}}\tau_i^{\frac{4\alpha}{\alpha+1}} + c_3\|\varpi\|^{\frac{2(\alpha+\beta)}{\alpha+1}}\tau_i^{\frac{2(\alpha+\beta)}{\alpha+1}} + c_4\chi^2 + c_5\chi^{\frac{4\alpha}{\alpha+1}} + c_6\chi^{\frac{2(\alpha+\beta)}{\alpha+1}} = \upsilon\theta_i|\phi_i|^{2\alpha} \quad (43)$$

where $\|\hat{\omega}\| \le \chi$. From (43), we know that τ_i is positive if $\phi_i \neq 0$. However, if ϕ_i converge to 0 in fixed time, τ_i is not strictly positive. Therefore, the Zeno behavior cannot be totally ruled out in this case.

Remark 6: From Theorem 1, we have proved that $\lim_{t\to T_1} \tilde{\eta}_i = 0$ can be achieved in fixed time $T_1 = ((\alpha+1)/(\vartheta_{\min}(1-\alpha))) + ((\alpha+1)/(\vartheta_{\min}3^{((1-\beta)/(\alpha+1))}(\beta-1)))$. From (9b), it is clear that $\lim_{t\to T_\phi}\phi_i = 0$ in fixed time $T_\phi = (1/(\gamma_{\min}(1-\alpha))) + (1/(\sigma_{\min}(\beta-1)))$, where $\gamma_{\min} = \min\{\gamma_1, \ldots, \gamma_N\}$ and $\sigma_{\min} = \min\{\sigma_1, \ldots, \sigma_N\}$. Therefore, the Zeno behavior can be ruled out when $T_1 < T_\phi$. We can choose the parameters associated with T_1, T_ϕ to guarantee $T_1 < T_\phi$ to excluded the Zeno behavior. This is under the condition of $T_1 < T_\phi$, and τ_i is positive over each interval $[t_k^i, t_{k+1}^i)$, which implies that the Zeno behavior is conditionally excluded over each interval $[t_k^i, t_{k+1}^i)$.

Theorem 2: Consider the MASs described by (1) and (2), and under Assumptions 1–5, the output regulation problem can be addressed using the following distributed state feedback controller (44) with the distributed fixed-time event-triggered compensator (4) and event-triggering mechanism (9):

$$u_i = K_{1i}x_i + K_{2i}\eta_i - \delta_1 K_{3i}\mathrm{sig}\big(x_i - X_i^*\eta_i\big)^{\ell_1} - \delta_2 K_{3i}\mathrm{sig}\big(x_i - X_i^*\eta_i\big)^{\ell_2} \quad (44)$$

where $\delta_1, \delta_2 > 0$. $\ell_1 \in (0, 1)$, $\ell_2 > 1$. K_{1i}, K_{2i}, and K_{3i} are the constant matrices meeting the condition that $A_i + B_i K_{1i}$ is Hurwitz and $B_i K_{3i} = I_n$. $K_{2i} = U_i^* - K_{1i}X_i^*$, and (X_i^*, U_i^*) is the solution of (3).

Proof: If $\tilde{x}_i = x_i - X_i^*\upsilon$, $\tilde{u}_i = u_i - U_i^*\upsilon$ and $K_{2i} = U_i^* - K_{1i}X_i^*$, for $t \ge T_1$, then we have

$$\begin{aligned}\tilde{u}_i &= K_{1i}\tilde{x}_i + K_{2i}\tilde{\eta}_i - \delta_1 K_{3i}\mathrm{sig}\big(\tilde{x}_i - X_i^*\tilde{\eta}_i\big)^{\ell_1} - \delta_2 K_{3i}\mathrm{sig}\big(\tilde{x}_i - X_i^*\tilde{\eta}_i\big)^{\ell_2} \\ &= K_{1i}\tilde{x}_i - \delta_1 K_{3i}\mathrm{sig}(\tilde{x}_i)^{\ell_1} - \delta_2 K_{3i}\mathrm{sig}(\tilde{x}_i)^{\ell_2}. \end{aligned} \quad (45)$$

Based on (45), for $t \ge T_1$, we can obtain

$$\begin{aligned}\dot{\tilde{x}}_i &= A_i\tilde{x}_i + B_i\tilde{u}_i + \big(A_iX_i^* + B_iU_i^* + F_i - X_i^*S_0\big)\upsilon \\ &= A_i\tilde{x}_i + B_i\big[K_{1i}\tilde{x}_i - \delta_1 K_{3i}\mathrm{sig}(\tilde{x}_i)^{\ell_1} - \delta_2 K_{3i}\mathrm{sig}(\tilde{x}_i)^{\ell_2}\big] \\ &= T_i\tilde{x}_i - \delta_1\mathrm{sig}(\tilde{x}_i)^{\ell_1} - \delta_2\mathrm{sig}(\tilde{x}_i)^{\ell_2} \end{aligned} \quad (46)$$

where $T_i = A_i + B_iK_{1i}$, and we have used the fact that $B_iK_{3i} = I_n$.

Construct the Lyapunov function $V_4 = \tilde{x}_i^T\tilde{x}_i$, and due to the fact that $T_i = A_i + B_iK_{1i}$ is Hurwitz, the derivative of V_4 along (46) is

$$\begin{aligned}\dot{V}_4 &= 2\tilde{x}_i^T T_i\tilde{x}_i - 2\delta_1\big(\tilde{x}_i^T\tilde{x}_i\big)^{\frac{1+\ell_1}{2}} - 2\delta_2\big(\tilde{x}_i^T\tilde{x}_i\big)^{\frac{1+\ell_2}{2}} \\ &\le -2\delta_1 V_4^{\frac{1+\ell_1}{2}} - 2\delta_2 V_4^{\frac{1+\ell_2}{2}}. \end{aligned} \quad (47)$$

By Lemma 3, $\lim_{t\to T_1+T_2}\tilde{x}_i = 0$ in fixed time, where $T_2 = (1/(\delta_1(1-\ell_1))) + (1/(\delta_2(\ell_2-1)))$. It implies that $\lim_{t\to T_1+T_2}\tilde{u}_i = 0$.

Finally, by (3), one has

$$\tilde{e}_i = C_i\tilde{x}_i + D_i\tilde{u}_i. \quad (48)$$

Thus, following from $\lim_{t\to T_1+T_2}\tilde{x}_i = 0$ and $\lim_{t\to T_1+T_2}\tilde{u}_i = 0$, we can conclude that $\lim_{t\to T_1+T_2}\tilde{e}_i = 0$ in fixed time $T_1 + T_2$. This completes the proof.

Remark 7: The following procedure is used to describe the key idea of proving Theorems 1 and 2: 1) in Theorem 1, consider the dynamic compensator (4) and (5) and exosystem (2), the compensator state can estimate the exosystem state for each agent in fixed time is verified, that is, $\lim_{t\to T_1}(\eta_i - \upsilon) = 0$, for $i = 1, \ldots, N$; 2) consider the event-triggered mechanism (8) and show that the Zeno behavior can be conditionally excluded; and 3) in Theorem 2, consider the MASs (1), exosystem (2), distributed controller (4) and (44), and event-triggered mechanism (9) and show that the regulated output for each agent converge to zero in fixed time, that is, $\lim_{t\to T_1+T_2}\tilde{e}_i = 0$, for $i = 1, \ldots, N$.

Remark 8: It is remarkable that the proposed distributed event-triggered control strategy in this article includes controller (4) and (44) and event-triggered mechanism (9), which makes the estimation error and regulated output of each agent tend to zero within fixed time. This strategy has significant advantages over the existing event-triggered control strategies in [18]–[20] and [33]–[35].

Remark 9: Leader-following consensus and synchronization of linear MASs can be treated as special cases of the considered cooperative output regulation problem in this article. As applications, the result shows that the strategy in this article is suitable for the leader-following consensus and synchronization problems of some linearized MASs, such as multiple nonholonomic mobile robots [36].

As a reminder, the controller (44) is designed based on the system states. However, due to practical constraints such as technical or external disturbances, not all agents' states are available. In view of this case, Theorem 3 is the output regulation problem with the distributed observer-based output feedback controller.

Theorem 3: Consider the MASs described by (1) and (2), and under Assumptions 1–5, the output regulation problem can be addressed using the following observer-based output feedback controller (49) with the distributed fixed-time event-triggered compensator (4) and event-triggering mechanism (9):

$$\begin{aligned} u_i &= K_{1i}\zeta_i + K_{2i}\eta_i - \delta_1 K_{3i}\mathrm{sig}\big(\zeta_i - X_i^*\eta_i\big)^{\ell_1} \\ &\quad - \delta_2 K_{3i}\mathrm{sig}\big(\zeta_i - X_i^*\eta_i\big)^{\ell_2} \\ \dot{\zeta}_i &= A_i\zeta_i + B_i u_i + F_i\eta_i + J_i(C_{mi}\zeta_i + D_{mi}u_i + F_{mi}\eta_i - y_{mi}) \\ &\quad - \varepsilon_{i1}\mathrm{sig}(R_i(C_{mi}\zeta_i + D_{mi}u_i + F_{mi}\eta_i - y_{mi}))^{\rho_1} \\ &\quad - \varepsilon_{i2}\mathrm{sig}(R_i(C_{mi}\zeta_i + D_{mi}u_i + F_{mi}\eta_i - y_{mi}))^{\rho_2} \end{aligned} \tag{49}$$

where $\zeta_i \in \mathbb{R}^n$ is the observer state; $\delta_1, \delta_2, \varepsilon_{i1}, \varepsilon_{i2} > 0$. $\ell_1, \rho_1 \in (0,1)$. $\ell_2, \rho_2 > 1$; and $K_{1i}, K_{2i}, K_{3i}, J_i$, and R_i are the constant matrices meeting the condition that $A_i + B_iK_{1i}$ and $A + J_iC_{mi}$ are Hurwitz, $B_iK_{3i} = I_n$, and $R_iC_{mi} = I_n$. $K_{2i} = U_i^* - K_{1i}X_i^*$, and (X_i^*, U_i^*) is the solution of (3).

Proof: If $\hat{x}_i = \zeta_i - x_i$, one has

$$\begin{aligned} \dot{\hat{x}}_i &= A_i\hat{x}_i + F_i\tilde{\eta}_i + J_i(C_{mi}\hat{x}_i + F_{mi}\tilde{\eta}_i) \\ &\quad - \varepsilon_{i1}\mathrm{sig}(R_i(C_{mi}\hat{x}_i + F_{mi}\tilde{\eta}_i))^{\rho_1} \\ &\quad - \varepsilon_{i2}\mathrm{sig}(R_i(C_{mi}\hat{x}_i + F_{mi}\tilde{\eta}_i))^{\rho_2} \\ &= \Gamma_i\hat{x}_i + (F_i + J_iF_{mi})\tilde{\eta}_i - \varepsilon_{i1}\mathrm{sig}(\hat{x}_i + R_iF_{mi}\tilde{\eta}_i)^{\rho_1} \\ &\quad - \varepsilon_{i2}\mathrm{sig}(\hat{x}_i + R_iF_{mi}\tilde{\eta}_i)^{\rho_2} \end{aligned} \tag{50}$$

where $\Gamma_i = A + J_iC_{mi}$, and we have used the fact that $B_iK_{3i} = I_n$. According to Theorem 1, we have that $\tilde{\eta}_i = 0$ holds in fixed time T_1, that is, for $t \geq T_1$, we can get

$$\dot{\hat{x}}_i = \Gamma_i\hat{x}_i - \varepsilon_{i1}\mathrm{sig}(\hat{x}_i)^{\rho_1} - \varepsilon_{i2}\mathrm{sig}(\hat{x}_i)^{\rho_2}. \tag{51}$$

As $\Gamma_i = A + J_iC_{mi}$ is Hurwitz and similar to the analysis of (46), we can deduce $\lim_{t\to T_1+T_3}\hat{x}_i = 0$ in fixed time, where $T_3 = (1/(\varepsilon_{\min}(1-\rho_1))) + (1/(\varepsilon_{\min}(\rho_2 - 1)))$ and $\varepsilon_{\min} = \min\{\varepsilon_{11}, \varepsilon_{21}, \ldots, \varepsilon_{N1}, \varepsilon_{12}, \varepsilon_{22}, \ldots, \varepsilon_{N2}\}$.

If $\tilde{x}_i = x_i - X_i^*\upsilon$, $\tilde{u}_i = u_i - U_i^*\upsilon$ and $K_{2i} = U_i^* - K_{1i}X_i^*$, for $t \geq T_1 + T_3$, we have

$$\begin{aligned} \tilde{u}_i &= K_{1i}\hat{x}_i + K_{1i}\tilde{x}_i + K_{2i}\tilde{\eta}_i - \delta_1K_{3i}\mathrm{sig}\big(\tilde{x}_i + \hat{x}_i - X_i^*\tilde{\eta}_i\big)^{\ell_1} \\ &\quad - \delta_2K_{3i}\mathrm{sig}\big(\tilde{x}_i + \hat{x}_i - X_i^*\tilde{\eta}_i\big)^{\ell_2} \\ &= K_{1i}\tilde{x}_i - \delta_1K_{3i}\mathrm{sig}(\tilde{x}_i)^{\ell_1} - \delta_2K_{3i}\mathrm{sig}(\tilde{x}_i)^{\ell_2}. \end{aligned} \tag{52}$$

Based on (52), for $t \geq T_1 + T_3$, we can obtain

$$\begin{aligned} \dot{\tilde{x}}_i &= A_i\tilde{x}_i + B_i\tilde{u}_i + \big(A_iX_i^* + B_iU_i^* + F_i - X_i^*S_0\big)\upsilon \\ &= A_i\tilde{x}_i + B_i\big[K_{1i}\tilde{x}_i - \delta_1K_{3i}\mathrm{sig}(\tilde{x}_i)^{\ell_1} - \delta_2K_{3i}\mathrm{sig}(\tilde{x}_i)^{\ell_2}\big] \\ &= (A_i + B_iK_{1i})\tilde{x}_i - \delta_1\mathrm{sig}(\tilde{x}_i)^{\ell_1} - \delta_2\mathrm{sig}(\tilde{x}_i)^{\ell_2}. \end{aligned} \tag{53}$$

As $A_i + B_iK_{1i}$ is Hurwitz and similar to the analysis of (46), we can deduce $\lim_{t\to T_1+T_2+T_3}\tilde{x}_i = 0$ in fixed time. It implies that $\lim_{t\to T_1+T_2+T_3}\tilde{u}_i = 0$. Then, we can conclude that $\lim_{t\to T_1+T_2+T_3}\tilde{e}_i = 0$ in fixed time $T_1 + T_2 + T_3$. This completes the proof.

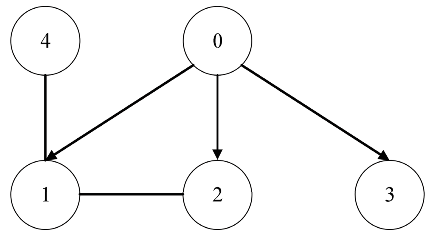

Fig. 1. Communication topology.

Remark 10: We can see that the designed controller (49) proposed in Theorem 3 requires strict of system dynamic, that is, when the agents' states are not available, the system dynamic is required to satisfy $B_iK_{3i} = I_n$ and $R_iC_{mi} = I_n$. Therefore, how to relax these requirements is the important aspect to investigate in the future.

IV. Simulation Example

Consider the cooperative output regulation problem of multiple wheeled mobile robots borrowed from [45], where the dynamics of each wheeled mobile robots are modeled by

$$\begin{aligned} \dot{x}_i &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & k_i \\ 0 & -m_i & -n_i \end{bmatrix} x_i + \begin{bmatrix} s_i & r_i & 0 \\ 0 & d_i & 0 \\ 0 & 0 & -g_i \end{bmatrix} u_i + F_i\upsilon \\ y_{mi} &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x_i \\ \tilde{e}_i &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x_i + E_i\upsilon, \quad i = 1, \ldots, 4 \end{aligned}$$

and the exosystem

$$\dot{\upsilon} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \upsilon$$

where $F_i = [-0.5i \;\; 0;\; -1 \;\; 0.5i;\; 0 \;\; 0]$, $i = 1, \ldots, 4$; $E_1 = E_2 = [-1 \;\; 0]$; $E_3 = [-1.5 \;\; -1]$; $E_4 = [-2 \;\; -1]$; and parameters $\{k_i, m_i, n_i, s_i, r_i, d_i, g_i\}$, $i = 1, \ldots, 4$ are selected as $\{1, 0, 1, 1, 0, 1, 1\}$, $\{1, 0, 10, 1, 0, 1, 1\}$, $\{1, 10, 2, 1, 0, 1, 1\}$ and $\{1, 1, 2, 1, 0, 1, 1\}$. Let agent 0 be denoted the exosystem. The communication graph of agents is shown in Fig. 1. Assumptions 1–5 are evidently satisfied. Thus, the distributed fixed-time event-based protocols can be used to solve the cooperative output regulation problem for heterogeneous linear MASs. Solving the regulator equations (3) yields $X_1^* = [1 \;\; 0;\; 0 \;\; 1;\; 1 \;\; -1.5]$, $X_2^* = [1 \;\; 0;\; 0 \;\; 1;\; 1 \;\; 1]$, $X_3^* = [1.5 \;\; 1;\; 0 \;\; 1;\; 1 \;\; 1]$, $X_4^* = [2 \;\; 1;\; 0 \;\; 1;\; 1 \;\; 1]$, $U_1^* = [0.5 \;\; 0;\; 0 \;\; 1;\; 1 \;\; -0.5]$, $U_2^* = [1 \;\; 0;\; 0 \;\; -2;\; 10 \;\; 11]$, $U_3^* = [1.5 \;\; 0.5;\; 0 \;\; -2.5;\; 2 \;\; 13]$, and $U_4^* = [2 \;\; 1;\; 0 \;\; -3;\; 2 \;\; 4]$. By solving the inequation (10), we can obtain $P = [0.9102 \;\; 0.4142;\; 0.4142 \;\; 1.2872]$ and $G = -P$. The gain matrices in (44) are given as $K_{1i} = [-28 \;\; -28 \;\; -28;\; 0 \;\; -28 \;\; -1;\; 0 \;\; -28 \;\; -28]$, $K_{2i} = U_i^* - K_{1i}X_i^*$, and $K_{3i} = [1 \;\; 0 \;\; 0;\; 0 \;\; 1 \;\; 0;\; 0 \;\; 0 \;\; 1]$, $i = 1, \ldots, 4$. Moreover, the other parameters are set as $\mu_1 = 4.2$, $\mu_2 = 0.8$, $\mu_3 = 1.2$, $\alpha = 0.25$, $\beta = 1.2$, $\omega = 0.0015$, $\nu = 100$, $\theta_i = 3.5$, $\sigma_i = 0.003$, $\gamma_i = 5$, $\ell_1 = 0.5$, $\ell_2 = 1.5$, $\delta_1 = 2$, and $\delta_2 = 2$.

The initial states $x(0)$, $\upsilon(0)$, and η are randomly chosen over $(0, 1)$ and the simulation results are given in Figs. 2–4.

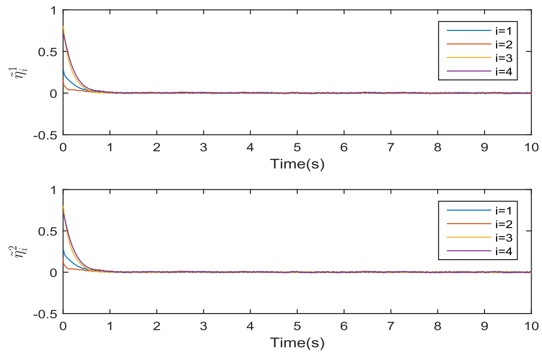

Fig. 2. Compensator errors $\eta_i - \upsilon = [\tilde{\eta}_i^1; \tilde{\eta}_i^2]$, $i = 1, \ldots, 4$.

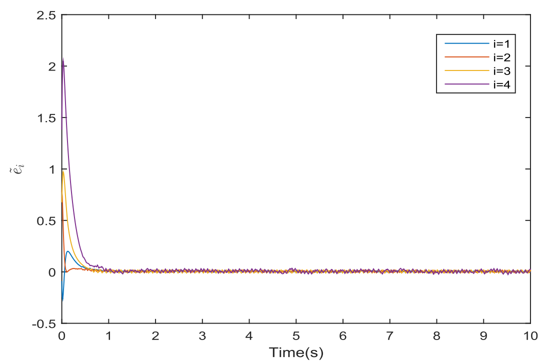

Fig. 3. Regulated outputs $\tilde{e}_i$ of four agents, $i = 1, \ldots, 4$.

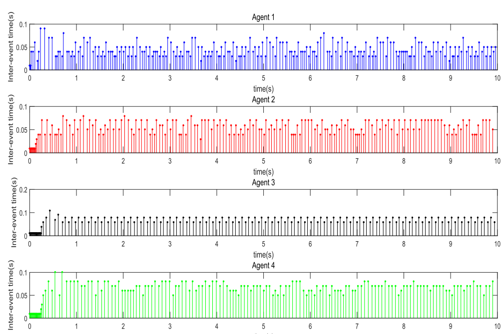

Fig. 4. Interevent times of four agents.

Fig. 2 shows the compensator errors of four agents, that is, $\eta_i - \upsilon = [\tilde{\eta}_i^1; \tilde{\eta}_i^2]$, $i = 1, \ldots, 4$. We then deduce that the compensator errors approach to zero in fixed time. Fig. 3 shows the regulated outputs of four agents. We then see that the regulated outputs approach to zero in fixed time. Furthermore, Fig. 4 shows the interevent times of four agents in the [0 s, 10 s] and the minimum interevent times of agent $i = 1, \ldots, 4$ are 0.04, 0.07, 0.08, and 0.07 s, respectively, which implies that the Zeno behavior is excluded.

V. Conclusion

This article addressed the fixed-time cooperative output regulation problem for linear MASs via the distributed event-triggered control. A novel distributed fixed-time event-triggered compensator is presented for each agent, which overcomes the problem that only the follower agent connected to the leader can obtain the information of the leader. Moreover, with the novel triggering function, the measurement error and the relative estimate are needed to be monitored by the agent, and continuous communication between agents and their neighbors is entirely avoidable. Then, a novel fixed-time event-triggered control protocol is proposed using a dynamic compensator method and the cooperative output regulation problem is addressed in fixed time without regard to any initial conditions. As an extension, for the case that the states are not available, the output regulation problem can further be addressed by the distributed observer-based output feedback controller with the event-triggered mechanism in fixed time. Finally, a simulation example has been made to demonstrate the theoretical results. In our work, we considered the fixed topology. Switching topology communication will be investigated in the future.

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