Resilient Bipartite Consensus of Second-Order Multiagent Systems With Event-Triggered Communication

Jiahui Wu, Yunru Zhu, Yuanshi Zheng, *Member, IEEE*, and Huaizhu Wang

***Abstract*—This article investigates the resilient bipartite consensus problem for continuous-time second-order multiagent systems in the presence of totally bounded malicious nodes under signed digraphs. An event-based resilient impulsive algorithm is employed, which cannot only mitigate the malicious nodes' influence on the convergence of normal ones but also reduce the communication loads of agents. A necessary and sufficient condition related to the network topology is established for solving resilient bipartite consensus by using system transformation. A numerical simulation illustrates the effectiveness of the result.**

***Index Terms*—Event-triggered communication, impulsive control, resilient bipartite consensus, second-order dynamics.**

I. Introduction

MULTIAGENT systems (MASs) can accomplish complex tasks through distributed collaboration of a group of agents with relatively limited capabilities. The coordination control of MASs has received a lot of attention owing to its wide application in consensus [1]–[3], formation control [4], clock synchronization [5], rendezvous [6], etc. In recent years, network and information technology have developed rapidly and researchers have attached more and more importance to the security of the network. Some fragile agents can easily become malicious agents that do not follow the predefined protocol in an unreliable environment. These malicious agents may try to mislead normal nodes to fail to converge or enter an unsafe range. As such, the resilience of consensus with malicious agents has aroused widespread concern. Particularly, it aims to design resilient algorithms to reduce malicious' influence and ensure the agreement among normal agents.

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The resilient consensus problems are essentially fault-tolerant problems. Redundant nodes and redundant connection edges are the primary means to tolerate malicious agents. In the early researches [7], [8], connectivity was used to characterize network topology conditions with sufficient redundant information. However, the traditional connectivity metric is not sufficient to describe networks with only local interaction. In [9], a new metric named *graph robustness* is proposed by Leblancn *et al.* to characterize the network structure. Using the mean-subsequence-reduced (MSR) algorithm, sufficient and necessary conditions using the notion of graph robustness were presented to ensure the resilient consensus for first-order MASs in discrete-time domain. In [10], the authors presented a quantized version of MSR algorithm for first-order MASs. The resilient group consensus was addressed based on MSR-type algorithm in [11]. In [12], the resilient consensus in a leader–follower architecture was investigated. In [13] and [14], Zheng *et al.* proposed the concept of hybrid consensus for discrete- and continuous-time dynamical agents, and then the resilient hybrid consensus for first-order hybrid MASs is presented in [15]. For a switched MAS, consensus problem is considered using Lyapunov function in [16] and resilient consensus problem is considered using a filtering algorithm in [17]. In [18] and [19], the resilient consensus problems are investigated for second-order MASs without and with time delays, respectively. In [20], a resilient impulsive algorithm is proposed for consensus of a multiagent network with second-order agents. In [21], resilient consensus condition was obtained for heterogeneous MASs composed of first-order agents and second-order agents.

Although there are numerous existing results on resilient consensus for MASs with various dynamics and network topologies, as far as we know, there is little work on event-triggered strategy for resilient consensus. In fact, when the sampled data fluctuates a little, such a time-triggered sampling mechanism can lead to excessive consumption of communication and computing resources. For this shortcoming, event-triggered idea provides a solution. Its basic idea is to drive some actions of the system through events rather than time. Tabuada *et al.* [22] first proposed the event-triggered idea to reasonably allocate the energy consumption in the network control system. Dimarogonasn *et al.* [23] then introduced it to solve the consensus of MASs. Roughly speaking, according to the trigger response's action, event-triggered schemes include event-triggered sampling, event-triggered control, event-triggered communication. Based on event-triggered sampling, Li *et al.* [24] studied the leader-following consensus. In [25], consensus of MASs with communication delays was investigated by employing event-triggered control. In [26], both event-triggered communication and event-triggered control were adopted to solve average consensus. Recently, there are some work employing event-triggered strategy to solve resilient consensus for MASs under different attacks (see [27]–[32]). In [27] and [28], resilient consensus protocols under event-based communication were designed for discrete-time first-order MAS in the presence of malicious nodes without and with quantized information, respectively.

In the (resilient) consensus problem, (normal) agents are all cooperative, which is characterized by nonnegative weight edges. In the real world, however, the relationship between agents is more complicated. It makes more sense to think of networks that have both cooperative and competitive relationships. Such cooperative-competitive networks are ubiquitous on social networks, such as two teams competing on a sports field and two companies in the same business field. The interaction among agents in cooperative-competitive networks is generally described by signed graphs, where some edges weights can be negative. In 2013, Altafini [33] studied the consensus of first-order MASs on signed graphs and proposed the concept of bipartite consensus, which refers to all agents converge to two opposite values. The author showed that under a connected signed graph, the structurally balanced graph is necessary and sufficient for achieving bipartite consensus. In [34], the authors made an analysis on equivalence of bipartite and ordinary consensus for linear MASs. In [35], a nonlinear consensus protocol was proposed for finite-time bipartite consensus. Until now, many interesting results on the bipartite consensus problem have been obtained under different contexts, such as input saturation [36], time-varying network [37], time delay [38], quantization [39], and heterogeneous dynamics [40].

Motivated by the abovementioned work, we studies the resilient bipartite consensus of second-order MASs under signed graphs with up to f malicious agents. The event-triggered communication strategy and impulsive control protocol based on only-position measurement are applied to the system. The existence of malicious agents, more complicated dynamics of agents and network topology, and the state errors causing by the event-triggered communication make the problem difficult. The main contributions of this article are three aspects: First, the resilient consensus of MASs under cooperative network is extended to the cooperative-competitive network, which is more practical in real-world applications. Second, an impulsive resilient algorithm with event-triggered communication is proposed for resilient bipartite consensus. Compared with resilient impulsive algorithm in [20], our algorithm significantly reduces the communication loads of agents. Different from [27], we deal with agents with continuous-time second-order dynamics, which are more complicated compared with the first-order case. Finally, a necessary and sufficient condition for normal agents to achieve bipartite consensus is obtained through system transformation.

The rest of this article is organized as follows. In Section II, we introduce the preliminaries and problem set. Section III contains our main results. Section IV gives a simulation example. Finally, Section V concludes this article.

Notation: Throughout this article, the sets of integers, real numbers, nonnegative integers and nonnegative real numbers are denoted by $\mathbb{Z}$, $\mathbb{R}$, $\mathbb{Z}_{\geq 0}$, and $\mathbb{R}_{\geq 0}$, respectively. The number of elements in the set S is represented by $|S|$. Let $S_1 \backslash S_2 = \{s : s \in S_1 \cap s \notin S_2\}$. The empty set is denoted $\emptyset$. $\operatorname{sgn}(\cdot)$ is the sign function.

II. Preliminaries and Problem Set

A. Signed Digraph

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ be a signed digraph, where $\mathcal{V} = \{1, 2, \ldots, n\}$ is the node set, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n \times n}$ is the adjacency weight matrix. If the edge $(j, i) \in \mathcal{E}$, then $a_{ij} \neq 0$, otherwise $a_{ij} = 0$. We assume that $a_{ii} = 0$, $\forall i \in \mathcal{V}$. Unlike unsigned digraphs, the edge weights in signed digraphs can be positive or negative. $\mathcal{G}$ is digon sign-symmetric if $a_{ij} a_{ji} \geq 0$, $\forall i, j \in \mathcal{V}$. In this article, we always hold that the $\mathcal{G}$ is digon sign-symmetric. Let $N_i = \{j : (j, i) \in \mathcal{E}\}$ be the set of in-neighbors of node i, $N_i^{out} = \{j : (i, j) \in \mathcal{E}\}$ be the set of out-neighbors of node i. For a subset $\mathcal{S} \subseteq \mathcal{V}$, a node $i \in \mathcal{S}$, if $j \in N_i \cap (\mathcal{V} \backslash \mathcal{S})$, then j is called the outer in-neighbor of i about $\mathcal{S}$, if $j \in N_i \cap \mathcal{S}$, then j is called the inner in-neighbor of i about $\mathcal{S}$. A *path* is a sequence of edges $(i_{s_1}, i_{s_2}), (i_{s_2}, i_{s_3}), \ldots, (i_{s_k}, i_{s_{k+1}})$, where $i_{s_j} \in \mathcal{V}$. $\mathcal{G}$ is said to have a *spanning tree* if there is a node in $\mathcal{G}$, such that from it to every other node, there exists a path.

Definition 1 (see [33]): Consider a signed digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, we say it is structurally balanced if there exist two disjoint subsets $\mathcal{V}_1, \mathcal{V}_2 \in \mathcal{V}$ with $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$, such that $a_{ij} \geq 0$ if i, j in the same subset, and $a_{ij} \leq 0$, otherwise.

Lemma 1 (see [33]): A signed digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ is structurally balanced if and only if there exists a diagonal matrix P with diagonal elements either $+1$ or -1 such that all elements of P^{-1}AP are nonnegative, that is, $\sigma_i \sigma_j a_{ij} \geq 0$.

Next, we will introduce the related concept of graph robustness. It plays an essential role in the following proof of resilient bipartite consensus.

Definition 2 (see [9]): A digraph $\mathcal{G}$ is (r, s)-robust if for any two nonempty and disjoint subsets $S_1, S_2 \subseteq \mathcal{V}$, at least one of the three conditions is satisfied

1) $|\mathcal{X}_{S_1}^r| = |S_1|$; 2) $|\mathcal{X}_{S_2}^r| = |S_2|$; 3) $|\mathcal{X}_{S_1}^r| + |\mathcal{X}_{S_2}^r| \geq s$ where $\mathcal{X}_{S_i}^r = \{i \in S_i : |N_i \backslash S_i| \geq r\}, i \in \{1, 2\}$.

Lemma 2 (see [10]): $\mathcal{G}$ is (1,1)-robust if and only if $\mathcal{G}$ has a spanning tree.

B. Event-Based Bipartite Consensus Protocol

Consider a group of agents with double integrator dynamics

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = u_i(t) \end{cases} \tag{1}$$

where $i \in \mathcal{V}$, $x_i(t)$, $v_i(t)$, $u_i(t) \in \mathbb{R}$ are the position state, velocity state, and control input of node i at time t, respectively.

All the results in this article still hold for $x_i(t)$, $v_i(t)$, $u_i(t) \in \mathbb{R}^m (m > 0)$ by using the property of Kronecker product.

Impulsive control strategy, with the virtue of robustness, fast transient and less energy, has been widely used for solving consensus. Considering that in some scenarios, the velocity state values of neighbor nodes are not available for technical reasons, the following impulsive control protocol using position-only measurements is proposed in [41]

$$u_i(t) = \sum_{k=1}^{\infty}\left[- k_1\sum_{j\in N_i} a_{ij}\left(x_i(t) - x_j(t)\right) - k_2\left(x_i(t) - x_i(t_{k-1})\right)\right]\delta(t - t_k) \tag{2}$$

where $k_1 > 0$, $k_2 > 0$ are control gains, $\delta(\cdot)$ is Dirac function, t_k is impulsive instant. The impulsive instants sequence is denoted as $\{t_k|_{k=1}^{\infty}\}$, where $0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots$, $\lim_{k\to+\infty} t_k = \infty$. Let $t_{k+1} - t_k \equiv h > 0$ for $k \in \mathbb{Z}_{\geq 0}$, $h > 0$ is the sampling interval.

Although the impulsive control protocol can avoid continuous communication, it is generally believed that such a time-triggered sampling method still has the disadvantage of causing excessive consumption of communication resources and computing resources, especially when the system states are almost in equilibrium [22]. To avoid this shortcoming, we adopt an event-triggered communication mechanism, which can significantly reduce the communication loads of nodes.

Different from impulsive control protocol (2), where node i sends its state value to all of its out-neighbors at every impulsive instant, in the proposed event-based impulsive control protocol (4), we design an event-triggered communication rule, where node i sends its state value to its out-neighbors only at the impulsive instants when the event-triggered condition (ETC) is satisfied. Let $\hat{x}_i(t_k)$ store the latest broadcast value of i at time t_k, that is, the state value of node i at the last event trigger time. Denoted by $t_k^i, k = 0, 1, \ldots$ the time when event of node i is activated, which satisfies $0 = t_0^i < t_1^i < \ldots$. Thus, we have $\hat{x}_i(t_k) = x_i(t_l^i)$, $t_k \in [t_l^i, t_{l+1}^i)$. More specifically, the outline of the scheme is as follows.

At each impulsive instant t_k, each normal node i checks if its ETC is satisfied. If the ETC is satisfied, it will broadcast its state value $x_i(t_k)$ to all out-neighbors, then $\hat{x}_i(t_k) = x_i(t_k)$. Otherwise, it will not broadcast $x_i(t_k)$ to out-neighbors, thus, $\hat{x}_i(t_k) = \hat{x}_i(t_{k-1})$. Dependent on its own state $x_i(t_k)$, $x_i(t_{k-1})$ and the latest received values of neighbors $\hat{x}_j(t_k)$, $j \in N_i$, normal agent i updates its control input $u_i(t_k)$. We assume $\hat{x}_i(t_0) = x_i(t_0)$, $i \in \mathcal{V}$. Now, we introduce the event-triggered function. Define the position measurement error $e_i(t_k) = \hat{x}_i(t_k) - x_i(t_{k+1})$ for $k \geq 0$. If $|e_i(t_k)|$ is small, it means that the new state $x_i(t_{k+1})$ of node i has almost the same effect on its out-neighbors as the latest information it sent. Therefore, it is not necessary to broadcast the new state. Inspired by the ETC in [42], we adopt the following ETC:

$$f_i(t_k) = |e_i(t_k)| - \left(c_0 + c_1 e^{-\alpha k}\right) > 0 \tag{3}$$

where c_0, c_1, and $\alpha \geq 0$ are constants predefined according to actual requirements.

Considering the cooperative-competitive network denoted by a signed digraph and the above event-triggered communication scheme, we propose the following control protocol based on protocol (2)

$$u_i(t) = \sum_{k=1}^{\infty}\Bigg[- k_1\sum_{j\in N_i} |a_{ij}|\left(x_i(t) - \operatorname{sgn}(a_{ij})\hat{x}_j(t)\right) - k_2\left(x_i(t) - x_i(t_{k-1})\right)\Bigg]\delta(t - t_k) \tag{4}$$

where $|a_{ij}|$ is the absolute value of the edge weight, $\hat{x}_j(t)$ represents node is the latest broadcast state value at t, and the meaning of other parameters is shown in protocol (2).

C. Threat Model and Resilient Bipartite Consensus Algorithm

In practical applications, there may be malicious nodes in the MAS. Therefore, two types of nodes are considered in this article: normal nodes and malicious nodes. Let $\mathcal{V}^N$ be the set of normal nodes, which always follow the protocol (4), $\mathcal{V}^M$ be the set of malicious nodes, which can update arbitrarily. Assume they satisfy $\mathcal{V}^N \cup \mathcal{V}^M = \mathcal{V}$, $\mathcal{V}^N \cap \mathcal{V}^M = \emptyset$, $|\mathcal{V}^N| = m$, $|\mathcal{V}^M| = n - m$.

Definition 3 (see [9]): The network $\mathcal{G}$ is said to under the f-total threat model if there are no more than f malicious nodes in the entire network, i.e., $|\mathcal{V}^M| \leq f$, $f \in \mathbb{Z}_{\geq 0}$.

Definition 4: Given a constant $e \geq 0$, we say the MAS (1) with malicious nodes achieves resilient bipartite consensus at the error level e if the following conditions are satisfied for any initial state values.

C1) *Safety condition:* There exists an interval Ω such that $x_i(t) \in \Omega$ for $t \geq 0, \forall i \in \mathcal{V}^N$.

C2) *Bipartite consensus condition:* For $\forall i, j \in \mathcal{V}^N$, there is

$$\limsup_{t\to\infty} ||x_i(t)| - |x_j(t)|| \leq e. \tag{5}$$

When malicious nodes appear in the network, the bipartite consensus problem becomes more complicated. The MSR algorithm with the idea of ignoring extreme values in [9] is a popular way to alleviate the impact of malicious nodes. Inspired by the idea, Algorithm 1 is presented for resilient bipartite consensus with event-triggered communication. Note that some malicious nodes' information may not be deleted in Algorithm 1, but the undeleted value must not be greater (less) than the maximum (minimum) value of normal nodes. Such nodes are generally considered to be incapable of attack, that is, they can be temporarily considered as normal nodes.

III. Main Results

In this section, we will analyze the convergence of system (1) under the designed event-based resilient impulsive control protocol.

Algorithm 1:

Require: $x(0)$, $v(0)$, $\mathcal{V}^N$, $\mathcal{V}^M$, $\mathcal{A}$=$[a_{ij}]$;
for i in $\mathcal{V}^N$ **do**
if $t = t_k$ **then**

1) Check if its ETC is satisfied. If it is satisfied, node i sends its state value $x_i(t_k)$ to its all out-neighbors. If not, not. Then, $\hat{x}_i(t_k)$ is set as

$$\hat{x}_i(t_k) = \begin{cases} x_i(t_k) & \text{if } f_i(t_{k-1}) > 0 \\ \hat{x}_i(t_{k-1}) & \text{otherwise.} \end{cases} \tag{6}$$

2) Collect the values $\operatorname{sgn}(a_{ij})\hat{x}_j(t_k)$, $j \in N_i$, and sort them in ascending order, denoted by $\mathcal{F}_i(t_k)$.

3) Agent i ignores the largest f values that are strictly greater than $x_i(t_k)$ in $\mathcal{F}_i(t_k)$. If there are less than f such values, agent i ignores all of these values. Similarly, agent i ignores the smallest f values that are strictly less than $x_i(t_k)$ in $\mathcal{F}_i(t_k)$. If there are fewer than f such values, agent i ignores all of these values. Let $\mathcal{Q}_i(t_k)$ be the set of nodes whose values were remained.

4) Use the following control input:

$$u_i(t) = \sum_{k=1}^{\infty}\left[-k_1 \sum_{j\in\mathcal{Q}_i(t_k)} |a_{ij}|\left(x_i(t) - \operatorname{sgn}(a_{ij})\hat{x}_j(t)\right) - k_2\left(x_i(t) - x_i(t_{k-1})\right)\right]\delta(t - t_k). \tag{7}$$

else
$u_i(t) = 0.$
end if
end for

Under control input (7), MAS (1) can be rewritten as follows:

$$\begin{cases} \dot{x}_i(t) = v_i(t), \\ \dot{v}_i(t) = 0, \quad t \in (t_k, t_{k+1}] \\ \Delta v_i(t_k) = -k_1 \sum\limits_{j\in\mathcal{Q}_i(t_k)} |a_{ij}|\left(x_i(t_k) - \operatorname{sgn}(a_{ij})\hat{x}_j(t_k)\right) \\ \quad -k_2\left(x_i(t_k) - x_i(t_{k-1})\right) \end{cases} \tag{8}$$

where $\Delta v_i(t_k) = v_i(t_k^+) - v_i(t_k)$, $v_i(t_k^+) = \lim_{t\to t_k^+} v_i(t)$. For simplicity, we assume that $v_i(t)$ is left-hand continuous at $t = t_k$, then $v_i(t_k^+) = v_i(t_{k+1})$ for $\forall k \in \mathbb{Z}_{\geq 0}$.

Since $x_i(t)$ is continuous and $v_i(t)$ is left-hand continuous at $t{=}t_k$, from (8), we have

$$x_i(t_{k+1}) = x_i(t_k) + hv_i\left(t_k^+\right) \tag{9}$$

and

$$\begin{aligned} v_i\left(t_k^+\right) &= v_i(t_k) - k_1 \sum_{j\in\mathcal{Q}_i(t_k)} |a_{ij}|\left(x_i(t_k) - \operatorname{sgn}(a_{ij})\hat{x}_j(t_k)\right) \\ &\quad - k_2(x_i(t_k) - x_i(t_{k-1})) \\ &= -k_1 \sum_{j\in\mathcal{Q}_i(t_k)} |a_{ij}|\left(x_i(t_k) - \operatorname{sgn}(a_{ij})\hat{x}_j(t_k)\right) \\ &\quad + \left(\frac{1}{h} - k_2\right)(x_i(t_k) - x_i(t_{k-1})). \end{aligned} \tag{10}$$

Then, we can arrive at

$$\begin{aligned} x_i(t_{k+1}) = {}& \left(2 - k_2 h - k_1 h \sum_{j\in\mathcal{Q}_i(t_k)} |a_{ij}|\right) x_i(t_k) \\ & + k_1 h \sum_{j\in\mathcal{Q}_i(t_k)} a_{ij}\hat{x}_j(t_k) + (k_2 h - 1)x_i(t_{k-1}). \end{aligned} \tag{11}$$

According to Lemma 1, we can choose a matrix $P = \operatorname{diag}\{\sigma_1, \sigma_2, \ldots, \sigma_n\}$, $\sigma_i \in \{\pm 1\}$ such that $\sigma_i\sigma_j a_{ij} = |a_{ij}|$. Let $y_i = \sigma_i x_i$ and $\hat{y}_i = \sigma_i \hat{x}_i$, then we have $\sigma_i a_{ij}\hat{x}_j(t_k) = \sigma_i\sigma_j a_{ij}\sigma_j\hat{x}_j(t_k) = |a_{ij}|\hat{y}_j(t_k)$. Thus, the position dynamic of each node after coordinate conversion is given by

$$\begin{aligned} y_i(t_{k+1}) = {}& \left(2 - k_2 h - k_1 h \sum_{j\in\mathcal{Q}_i(t_k)} |a_{ij}|\right) y_i(t_k) \\ & + k_1 h \sum_{j\in\mathcal{Q}_i(t_k)} |a_{ij}|\hat{y}_j(t_k) + (k_2 h - 1)y_i(t_{k-1}). \end{aligned} \tag{12}$$

Remark 1: Note that before the coordinate conversion, the node i deletes f extreme values strictly larger than x_i and f extreme values strictly smaller than x_i from the set $\{\operatorname{sgn}(a_{ij})\hat{x}_j : j \in N_i\}$ in Algorithm 1. After the coordinate transformation, the corresponding operation in Algorithm 1 is that the node i deletes f extreme values larger than y_i and f extreme values smaller than y_i in the set $\{\hat{y}_j : j \in N_i\}$ since $\operatorname{sgn}(a_{ij}) = \sigma_i\sigma_j$, $y_i = \sigma_i x_i$, $\forall i, j \in \mathcal{V}$. Hence, the set $\mathcal{Q}_i$ in (11) is the same as that in (12).

Next, we will give the main result. Define $\bar{m}(t_k) = \min_{i\in\mathcal{V}^N}\{y_i(t_k), y_i(t_{k-1})\}$, $\bar{M}(t_k) = \max_{i\in\mathcal{V}^N}\{y_i(t_k), y_i(t_{k-1})\}$, $m(t_k) = \min_{i\in\mathcal{V}^N}\{y_i(t_k), y_i(t_{k-1}), \hat{y}_i(t_k)\}$, $M(t_k) = \max_{i\in\mathcal{V}^N}\{y_i(t_k), y_i(t_{k-1}), \hat{y}_i(t_k)\}$, and the set $S(t_k) = [m(t_k), M(t_k)]$ for $k = 1, 2, \ldots$.

Theorem 1: Consider a network $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ under the f-total thereat model. Support $\mathcal{G}$ is structurally balanced and $\frac{1}{h} < k_2 < \frac{2}{h} - k_1 \max_{i\in\mathcal{V}^N}\sum_{j\in N_i}|a_{ij}|$. The MAS (1) with protocol (7) can achieve resilient bipartite consensus at the error level e if and only if the digraph $\mathcal{G}$ is $(f+1, f+1)$-robust and the bipartite consensus error level e is achieved if the parameter c_0 in ETC (3) satisfies $c_0 \leq \frac{\mu^m e}{4mk_1 h}$.

Proof: (Sufficiency): To prove sufficiency, we take two steps to show that (C1) and (C2) in Definition 4 are satisfied.

step 1: From $\frac{1}{h} < k_2 < \frac{2}{h} - k_1 \max_{i\in\mathcal{V}^N}\sum_{j\in N_i}|a_{ij}|$, it has the coefficients of system (12) are nonnegative and the sum of coefficients is less than 1. Hence, $y_i(t_{k+1})$ is a convex combination of values in $\{y_i(t_k), y_i(t_{k-1})\}\bigcup\{\hat{y}_j(t_k)\}_{j\in\mathcal{Q}_i(t_k)}$ for $k \geq 1$. Denote by η the smallest value among all nonzero coefficients, then $0 < \eta < 1$. Note the values in $\{\hat{y}_j(t_k)\}_{j\in\mathcal{Q}_i(t_k)}$ lie in $S(t_k)$, since the malicious nodes with values outside $S(t_k)$ will be ignored in step 3 of Algorithm 1. It follows that $y_i(t_{k+1}) \in S(t_k)$. Moreover, according to (6), $\hat{y}_i(t_{k+1})$ takes value in $\{y_i(t_{k+1}), \hat{y}_i(t_k)\}$, which implies that

$\hat{y}_i(t_{k+1}) \in S(t_k)$. Hence, we have $S(t_{k+1}) \subseteq S(t_k)$. Therefore, it can be concluded that $y_i(t_k) \in S(t_1)$ for $\forall k \geq 0, i \in \mathcal{V}^N$. Define $\bar{H} = \max_{i \in \mathcal{V}^N}\{|x_i(t_1)|, |x_i(t_0), |\hat{x}_i(t_1)|\}$. From $y_i(t_k) = \sigma_i x_i(t_k)$, it is easy to obtain that $x_i(t_k) \in [-\bar{H}, \bar{H}]$ for $\forall k \geq 0$, $i \in \mathcal{V}^N$. From (8), we have $x_i(t_{k-1}) \leq x_i(t) \leq x_i(t_k)$ for $i \in \mathcal{V}^N, t \in (t_k, t_{k+1}]$. Thus, $x_i(t) \in [-\bar{H}, \bar{H}]$ for $\forall t \geq 0, i \in \mathcal{V}^N$. This completes the proof of step 1.

Step 2: Let $\hat{e}_i(t_k) = \hat{y}_i(t_k) - y_i(t_{k+1}) = \sigma_i e_i(t_k)$, $\hat{f}_i(t_k) = |\hat{e}_i(t_k)| - (c_0 + c_1 e^{-\alpha k}) = f_i(t_k)$. From (6), we can know that if $\hat{f}_j(t_{k-1}) > 0$, then $\hat{y}_j(t_k) = y_j(t_k)$, otherwise $\hat{y}_j(t_k) = \hat{y}_j(t_{k-1}) = y_j(t_k) + \hat{e}_j(t_{k-1})$. Let

$$\bar{e}_j(t_{k-1}) = \begin{cases} 0 & \text{if } \hat{f}_j(t_{k-1}) > 0 \\ \hat{e}_j(t_{k-1}) & \text{otherwise.} \end{cases} \tag{13}$$

Then, $\hat{y}_j(t_k) = y_j(t_k) + \bar{e}_j(t_{k-1})$ always holds for $k = 1, 2, \ldots$. Furthermore, we have

$$|\bar{e}_j(t_{k-1})| \leq |\hat{e}_j(t_{k-1})| \leq c_0 + c_1 e^{-\alpha(k-1)}. \tag{14}$$

Thus, system (12) can be rewritten as

$$\begin{aligned} y_i(t_{k+1}) = &\left(2 - k_2 h - k_1 h \sum_{j \in \mathcal{Q}_i(t_k)} |a_{ij}|\right) y_i(t_k) \\ &+ (k_2 h - 1) y_i(t_{k-1}) \\ &+ k_1 h \sum_{j \in \mathcal{Q}_i(t_k)} |a_{ij}|(y_j(t_k) + \bar{e}_j(t_{k-1})) \\ &\leq \bar{M}(t_k) + k_1 h |\bar{e}_j(t_{k-1})| \\ &\leq \bar{M}(t_k) + k_1 h c_0 + k_1 h c_1 e^{-\alpha(k-1)}. \end{aligned} \tag{15}$$

Hence, it can be obtained that

$$\bar{M}(t_{k+1}) \leq \bar{M}(t_k) + k_1 h c_0 + k_1 h c_1 e^{-\alpha(k-1)}. \tag{16}$$

Similarly, $\bar{m}(t_{k+1}) \geq \bar{m}(t_k) - k_1 h c_0 - k_1 h c_1 e^{-\alpha(k-1)}$ can be obtained.

Next, we introduce $V(t_k) = \bar{M}(t_k) - \bar{m}(t_k)$ for $k = 1, 2, \ldots$ and analyze the upper bound of $V(t_k)$ to get the error level e. Inspired by (16), we create two variables as follows to find the upper bound of $\bar{M}(t_k)$ and the lower bound of $\bar{m}(t_k)$

$$\begin{aligned} \Phi(k+1) &= \Phi(k) + k_1 h c_0 + k_1 h c_1 e^{-\alpha(k-1)} \\ \phi(k+1) &= \phi(k) - k_1 h c_0 - k_1 h c_1 e^{-\alpha(k-1)} \end{aligned} \tag{17}$$

where $k = 1, 2, \ldots$, $\Phi(1) = \bar{M}(t_1)$ and $\phi(1) = \bar{m}(t_1)$. It easy to know that $\bar{M}(t_k) \leq \Phi(k)$ and $\bar{m}(t_k) \geq \phi(k)$.

Define a recursive sequence $\{\epsilon_k\}$ as

$$\epsilon_{k+1} = \eta \epsilon_k, \quad k = 1, 2, \ldots \tag{18}$$

where $\epsilon_1 = \frac{1}{2} V(t_1)$. It is easy to show that $0 < \epsilon_{k+1} < \epsilon_k$ since $0 < \eta < 1$.

Next, define the following two sets with $\Phi(k)$, $\phi(k)$ and ϵ_k:

$$\begin{cases} \overline{S}(t_k, \epsilon_k) := \{i \in \mathcal{V} : y_i(t_k) > \Phi(k) - \epsilon_k\} \\ \underline{S}(t_k, \epsilon_k) := \{i \in \mathcal{V} : y_i(t_k) < \phi(k) + \epsilon_k\}. \end{cases} \tag{19}$$

For these two sets, the following two conclusions hold.

1) $\overline{S}(t_k, \epsilon_k)$ and $\underline{S}(t_k, \epsilon_k)$ are disjoint.

2) At least one of sets $\overline{S}(t_{m+1}, \epsilon_{m+1}) \cap \mathcal{V}^N$ and $\underline{S}(t_{m+1}, \epsilon_{m+1}) \cap \mathcal{V}^N$ is empty.

For conclusion (1), we get the result by $\Phi(k) - \epsilon_k > \phi(k) + \epsilon_k$. From (17) and (18), we can obtain

$$\begin{aligned} &(\Phi(k) - \epsilon_k) - (\phi(k) + \epsilon_k) \\ &= \left(\Phi(1) + (k-1) k_1 h c_0 + k_1 h c_1 \frac{1 - e^{-\alpha(k-1)}}{1 - e^{-\alpha}}\right) \\ &\quad - \left(\phi(1) - (k-1) k_1 h c_0 - k_1 h c_1 \frac{1 - e^{-\alpha(k-1)}}{1 - e^{-\alpha}}\right) - 2\eta^{k-1}\epsilon_1 \\ &= (1 - \eta^{k-1}) V(t_1) + 2(k-1) k_1 h c_0 + 2 k_1 h c_1 \frac{1 - e^{-\alpha(k-1)}}{1 - e^{-\alpha}}. \end{aligned} \tag{20}$$

From $0 < \eta < 1$, it is easy to have $(\Phi(k) - \epsilon_k) - (\phi(k) + \epsilon_k) > 0$. Conclusion (1) is established.

For conclusion (2), we first consider sets $\overline{S}(t_1, \epsilon_1)$ and $\underline{S}(t_1, \epsilon_1)$. We suppose that $\overline{S}(t_1, \epsilon_1) \cap \mathcal{V}^N$ and $\underline{S}(t_1, \epsilon_1) \cap \mathcal{V}^N$ are not empty (If one of them is an empty set, the conclusion can be drawn directly). From conclusion (1), we know that these two sets are disjoint. Since the digraph is $(f+1, f+1)$-robust, one has that there is one or more normal nodes in $\overline{S}(t_1, \epsilon_1) \cup \underline{S}(t_1, \epsilon_1)$ with no less than $f+1$ in-neighbors about its set. Assume that the normal node $i \in \overline{S}(t_1, \epsilon_1)$ has this property. In Algorithm 1, at most f values from these outer in-neighbors are deleted. Thus, at least one normal node from set $\mathcal{Q}_i(t_1) \backslash \overline{S}(t_1, \epsilon_1)$ is used to update node i. Therefore, node i is updated according to

$$\begin{aligned} y_i(t_2) = &\left(2 - k_2 h - k_1 h \sum_{j \in \mathcal{Q}_i(t_1)} |a_{ij}|\right) y_i(t_1) + (k_2 h - 1) y_i(t_0) \\ &+ k_1 h \sum_{j \in \mathcal{Q}_i(t_1) \cap \overline{S}(t_1, \epsilon_1)} |a_{ij}| y_j(t_1) + k_1 h \sum_{j \in \mathcal{Q}_i(t_1) \backslash \overline{S}(t_1, \epsilon_1)} |a_{ij}| y_j(t_1) \\ &+ k_1 h \sum_{j \in \mathcal{Q}_i(t_1)} |a_{ij}| \bar{e}_j(t_0) \\ &\leq (1 - \eta) \bar{M}(t_1) + \eta \max_{j \in \mathcal{Q}_i(t_1) \backslash \overline{S}(t_1, \epsilon_1)} y_j(t_1) + k_1 h (c_0 + c_1) \\ &\leq (1 - \eta) \Phi(1) + \eta (\Phi(1) - \epsilon_1) + k_1 h (c_0 + c_1) \\ &\leq \Phi(1) - \eta \epsilon_1 + k_1 h c_0 + k_1 h c_1 \\ &= \Phi(2) - \epsilon_2. \end{aligned} \tag{21}$$

It shows that node i will be outside the set $\overline{S}(t_2, \epsilon_2)$ after updating.

For normal node $i \notin \overline{S}(t_1, \epsilon_1)$, we have

$$\begin{aligned} y_i(t_2) = &\left(2 - k_2 h - k_1 h \sum_{j \in \mathcal{Q}_i(t_1)} |a_{ij}|\right) y_i(t_1) \\ &+ (k_2 h - 1) y_i(t_0) \\ &+ k_1 h \sum_{j \in \mathcal{Q}_i(t_1)} |a_{ij}| y_j(t_1) + k_1 h \sum_{j \in \mathcal{Q}_i(t_1)} |a_{ij}| \bar{e}_j(t_0) \end{aligned}$$

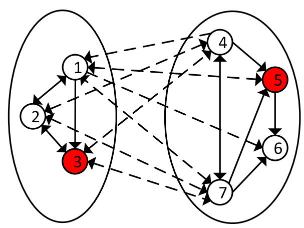

Fig. 1. (3,3)-robust digraph. It is structurally balanced. The solid line represents cooperative interaction, the dotted line represents competitive interaction.

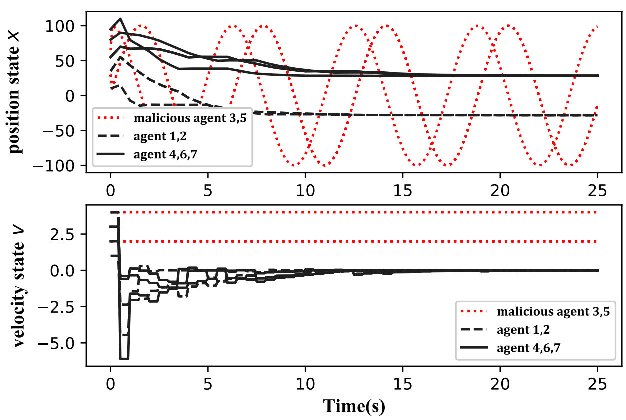

Fig. 2. Trajectory of the position and velocity state of agents under $c_0 = 0$, $c_1 = 1$.

$$\begin{aligned} &\leq \eta(\Phi(1) - \epsilon_1) + (1-\eta)\Phi(1) + k_1 h(c_0 + c_1) \\ &\leq \Phi(2) - \epsilon_2. \end{aligned} \tag{22}$$

It shows that all normal nodes outside the set $\overline{S}(t_1, \epsilon_1)$ will not enter the set $\overline{S}(t_2, \epsilon_2)$ after updating. Thus, it has that $|\overline{S}(t_2, \epsilon_2) \cap \mathcal{V}^N| < |\overline{S}(t_1, \epsilon_1) \cap \mathcal{V}^N|$. Similar arguments can be used to show $|\underline{S}(t_2, \epsilon_2) \cap \mathcal{V}^N| < |\underline{S}(t_1, \epsilon_1) \cap \mathcal{V}^N|$. Due to $|\mathcal{V}^N| = m$, that is, the number of normal agents is limited, by repeating the same analysis as above and after at most m steps, we have $|\overline{S}(t_{m+1}, \epsilon_{m+1}) \cap \mathcal{V}^N| = 0$ or $|\underline{S}(t_{m+1}, \epsilon_{m+1}) \cap \mathcal{V}^N| = 0$. The proof of conclusion (2) is completed.

We assume $\overline{S}(t_{m+1}, \epsilon_{m+1}) \cap \mathcal{V}^N = \emptyset$ (The same result can be obtained if $\underline{S}(t_{m+1}, \epsilon_{m+1}) \cap \mathcal{V}^N = \emptyset$). It follows that $\bar{M}(t_{m+1}) \leq \Phi(m+1) - \epsilon_{m+1}$. Thus, we have

$$\begin{aligned} V(t_{m+1}) &= \bar{M}(t_{m+1}) - \bar{m}(t_{m+1}) \\ &\leq \Phi(m+1) - \epsilon_{m+1} - \phi(m+1) \\ &\leq \Phi(1) - \phi(1) + 2\, m k_1 h c_0 + 2 k_1 h c_1 \frac{1-e^{-\alpha m}}{1-e^{-\alpha}} - \eta^m \epsilon_1 \\ &\leq \left(1 - \frac{1}{2}\eta^m\right) V(t_1) + 2\, m k_1 h c_0 + 2 k_1 h c_1 \frac{1-e^{-\alpha m}}{1-e^{-\alpha}}. \end{aligned} \tag{23}$$

Similarly, the abovementioned results can be extended further as

$$\begin{aligned} V(t_{lm+1}) &\leq \left(1 - \frac{1}{2}\eta^m\right) V\left(t_{(l-1)m+1}\right) + 2\, m k_1 h c_0 \\ &\quad + 2 k_1 h c_1 \frac{1-e^{-\alpha m}}{1-e^{-\alpha}} e^{-\alpha(l-1)m} \\ &\leq \left(1 - \frac{1}{2}\eta^m\right)^l V(t_1) + 2\, m k_1 h c_0 \frac{1 - \left(1 - \frac{1}{2}\eta^m\right)^l}{1 - \left(1 - \frac{1}{2}\eta^m\right)} \\ &\quad + 2 k_1 h c_1 \frac{1-e^{-\alpha m}}{1-e^{-\alpha}} \frac{\left(1-\frac{1}{2}\eta^m\right)^{l-1} \left(1 - \left(1-\frac{1}{2}\eta^m\right)^{-l} e^{-\alpha m l}\right)}{1 - \left(1-\frac{1}{2}\eta^m\right)^{-1} e^{-\alpha m}}. \end{aligned} \tag{24}$$

Then, it has

$$\limsup_{l\to\infty} V(t_{lm+1}) \leq \frac{4 m k_1 h c_0}{\eta^m}. \tag{25}$$

Following the similar analysis mentioned above, we can get $\limsup_{l\to\infty} V(t_{lm+1+d}) \leq \frac{4mk_1hc_0}{\eta^m}$, $d = 1, 2, \ldots, m$. Therefore, we have

$$\limsup_{k\to\infty} V(t_k) \leq \frac{4 m k_1 h c_0}{\eta^m} \leq e \tag{26}$$

which implies that $\limsup_{k\to\infty} |y_i(t_k) - y_j(t_k)| \leq e$ for all $i, j \in \mathcal{V}^N$. It follows that $\limsup_{k\to\infty} ||x_i(t)| - |x_j(t)|| \leq e$. The proof of sufficiency is completed.

Necessity: In the interest of space we omit the proof, which follows from Theorem 1 for the first-order case presented in [9] by setting $c_0 = c_1 = 0$.

Remark 2: Note that the error level e is related to the parameter c_0 of the event-triggered function. Choosing $c_0 = 0$, we have $\limsup_{k\to\infty} ||x_i(t)| - |x_j(t)|| \leq 0$. From (9), there is $y_i(t_{k+1}) = y_i(t_k) + h\sigma_i v_i(t_k^+)$. Combining (16), we can get

$$\begin{aligned} &\limsup_{k\to\infty} \sigma_i v_i(t_k) = \limsup_{k\to\infty} \sigma_i v_i(t_{k-1}^+) \\ &= \limsup_{k\to\infty} \frac{y_i(t_k) - y_i(t_{k-1})}{h} \\ &\leq \limsup_{k\to\infty} \frac{\bar{M}(t_k) - \bar{m}(t_{k-1})}{h} \\ &\leq \limsup_{k\to\infty} \frac{\bar{M}(t_{k-1}) + k_1 h c_0 + k_1 h c_1^{-\alpha(k-2)} - \bar{m}(t_{k-1})}{h} \\ &\leq \frac{4mk_1}{\eta^m} c_0 + k_1 c_0. \end{aligned} \tag{27}$$

Similarly, $\limsup_{k\to\infty} \sigma_i v_i(t_k) \geq -\frac{4mk_1}{\eta^m} c_0 - k_1 c_0$ can be obtained. Thus, we have

$$\limsup_{k\to\infty} |v_i(t_k)| \leq \left(\frac{4mk_1}{\eta^m} + k_1\right) c_0 = 0 \tag{28}$$

that is the system achieves exact resilient static bipartite consensus.

Remark 3: We consider the periodic sampling in this article. In fact, for the aperiodic sampling, that is, $t_{k+1} - t_k = h_k$, where $h_m < h_k < h_M$ for $k \in \mathbb{Z}_{\geq 0}$, the corresponding (12) can be written in the following form:

$$y_i(t_{k+1}) = \left(1 + \frac{h_k}{h_{k-1}} - k_2 h_k - k_1 h_k \sum_{j \in \mathcal{Q}_i(t_k)} |a_{ij}|\right) y_i(t_k)$$

TABLE I
EVENT-TRIGGERED TIMES AND CONSENSUS ERROR

normal nodes	event-triggered times in 25s		
	$c_0 = 0$, $c_1 = 0$	$c_0 = 0$, $c_1 = 1$	$c_0 = 1$, $c_1 = 1$
1	50	11	8
2	50	16	15
4	50	19	14
6	50	23	17
7	50	13	10
average times	50	16.4	12.8
consensus error at 100 s	5.63×10^{-10}	3.68×10^{-8}	2.67×10^{-7}

$$+k_1 h_k \sum_{j\in\mathcal{Q}_i(t_k)} |a_{ij}|\hat{y}_j(t_k)+\left(k_2 h_k - \frac{h_k}{h_{k-1}}\right) y_i(t_{k-1}). \quad (29)$$

The results of this article still hold if $\frac{1}{h_m} < k_2 < \frac{2}{h_M} - k_1 \max_{i\in\mathcal{V}^N} \sum_{j\in\mathcal{N}_i} |a_{ij}|$.

Corollary 1: Consider the MAS (1) with no malicious agents and assume that its corresponding network topology $\mathcal{G}$ is structurally balanced. Under $\frac{1}{h} < k_2 < \frac{2}{h} - k_1 \max_{i\in\mathcal{V}^N} \sum_{j\in\mathcal{N}_i} |a_{ij}|$, system (1) with protocol (4) can achieve bipartite consensus if and only if the digraph $\mathcal{G}$ has a spanning tree.

Proof: In this case, $f = 0$. The system can achieve bipartite consensus if $\mathcal{G}$ is (1,1)-robust from Theorem 1. Then, it follows straightforwardly from Lemma 2.

IV. NUMERICAL SIMULATIONS

Consider a structurally balanced digraph with seven nodes shown in Fig. 1. We assume that node 3 and node 5 are malicious nodes with dynamics $x_3 = 100\sin(0.1t)$, $v_3 = 2$, $x_5 = 100\cos(0.1t)$, $v_5 = 4$. Other normal agents updates according to the local protocol (7). Divide the set $\mathcal{V}$ into two subsets $\mathcal{V}_1 = [1, 2, 3]$, $\mathcal{V}_2 = [4, 5, 6, 7]$. Take $P = \text{diag}\{1, 1, 1, -1, -1, -1, -1\}$. Set $x(0) = \hat{x}(0) = [10, 35, 30, 55, 65, 80, 90]$, $v(0) = [1, 4, 2, 3, 4, 2, 3]$. Take the impulsive interval $h = 0.5$ and the control gains $k_1 = 1$, $k_2 = 2.4$. For the parameters of the event-triggered function, we always set $\alpha = 0.01$, and set c_0 and c_1 according to Table I. The trigger times of each normal node in 25s and the consensus error at $100s$ are given in Table I. Fig. 2 gives the dynamic trajectories of all nodes with $c_0 = 0$, $c_1 = 1$. The specific trigger instants of each normal agent is shown in Fig. 3.

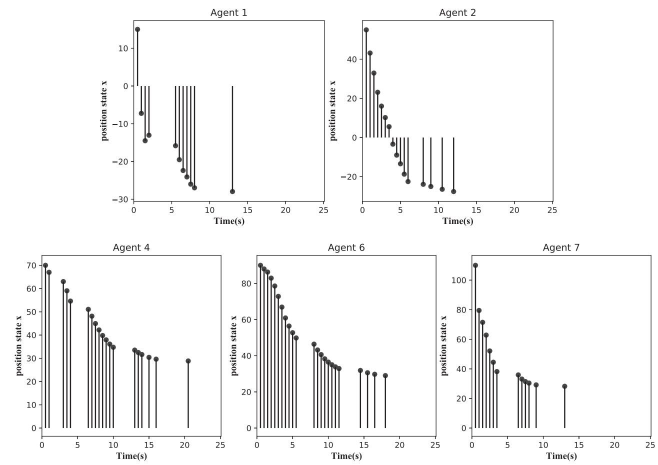

Fig. 3. Trigger instants of normal nodes. The time with the vertical line represents the trigger instant, and the black dot represents the position state of the agent at that trigger instant.

V. CONCLUSION

This article studied the resilient bipartite consensus of second-order MASs with malicious agents. An event-based resilient impulsive algorithm was employed not only to mitigate the effects of malicious nodes but also greatly reduce communication loads of agents. Under the f-total threat model, two groups of competing normal nodes in structurally balanced signed digraph can successfully converge to two opposing values if and only if the digraph is (f, f)-robust. Our future research will focus on finite-time (bipartite) resilient consensus with even-triggered communication.

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