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Sampled-data based resilient consensus of heterogeneous multiagent systems

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Summary
This article studies consensus of a group of heterogeneous agents with first-order and second-order integrator dynamics in presence of malicious agents. We employ the algorithm where each normal agent ignores large and small relative state values of its neighbors to mitigate the effects of malicious agents. Assuming that the maximum number of malicious agents in the neighborhood of each agent is known, sufficient topological condition is obtained to guarantee resilient consensus in directed networks. The result is further extended to heterogeneous multiagent systems with bounded communication delays. Moreover, impulsive control strategy is introduced in the update schemes and sufficient condition in terms of graph robustness is provided for resilient consensus. Numerical examples are provided to illustrate the effectiveness of the theoretical results.

KEYWORDS
communication delays, heterogeneous multiagent systems, resilient consensus, sampled-data

1 | INTRODUCTION

Considerable attention has been paid to coordination of multiagent systems in the past decades because of its wide application in many fields, such as distributed computation, mobile robots formation, and intelligent transportation systems. Consensus problem, which is a fundamental problem in multiagent systems, aims at making a group of locally interacting agents reach an agreement upon some quantities of interest. So far, lots of work has been done under different contexts, such as communication delays,[1] noise,[2] quantization,[3] states constraints,[4] and fast consensus.[5]

In most of the existing literature on the consensus problem, the agents are cooperative to achieve the global goal. However, some agents may become noncooperative or even malicious when the network is suffering malicious attack or platform-level failures, which will lead to degradation of system performance and even failure of the global goal. Therefore, it is of great importance to consider how to improve the algorithms to avoid system performance being influenced by such compromised agents. Resilient consensus, as a special case of consensus, has long been studied in computer science. A class of algorithms where each normal agent disregards the most deviated agents in the updates has been extensively used for resilient consensus and is often called the mean subsequence reduced (MSR) algorithms.[6] However, this strategy had been studied mostly under complete graphs. In Reference 7, a new concept in graph theory, termed r-robustness, was introduced to study resilient consensus in noncomplete networks. Afterward, a lot of excellent work emerged in discrete-time setting by employing the concept of network robustness and MSR-type algorithms. In References 8 and 9, resilient conditions were obtained for second-order multiagent systems under DP-MSR algorithm, which took an adapted form of the MSR-type algorithms for second-order multiagent systems. In Reference 10, SW-MSR algorithm, which extended MSR-type algorithm by introducing a sliding window, was introduced for multiagent systems with time-varying network topologies. To reduce the communication burden, E-MSR algorithm, which introduced event-based update rule to MSR-type algorithms, was proposed in Reference 11 for first-order multiagent systems to reach resilient consensus. In References 12-16, resilient consensus results were given in continuous-time setting. In Reference 12, ARC-P algorithm, which is a continuous-time variation of the MSR, was proposed to solve asymptotic consensus under totally bounded malicious model. In Reference 14, DRSC algorithm, which is an improved ARC-P algorithm, was proposed for nonlinear multiagent systems with communication delays to reach resilient consensus. In Reference 17, resilient consensus problem was studied for multiagent systems with both continuous-time and discrete-time subsystems.

Heterogeneity is an important feature of multiagent systems. The agents in one group often have different dynamics due to various restrictions in practical application. There has been a lot of work studying on coordination of heterogeneous multiagent systems. In Reference 18, the authors investigated output consensus problem for heterogeneous linear multiagent systems. In Reference 19, an event-based communication strategy was proposed for heterogeneous linear multiagent systems to solve leader-follower consensus problem. The authors of Reference 20 investigated containment control problem for heterogeneous linear multiagent systems. In Reference 21, adaptive algorithms were designed for a group of agents with nonidentical nonlinear dynamics to reach consensus. The authors of References 22 and 23 studied the consensus problem for hybrid multiagent systems consisting of continuous-time and discrete-time dynamic agents. In Reference 24, a game-theoretic approach was adopted to solve the consensus of hybrid multiagent systems. In Reference 25, multitarget tracking problem was studied for a group of heterogeneous inertial agents using the decomposition approach. This article focus on resilient consensus of heterogeneous multiagent systems composed of first-order and second-order integrator dynamics. A power transmission network including generator buses and load buses is a good example of such heterogeneous multiagent systems.[26] In Reference 27, the authors implemented a secondary control for paralleled inverters using this heterogeneous multiagent model to regulated the frequency. Due to its potential applications, a lot of work has been done on this type of multiagent systems. In References 28 and 29, the authors obtained some sufficient conditions for consensus of this type of heterogeneous multiagent systems in continuous-time setting in Reference 28 and discrete-time setting in Reference 29. The work in Reference 28 was improved to containment control in Reference 30. In References 31 and 32, group consensus was studied for the heterogeneous multiagent systems.

Inspired by the work above, we study the resilient consensus problem for heterogeneous multiagent systems in this article. We aim to make a group of normal agents to reach consensus in presence of malicious agents. The difficulty of this problem comes from the presence of malicious agents and more complex system dynamics. The main contribution of this article is threefold. First, the resilient consensus problem is investigated for the heterogeneous multiagent system composed of first-order and second-order integrator agents. Such system has many applications, but exhibits more complicated dynamics in comparison to the homogeneous systems with identical dynamics agents.[7-9] MSR-type algorithm is proposed and sufficient conditions are obtained for resilient consensus under the f-locally bounded model. When there are just second-order agents in heterogeneous multiagent systems, the constraint on control gain is looser than that in Reference 8. Second, considering the agents might not have access to the current data of all neighbors simultaneously in practice, we extend the results to heterogeneous multiagent systems with communication delays. Moreover, a modified update algorithm is introduced for resilient consensus with switching topologies. Third, sufficient conditions in terms of graph robustness are obtained for resilient of heterogeneous multiagent systems with impulsive control input and quantized relative information.

This article is organized as follows. In Section 2, some mathematical preliminaries are presented. The resilient consensus problem is discussed in Section 3. In Section 4, the simulation results are given to show the effectiveness of the obtained results. Section 5 is a brief conclusion.

2 | PRELIMINARIES

2.1 | Notation and graph theory

Throughout this article, we denote the set of real numbers by R and the set of nonnegative integers by N. Let $|S|$ be the cardinality of a set S. The set union and set difference operations of two sets S_1 and S_2 are denoted by $S_1 \cup S_2$ and $S_1 \setminus S_2$, respectively.

A directed graph is denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{1, 2, \dots, n\}$ is the node set, $\mathcal{E} = \{e_{ij}\} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set, and $\mathcal{A} = [a_{ij}]_{n\times n}$ is the weighted adjacency matrix. The edge $(j, i) \in \mathcal{E}$ indicates that node i can receive information from node j. If $(j, i) \in \mathcal{E}$, $a_{ij} > 0$ and $a_{ij} = 0$ otherwise. Note that for directed graph $(i, j) \in \mathcal{E}$ does not necessarily imply that $(j, i) \in \mathcal{E}$. The graph is assumed to be simple, namely, $(i, i) \notin \mathcal{E}, \forall i \in \mathcal{V}$. For node i, the set of neighbors is given by $\mathcal{N}_i = \{j : (j, i) \in \mathcal{E}\}$.

Definition 1 (7). For $r \in N$, a set $S \subset \mathcal{V}$ is said to be r-reachable if there exists a node $i \in S$ such that $|\mathcal{N}_i \setminus S| \geq r$.

Definition 2 (7). For $r \in N$, a graph $\mathcal{G}$ is said to be r-robust if for any pair of nonempty, disjoint subsets of $\mathcal{V}$, at least one of them is r-reachable.

2.2 | Model description

In this article, we consider a heterogeneous multiagent system composed of first-order and second-order integrator agents. The dynamics of first-order integrator agents is as follows:

$$\dot{x}_i(t) = u_i(t), \tag{1}$$

and the dynamics of second-order integrator agents is as follows:

$$\dot{x}_i(t) = v_i(t), \dot{v}_i(t) = u_i(t), \tag{2}$$

where $x_i, v_i \in R$ are, respectively, the position and velocity states of agent i, $u_i \in R$ is the input of agent i.

This article considers a partition of the agents. An agent i is called normal if it applies a predefined control input. Otherwise, it is called malicious and its input can be arbitrary. We assume there are n agents with communication graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, and there are at most f malicious agents in the neighborhood of each normal agent. Let $\mathcal{I}_1 = \{1, \dots, n_1\}$, $\mathcal{I}_2 = \{n_1 + 1, \dots, n_1 + n_2\}$, and $\mathcal{I}_3 = \{n_1 + n_2 + 1, \dots, n_1 + n_2 + n_3\}$, where $n_1 + n_2 + n_3 = n$ denote the sets of normal agents with first-order dynamics, normal agents with second-order dynamics and malicious agents.

We investigate resilient consensus problems in a sampled-data setting, where the sampling interval is h. First, we study the case that the control inputs are based on zero-order hold. Thus, the discretized model of normal agents can be written as:

$$\begin{cases} x_i(k) = x_i(k) + hu_i(k), i \in \mathcal{I}_1, \\ x_i(k) = x_i(k) + hv_i(k) + \frac{h^2}{2}u_i(k), v_i(k+1) = v_i(k) + hu_i(k), i \in \mathcal{I}_2, \end{cases} \tag{3}$$

where we omit the sampling time interval to simplify the notations.

Definition 3. The heterogeneous multiagent system is said to reach resilient consensus if for any set of malicious agents, any initial states and any malicious inputs, the following conditions are satisfied:

1. Safety condition: there exists an interval $S \subset R$ such that $x_i(k) \in S$ for all $i \in \mathcal{I}_1 \cup \mathcal{I}_2, k \geq 0$.
2. Agreement condition: there exists $c \in R$ such that $\lim_{k \to +\infty} x_i(k) = c, \forall i \in \mathcal{I}_1 \cup \mathcal{I}_2$ and $\lim_{k \to +\infty} v_i(k) = 0, \forall i \in \mathcal{I}_2$.

3 | MAIN RESULTS

In this section, we first outline the proposed algorithm and then present the conditions in terms of graph robustness for resilient consensus under the proposed algorithm.

For the case that there is no malicious agent, that is, $f = 0$, the following distributed protocol is often adopted for consensus

$$u_i(k) = \begin{cases} \sum\limits_{j \in \mathcal{N}_i} a_{ij}(x_j(k) - x_i(k)), & i \in \mathcal{I}_1, \\ \sum\limits_{j \in \mathcal{N}_i} a_{ij}(x_j(k) - x_i(k)) - \alpha v_i(k), & i \in \mathcal{I}_2, \end{cases} \tag{4}$$

where a_{ij} is the entry of adjacency matrix $\mathcal{A}$ and $\alpha > 0$ is a control parameter. In Reference 29, the authors showed that by properly choosing α and h, the heterogeneous multiagent systems can reach stationary consensus if the communication graph has a spanning tree. However, this distributed protocol cannot solve resilient consensus even if the communication graph has a spanning tree.

Inspired by the work in References 7 and 8, we propose the heterogeneous position-based mean subsequence reduced (HP-MSR) algorithm for resilient consensus. The specific steps are as follows:

1. At time step k, each normal agent i receives the relative position values of its neighbors, that is, $x_j(k)-x_i(k), j \in \mathcal{N}_i$ and its velocity value $v_i(k)$, and sorts the relative position values from the largest to the smallest.
2. Agent i removes the f largest values that are larger than zero in this list. If there are less than f such values, all of them are removed. The similar removal process is applied to the values that are smaller than zero. The set of neighbors which are removed by agent i at time step k is represented by $\mathcal{R}_i(k)$.
3. Agent i updates its state by:

$$u_i(k) = \begin{cases} \sum\limits_{j\in\mathcal{N}_i\setminus\mathcal{R}_i(k)} a_{ij}(x_j(k)-x_i(k)), & i \in \mathcal{I}_1, \\ \sum\limits_{j\in\mathcal{N}_i\setminus\mathcal{R}_i(k)} a_{ij}(x_j(k)-x_i(k)) - \alpha v_i(k), & i \in \mathcal{I}_2. \end{cases} \tag{5}$$

Let $s_1 = \min\left(\min\limits_{i\in\mathcal{I}_1\cup\mathcal{I}_2} x_i(0), \min\limits_{i\in\mathcal{I}_2}\left(x_i(0)+\frac{2}{\alpha}v_i(0)\right)\right)$, $s_2 = \max\left(\max\limits_{i\in\mathcal{I}_1\cup\mathcal{I}_2} x_i(0), \max\limits_{i\in\mathcal{I}_2}\left(x_i(0)+\frac{2}{\alpha}v_i(0)\right)\right)$ and $S=[s_1,s_2]$.

In the following, we will present the resilient consensus conditions and show that the safety condition holds with the interval S.

Theorem 1. *Suppose $h < \frac{1}{\sum_{j\in\mathcal{N}_i, i\in\mathcal{I}_1} a_{ij}}$ and $2\sqrt{\sum_{j\in\mathcal{N}_i, i\in\mathcal{I}_2} a_{ij}} < \alpha < \frac{\sqrt{5}-1}{h}$. Under the f-locally bounded model, the heterogeneous multiagent system using HP-MSR algorithm with control input (5) can solve the resilient consensus problem if the communication graph $\mathcal{G}$ is $(2f+1)$-robust.*

Proof. For normal agent $i \in \mathcal{I}_1$, the discretized model under (5) can be written as:

$$x_i(k+1) = x_i(k) + h\sum_{j\in\mathcal{N}_i\setminus\mathcal{R}_i(k)} a_{ij}(x_j(k)-x_i(k)). \tag{6}$$

For normal agent $i \in \mathcal{I}_2$, define $x_{n_2+n_3+i}(k) = x_i(k) + \frac{2}{\alpha}v_i(k)$ and the discretized model under control input (5) can be written as:

$$\begin{aligned} x_i(k+1) &= x_i(k) + hv_i(k) + \frac{h^2}{2}\left(\sum_{j\in\mathcal{N}_i\setminus\mathcal{R}_i(k)} a_{ij}(x_j(k)-x_i(k)) - \alpha v_i(k)\right) \\ &= x_i(k) + \left(h - \frac{h^2}{2}\alpha\right)v_i(k) + \frac{h^2}{2}\sum_{j\in\mathcal{N}_i\setminus\mathcal{R}_i(k)} a_{ij}(k)(x_j(k)-x_i(k)) \\ &= x_i(k) + \left(\frac{h\alpha}{2} - \frac{h^2\alpha^2}{4}\right)(x_{n_2+n_3+i}(k)-x_i(k)) + \frac{h^2}{2}\sum_{j\in\mathcal{N}_i\setminus\mathcal{R}_i(k)} a_{ij}(k)(x_j(k)-x_i(k)), \end{aligned} \tag{7}$$

and

$$\begin{aligned} x_{n_2+n_3+i}(k+1) &= x_i(k+1) + \frac{2}{\alpha}v_i(k+1) \\ &= x_i(k) + \left(\frac{h\alpha}{2} - \frac{h^2\alpha^2}{4}\right)(x_{n_2+n_3+i}(k)-x_i(k)) + \frac{h^2}{2}\sum_{j\in\mathcal{N}_i\setminus\mathcal{R}_i(k)} a_{ij}(k)(x_j(k)-x_i(k)) \\ &\quad + \frac{2}{\alpha}v_i(k) + \frac{2h}{\alpha}\sum_{j\in\mathcal{N}_i\setminus\mathcal{R}_i(k)} a_{ij}(x_j(k)-x_i(k)) - 2hv_i(k) \end{aligned}$$

$$= x_{n_2+n_3+i}(k) + \left(\frac{h^2}{2} + \frac{2h}{\alpha}\right) \sum_{j\in\mathcal{N}_i\setminus\mathcal{R}_i(k)} a_{ij}(x_j(k) - x_i(k)) + (\frac{h\alpha}{2} + \frac{h^2\alpha^2}{4})(x_i(k) - x_{n_2+n_3+i}(k))$$

$$= x_{n_2+n_3+i}(k) + \left(\frac{h^2}{2} + \frac{2h}{\alpha}\right) \sum_{j\in\mathcal{N}_i\setminus\mathcal{R}_i(k)} a_{ij}(x_j(k) - x_{n_2+n_3+i}(k)) + \beta(k)(x_i(k) - x_{n_2+n_3+i}(k)), \tag{8}$$

where $\beta(k) = \frac{h\alpha}{2} + \frac{h^2\alpha^2}{4} - \frac{h^2}{2}\sum_{j\in\mathcal{N}_i\setminus\mathcal{R}_i(k)} a_{ij} - \frac{2h}{\alpha}\sum_{j\in\mathcal{N}_i\setminus\mathcal{R}_i(k)} a_{ij}$.

For $i \in \mathcal{I}_2$, from $2\sqrt{\sum_{j\in\mathcal{N}_i, i\in\mathcal{I}_2} a_{ij}} < \alpha$, it has $\frac{h^2\alpha^2}{4} - \frac{h^2}{2}\sum_{j\in\mathcal{N}_i\setminus\mathcal{R}_i(k)} a_{ij} > 0$ and $\frac{h\alpha}{2} - \frac{2h}{\alpha}\sum_{j\in\mathcal{N}_i\setminus\mathcal{R}_i(k)} a_{ij} > 0$. Hence, $\beta(k) > 0$. From $0 < \alpha < \frac{\sqrt{5}-1}{h}$, it has $0 < \frac{h\alpha}{2} < \frac{h\alpha}{2} + \frac{h^2\alpha^2}{4} < 1$. It follows that $0 < \frac{h\alpha}{2} - \frac{h^2\alpha^2}{4} < 1$, $\frac{h\alpha}{2} - \frac{h^2\alpha^2}{4} + \frac{h^2}{2}\sum_{j\in\mathcal{N}_i\setminus\mathcal{R}_i(k)} a_{ij} < \frac{h\alpha}{2} < 1$ and $\beta(k) + \left(\frac{h^2}{2} + \frac{2h}{\alpha}\right)\sum_{j\in\mathcal{N}_i\setminus\mathcal{R}_i(k)} a_{ij} = \frac{h\alpha}{2} + \frac{h^2\alpha^2}{4} < 1$.

Let $\mathcal{I}_4 = \{n+1, \dots, n+n_2\}$ and $x_i, i \in \mathcal{I}_4$ be the position state of agent i. Thus, there are $n_1 + 2n_2$ normal agents with dynamics shown in (6), (7), and (8). We use the directed graph $\mathcal{G}' = (\mathcal{V}', \mathcal{E}', \mathcal{A}')$ to describe the communication among agents, where $\mathcal{V}' = \mathcal{V} \cup \mathcal{I}_4$ and $\mathcal{A}' = [a'_{ij}]_{(n+n_2)\times(n+n_2)}$. According to the analysis above, we know the weights in $\mathcal{A}'$ satisfy that $h\sum_{j\in\mathcal{N}_i} a'_{ij} < 1$ for $i \in \mathcal{I}_1 \cup \mathcal{I}_2 \cup \mathcal{I}_4$, which implies that the position value of each normal agent in $\mathcal{G}'$ at $k+1$ is a convex combination of values of i and its neighbors in $\mathcal{G}'$ at time k. Let γ be the minimum nonzero element of ha'_{ij}, $i \in \mathcal{I}_1 \cup \mathcal{I}_2 \cup \mathcal{I}_4$.

We check the robustness of the graph $\mathcal{G}'$. For any pair of nonempty, disjoint subsets S_1 and S_2 of $\mathcal{V}'$, we consider the following two cases.

(i) The first case is that $S_i \subseteq \mathcal{I}_4$ for some $i \in \{1,2\}$. In the graph $\mathcal{G}'$, we know from (7) and (8) that each agent $i, i \in \mathcal{I}_2$ keeps all its neighbors in $\mathcal{G}$, which are also the neighbors of agent $n_2 + n_3 + i$. By $(2f+1)$-robustness, it is easy to know that each agent $i, i \in \mathcal{V}$ has at least $2f+1$ neighbors from $\mathcal{V}$ in $\mathcal{G}$. Thus, agent $n_2 + n_3 + i$ also has at least $2f+1$ neighbors from $\mathcal{V}$ in $\mathcal{G}'$. It follows that S_i is $(2f+1)$-reachable.

(ii) The second case is that $S_i \nsubseteq \mathcal{I}_4$ for all $i \in \{1,2\}$. It implies that the disjoint sets $S_1 \cap \mathcal{V}$ and $S_2 \cap \mathcal{V}$ are nonempty. Therefore, by $(2f+1)$-robustness of $\mathcal{G}$, there exists an agent i, $i \in S_i \cap \mathcal{V}$ for some $i \in \{1,2\}$, having at least $2f+1$ neighbors from the set $\mathcal{V} \setminus S_i$. Note each agent $i, i \in \mathcal{V}$ in $\mathcal{G}'$ keeps all its neighbors in $\mathcal{G}$ and $\mathcal{V} \subseteq \mathcal{V}'$. Thus, the agent i has at least $2f+1$ neighbors from the set $\mathcal{V}' \setminus S_i$ in $\mathcal{G}'$, that is, S_i is $(2f+1)$-reachable.

Thus, we can conclude that the graph $\mathcal{G}'$ is $(2f+1)$-robust.

We next show that the interval S satisfies the safety condition. Define $x_M(k) = \max_{i\in\mathcal{I}_1\cup\mathcal{I}_2\cup\mathcal{I}_4} x_i(k)$ and $x_m(k) = \min_{i\in\mathcal{I}_1\cup\mathcal{I}_2\cup\mathcal{I}_4} x_i(k)$. Thus, $S = [x_m(0), x_M(0)]$. According to HP-MSR algorithm, if agent i has some malicious neighbors with values outside $[x_m(k), x_M(k)]$ at time k, they will be removed. Thus, it holds that $x_i(k+1) \in [x_m(k), x_M(k)]$, which implies that $x_M(k)$ is nonincreasing and $x_m(k)$ is nondecreasing. Hence, it has $x_i(k) \in [x_m(0), x_M(0)]$ for $k \geq 0$, which means S is the safety interval.

In the rest of proof, we establish the agreement condition. Let $V(k) = x_M(k) - x_m(k)$. Take $\varepsilon_0 = \eta V(0)$ with $\eta \in \left(0, \frac{1}{2}\right]$ and $\varepsilon_{k+1} = \gamma\varepsilon_k$, $k \geq 1$. Define sets

$$H_1(k, \varepsilon_k) = \{j \in \mathcal{I}_1 \cup \mathcal{I}_2 \cup \mathcal{I}_4 : x_j(k) > x_M(0) - \varepsilon_k\},$$

and

$$H_2(k, \varepsilon_k) = \{j \in \mathcal{I}_1 \cup \mathcal{I}_2 \cup \mathcal{I}_4 : x_j(k) < x_m(0) + \varepsilon_k\}.$$

It is obvious that the sets $H_1(0, \varepsilon_0)$ and $H_2(0, \varepsilon_0)$ are nonempty and the sets $H_1(k, \varepsilon_k)$ and $H_2(k, \varepsilon_k)$ are disjoint by $\varepsilon_{k+1} \leq \varepsilon_k$.

If the disjoint sets $H_1(k, \varepsilon_k)$ and $H_2(k, \varepsilon_k)$ are nonempty, considering a normal agent $j \notin H_1(k, \varepsilon_k)$, that is, $x_j(k) \leq x_M(0) - \varepsilon_k$, it has

$$\begin{aligned} x_j(k+1) &\leq (1-\gamma)x_M(k) + \gamma(x_M(0) - \varepsilon_k) \\ &\leq (1-\gamma)x_M(0) + \gamma(x_M(0) - \varepsilon_k) \\ &= x_M(0) - \gamma\varepsilon_k, \\ &= x_M(0) - \varepsilon_{k+1}, \end{aligned}$$

where we have used the fact each normal agent in $\mathcal{G}'$ updates its position value by a convex combination of its own value and the values of its neighbors with coefficients bounded below by γ. Thus, $j \notin H_1(k+1, \varepsilon_{k+1})$. Similarly, we can show that if $j \notin H_2(k, \varepsilon_k)$, then $j \notin H_2(k+1, \varepsilon_{k+1})$.

Since graph $\mathcal{G}'$ is $(2f+1)$-robust, there exists an agent $j, j \in H_i(k, \varepsilon_k)$ for some $i \in \{1, 2\}$, having at least $2f+1$ neighbors outside of its set. Suppose $j \in H_1(k, \varepsilon_k)$. Note there are at most f malicious agents in the neighborhood of j. Therefore, agent j has at least $f+1$ normal neighbors outside $H_1(k, \varepsilon_k)$. According to HP-MSR algorithm, at least one normal neighbor outside $H_1(k, \varepsilon_k)$ will be used by agent j to update its state. Hence, we have

$$\begin{aligned}
x_j(k+1) &\leq (1-\gamma)x_M(k) + \gamma(x_M(0) - \varepsilon_k) \\
&\leq x_M(0) - \gamma\varepsilon_k, \\
&= x_M(0) - \varepsilon_{k+1},
\end{aligned}$$

which implies that $j \notin H_1(k+1, \varepsilon_{k+1})$. Likewise, if $j \in H_2(k, \varepsilon_k)$ has at least $2f+1$ neighbors outside of its set, then $j \notin H_2(k+1, \varepsilon_{k+1})$. Since there are $n_1 + 2n_2$ normal agents in the network, by following the steps above, we can obtain that there exists $T \leq (n_1 + 2n_2)$ such that for some $i \in \{1, 2\}$, the set $H_i(T, \varepsilon_T)$ becomes empty .

Suppose $H_1(T, \varepsilon_T)$ is empty, which implies that $x_M(T) \leq x_M(0) - \varepsilon_T$. By $x_M(k+1) \leq x_M(k)$ and $\varepsilon_{k+1} \leq \varepsilon_k$, it has $x_M(T+p) \leq x_M(T) \leq x_M(0) - \varepsilon_T \leq x_M(0) - \varepsilon_{T+p}, p \geq 1$. It follows that $x_M(n_1+2n_2) \leq x_M(0) - \varepsilon_{n_1+2n_2}$. Note that $x_m(n_1+2n_2) \geq x_m(0)$. Thus, it has

$$\begin{aligned}
V(n_1+2n_2) &= x_M(n_1+2n_2) - x_m(n_1+2n_2) \\
&\leq x_M(0) - \varepsilon_{n_1+2n_2} - x_m(0) \\
&= (1 - \eta\gamma^{n_1+2n_2})V(0).
\end{aligned}$$

Similarly, the inequality above holds if $H_2(T, \varepsilon_T)$ is empty. Follow the steps above and we have for $s \in N$, $V(s(n_1 + 2n_2)) \leq (1 - \eta\gamma^{n_1+2n_2})^s V(0)$.

Similar to the analysis above, it can be obtained that $V(s(n_1+2n_2)+l) \leq (1-\eta\gamma^{n_1+2n_2})^s V(l), l = 1, 2 \dots, n_1+2n_2-1$. Thus, it has $\lim_{s\to+\infty} V(s(n_1+2n_2)+l) = 0, l = 1, 2 \dots, n_1+2n_2-1$. It follows that $\lim_{k\to+\infty} V(k) = 0$, that is $\lim_{k\to+\infty}(x_M(k) - x_m(k)) = 0$. Since $x_M(k)$ and $x_m(k)$ are monotone functions and bounded by the interval $[x_M(0), x_m(0)]$, they have limits. Thus, it has $\lim_{k\to+\infty} x_M(k) = \lim_{k\to+\infty} x_m(k) = c$, which implies that $\lim_{k\to+\infty} x_i(k) = c, i \in \mathcal{I}_1 \cup \mathcal{I}_2 \cup \mathcal{I}_4$. Note for $i \in \mathcal{I}_2$, $x_{n_2+n_3+i}(k) = x_i(k) + \frac{2}{\alpha}v_i(k)$. Thus, it has $\lim_{k\to+\infty} v_i(k) = 0, i \in \mathcal{I}_2$. This completes the proof. ∎

Remark 1. We can modify the control input (5) as $u_i(k) = \sum_{j\in\mathcal{N}_i\setminus\mathcal{R}_i(k)} a_{ij}((x_j(k)-r_j)-(x_i(k)-r_i)), i \in \mathcal{I}_1$ and $u_i(t) = \sum_{j\in\mathcal{N}_i\setminus\mathcal{R}_i(k)} a_{ij}((x_j(k)-r_j)-(x_i(k)-r_i)) - \alpha v_i(k), i \in \mathcal{I}_2$, where r_i represents the desired relative position of agent i in a formation. Thus, the normal agents in heterogeneous multiagent will converge to the desired formation under $(2f+1)$-robust network.

Remark 2. It is worth noting the model transformation technique used in this article is not applicable for the case that the control inputs of normal agents are continuous, that is, $u_i(t) = \sum_{j\in\mathcal{N}_i\setminus\mathcal{R}_i(k)} a_{ij}(x_j(t)-x_i(t)), i \in \mathcal{I}_1$ and $u_i(t) = \sum_{j\in\mathcal{N}_i\setminus\mathcal{R}_i(k)} a_{ij}(x_j(t)-x_i(t)) - \alpha v_i(t), i \in \mathcal{I}_2$. This is because the network topology does not maintain $(2f+1)$-robust after transformation.

Communication delay is an important issue in information exchange for multiagent systems. In practical applications, the interference of communication delays is inevitable, which may cause the divergence or oscillation of the network system. Hence, we will study the resilient consensus with communication delays in the following. Considering the communication delay, control input (5) can be modified as:

$$u_i(k) = \begin{cases} \sum\limits_{j\in\mathcal{N}_i\setminus\mathcal{R}_i(k)} a_{ij}(x_j(k-\tau_{ij}(k)) - x_i(k)), & i \in \mathcal{I}_1, \\ \sum\limits_{j\in\mathcal{N}_i\setminus\mathcal{R}_i(k)} a_{ij}(x_j(k-\tau_{ij}(k)) - x_i(k)) - \alpha v_i(k), & i \in \mathcal{I}_2, \end{cases} \tag{9}$$

where the communication delay $\tau_{ij}(k) \in N$ and $0 \leq \tau_{ij}(k) \leq \tau_{max}$.

Let $s_1 = \min\left(\min_{i\in\mathcal{I}_1\cup\mathcal{I}_2,\tau\in[0,\tau_{max}]} x_i(-\tau), \min_{i\in\mathcal{I}_2,\tau\in[0,\tau_{max}]}\left(x_i(-\tau)+\frac{2}{\alpha}v_i(-\tau)\right)\right)$, $s_2 = \max\left(\max_{i\in\mathcal{I}_1\cup\mathcal{I}_2,\tau\in[0,\tau_{max}]} x_i(-\tau), \max_{i\in\mathcal{I}_2,\tau\in[0,\tau_{max}]}\left(x_i(-\tau)+\frac{2}{\alpha}v_i(-\tau)\right)\right)$, and $S=[s_1,s_2]$.

In the following, we will extend the results in Theorem 1 to the multiagent systems with time-varying communication delays and show the safety condition holds with the interval S.

Theorem 2. *Suppose* $h<\frac{1}{\sum_{j\in\mathcal{N}_i,i\in\mathcal{I}_1}a_{ij}}$ *and* $\sqrt{\sum_{j\in\mathcal{N}_i,i\in\mathcal{I}_2}a_{ij}}<\alpha<\frac{\sqrt{5}-1}{h}$. *Under the f-locally bounded model, the heterogeneous system using DH-MSR algorithm with control input (9) can solve resilient consensus problem if the communication graph* $\mathcal{G}$ *is* $(2f+1)$*-robust.*

Proof. For $i\in\mathcal{I}_2$, define $x_{n_2+n_3+i}(k)=x_i(k)+\frac{2}{\alpha}v_i(k)$. Thus, the discretized model of normal agents under input (9) can be written as:

$$\begin{cases} x_i(k+1)=x_i(k)+h\sum_{j\in\mathcal{N}_i\setminus\mathcal{R}_i(k)}a_{ij}(x_j(k-\tau_{ij}(k))-x_i(k)), i\in\mathcal{I}_1,\\ x_i(k+1)=x_i(k)+\left(\frac{h\alpha}{2}-\frac{h^2\alpha^2}{4}\right)(x_{n_2+n_3+i}(k)-x_i(k))+\frac{h^2}{2}\sum_{j\in\mathcal{N}_i\setminus\mathcal{R}_i(k)}a_{ij}(x_j(k-\tau_{ij}(k))-x_i(k)), i\in\mathcal{I}_2,\\ x_{n_2+n_3+i}(k+1)=x_{n_2+n_3+i}(k)+\beta(k)(x_i(k)-x_{n_2+n_3+i}(k))\\ \quad+\left(\frac{h^2}{2}+\frac{2h}{\alpha}\right)\sum_{j\in\mathcal{N}_i\setminus\mathcal{R}_i(k)}a_{ij}(x_j(k-\tau_{ij}(k))-x_{n_2+n_3+i}(k)), i\in\mathcal{I}_2, \end{cases} \tag{10}$$

where $\beta(k)=\frac{h\alpha}{2}+\frac{h^2\alpha^2}{4}-\frac{h^2}{2}\sum_{j\in\mathcal{N}_i\setminus\mathcal{R}_i(k)}a_{ij}-\frac{2h}{\alpha}\sum_{j\in\mathcal{N}_i\setminus\mathcal{R}_i(k)}a_{ij}$.

Let $\mathcal{I}_4=\{n+1,\ldots,n+n_2\}$ and $x_i, i\in\mathcal{I}_4$ be the position state of agent i. Thus, we get a multiagent systems of n_1+2n_2 normal agents with communication graph $\mathcal{G}'=(\mathcal{V}',\mathcal{E}',\mathcal{A}')$, where $\mathcal{V}'=\mathcal{V}\cup\mathcal{I}_4$, and $\mathcal{A}'=[a'_{ij}]$. Take the parameters h and α satisfying the condition in Theorem 2. It has been proved in Theorem 1 that $\mathcal{G}'$ is $(2f+1)$-robust and $\sum_{j\in\mathcal{N}_i}a'_{ij}<1$ for $i\in\mathcal{I}_1\cup\mathcal{I}_2\cup\mathcal{I}_4$.

We next show that the interval S satisfies the safety condition. Define $x_M(k)=\max_{i\in\mathcal{I}_1\cup\mathcal{I}_2\cup\mathcal{I}_4,\tau\in[0,\tau_{max}]}x_i(k-\tau)$ and $x_m(k)=\min_{i\in\mathcal{I}_1\cup\mathcal{I}_2\cup\mathcal{I}_4,\tau\in[0,\tau_{max}]}x_i(k-\tau)$. Then, $S=[x_m(0),x_M(0)]$. Note $x_i(k+1)$ is a convex combination of values of i and its neighbors from time k to $k-\tau_{max}$. If the values of agent i received from some malicious neighbors at time k are outside $[x_m(k),x_M(k)]$, they will be moved by HP-MSR. Thus, it holds that $x_i(k+1)\in[x_m(k),x_M(k)]$, which implies that $x_M(k)$ is nonincreasing and $x_m(k)$ is nondecreasing. Hence, it has $x_i(k)\in[x_m(0),x_M(0)]$ for $k\geq 0$, which means S is the safety interval.

In the rest of proof, we establish the agreement condition. Let $V(k)=x_M(k)-x_m(k)$. Take $\varepsilon_0=\eta V(0)$ with $\eta\in\left(0,\frac{1}{2}\right]$ and $\varepsilon_{k+1}=\gamma\varepsilon_k$ for $k\geq 0$. Define sets

$$H_1(k,\varepsilon_k)=\{j\in\mathcal{I}_1\cup\mathcal{I}_2\cup\mathcal{I}_4 : x_k(k)>x_M(0)-\varepsilon_k\}$$

and

$$H_2(k,\varepsilon_k)=\{j\in\mathcal{I}_1\cup\mathcal{I}_2\cup\mathcal{I}_4 : x_j(k)<x_m(0)+\varepsilon_k\}.$$

It is obvious that the sets $H_1(0,\varepsilon_0)$ and $H_2(0,\varepsilon_0)$ are nonempty and $H_1(k,\varepsilon_k)$ and $H_2(k,\varepsilon_k)$ are disjoint by $\varepsilon_{k+1}\leq\varepsilon_k$.

If the disjoint sets $H_1(k,\varepsilon_k)$ and $H_2(k,\varepsilon_k)$ are nonempty, considering a normal agent $j\notin H_1(k,\varepsilon_k)$, that is $x_j(k)\leq x_M(0)-\varepsilon_k$, it has

$$\begin{aligned} x_j(k+1) &\leq (1-\gamma)x_M(k)+\gamma(x_M(0)-\varepsilon_k)\\ &\leq x_M(0)-\varepsilon_{k+1}, \end{aligned}$$

that is, $j\notin H_1(k+1,\varepsilon_{k+1})$. Similarly, we can show that if $j\notin H_2(k,\varepsilon_k)$, then $j\notin H_2(k+1,\varepsilon_{k+1})$.

Since graph $\mathcal{G}'$ is $(2f+1)$-robust, there exists an agent $j, j\in H_i(k,\varepsilon_k)$ for some $i\in\{1,2\}$, having at least $2f+1$ neighbors outside of its set. Suppose $j\in H_1(k,\varepsilon_k)$. Since there are at most f malicious agents in the neighborhood, agent j has at least $f+1$ normal neighbors outside $H_1(k,\varepsilon_k)$. By HP-MSR algorithm, at least one normal neighbor outside $H_1(k,\varepsilon_k)$ will be used by agent j to update its state. Hence, we have

$$\begin{aligned} x_j(k+1) &\le (1-\gamma)x_M(k) + \gamma(x_M(0) - \varepsilon_k) \\ &\le x_M(0) - \varepsilon_{k+1}. \end{aligned}$$

Analogously, it has that $x_j(k+p) \le x_M(0) - \varepsilon_{k+p},\ p \ge 1$. From $\varepsilon_{k+1} < \varepsilon_k$, it is obtained that $j \notin H_1(k + \tau_{max} + 1, \varepsilon_{k+\tau_{max}+1})$. Likewise, if agent $j \in H_2(k,\varepsilon_k)$ has at least $2f+1$ neighbors outside of its set, we can obtain that $j \notin H_2(k + \tau_{max} + 1, \varepsilon_{k+\tau_{max}+1})$.

Follow the steps above and we can conclude that there exists a time $T \le (\tau_{max} + 1)(n_1 + 2n_2)$ such that for some $i \in \{1,2\}$, the set $H_i(T,\varepsilon_T)$ becomes empty. We assume $H_1(T,\varepsilon_T)$ is empty, that is $x_M(T) \le x_M(0) - \varepsilon_T$. Since $T \le (\tau_{max} + 1)(n_1 + 2n_2)$, it follows that $x_M((\tau_{max} + 1)(n_1 + 2n_2)) \le x_M(T) \le x_M(0) - \varepsilon_T \le x_M(0) - \varepsilon_{(\tau_{max}+1)(n_1+2n_2)}$. Thus, we have

$$\begin{aligned} V((\tau_{max} + 1)(n_1 + 2n_2)) &= x_M((\tau_{max} + 1)(n_1 + 2n_2)) - x_m((\tau_{max} + 1)(n_1 + 2n_2)) \\ &\le x_M(0) - \varepsilon_{(\tau_{max}+1)(n_1+2n_2)} - x_m(0) \\ &= (1 - \eta\gamma^{(\tau_{max}+1)(n_1+2n_2)-1})V(0). \end{aligned}$$

Similarly, the inequality above holds if $H_2(T,\varepsilon_T)$ is empty. Follow the steps above and we have for $s \in N$, $V(s(\tau_{max} + 1)(n_1 + 2n_2)) \le (1 - \eta\gamma^{(\tau_{max}+1)(n_1+2n_2)})^s V(0)$.

Similar to the analysis above, we can obtain the inequalities $V(s(\tau_{max} + 1)(n_1 + 2n_2) + l) \le (1 - \eta\gamma^{(\tau_{max}+1)(n_1+2n_2)})^s V(l), l = 1, 2\ \dots\ , (\tau_{max} + 1)(n_1 + 2n_2) - 1$. Thus, we have $\lim\limits_{s\to+\infty} V(s(n_1 + 2n_2) + l) = 0$. It follows that $\lim\limits_{k\to+\infty} V(k) = 0$. Similar to the deduce in Theorem 1, we can get the resilient consensus is achieved. This completes the proof. ∎

Remark 3. In fact, the results we have gotten can be extended to heterogeneous system with time delays under switching topology by making a modification to the proposed algorithm. Different from step 1 of HP-MSR algorithm, at time k, each normal agent i receives full relative position information from its neighbors during time interval $[k-q,k]$, $k \ge q$. In the case that normal agent i may receives more than one values from the same neighbor, agent i just stores the most recent received value from its neighbor for later use. Then follow the step 2 and step 3 in HP-SMR. Take $h < \frac{1}{\sum_{j\in\mathcal{N}_i, i\in\mathcal{I}_1} a_{ij}}$ and $\sqrt{\sum_{j\in\mathcal{N}_i, i\in\mathcal{I}_2} a_{ij}} < \alpha < \frac{\sqrt{5}-1}{h}$. Similar to the proofs above, we can get the heterogeneous multiagent system using the modified HP-MSR algorithm can solve the resilient consensus problem if the union of the switching topologies $\bigcup\limits_{\tau=0}^{q} \mathcal{G}((k-\tau)h)$ are $(2f+1)$-robust for $k \ge q$.

Impulsive control strategy enjoys many advantages, such as fast transient, less energy, and more flexible design. In Step 3 of HP-MSR, normal agent i can update its state using the following impulsive protocol with sampled information:

$$u_i(t) = \begin{cases} h_1 \sum\limits_{k=0}^{\infty} \left[\sum\limits_{j\in\mathcal{N}_i\backslash\mathcal{R}_i(t)} a_{ij}(x_j(t) - x_i(t)) \right] \delta(t - t_k), & i \in \mathcal{I}_1, \\ \sum\limits_{k=0}^{\infty} \left[\sum\limits_{j\in\mathcal{N}_i\backslash\mathcal{R}_i(t)} a_{ij}(x_j(t) - x_i(t)) \right] \delta(t - t_k) - \alpha v_i(t), & i \in \mathcal{I}_2, \end{cases} \tag{11}$$

where $\delta(\cdot)$ is the Dirac function, the impulsive instants satisfy $0 = t_0 < t_1 < \dots < t_k < \dots$, $\lim\limits_{k\to+\infty} t_k = +\infty$, $t_{k+1} - t_k \equiv h$ and $h_1, \alpha > 0$ are control parameters.

It should be noted that the system is running without any control when $t \ne t_k$. This is quite different from the control inputs based on zero-order hold, where the control inputs are applied continuously through each sampling period. Let $s_1 = \min\left(\min\limits_{i\in\mathcal{I}_1\cup\mathcal{I}_2} x_i(0), \min\limits_{i\in\mathcal{I}_2}(x_i(0) + v_i(0))\right)$, $s_2 = \max\left(\max\limits_{i\in\mathcal{I}_1\cup\mathcal{I}_2} x_i(0), \max\limits_{i\in\mathcal{I}_2}(x_i(0) + v_i(0))\right)$ and $S = [s_1,s_2]$.

Theorem 3. *Suppose $h_1 < \frac{1}{\sum_{j\in\mathcal{N}_i, i\in\mathcal{I}_1} a_{ij}}$ and $h\sum_{j\in\mathcal{N}_i, i\in\mathcal{I}_2} a_{ij} + \frac{1}{2} < \alpha < 1$. Under the f-locally bounded model, the heterogeneous system using DH-MSR algorithm with control input (11) can solve resilient consensus problem if the communication graph $\mathcal{G}$ is $(2f+1)$-robust.*

Proof. For $i \in \mathcal{I}_2$, define $x_{n_2+n_3+i}(k) = x_i(k) + v_i(k)$. Thus, the discretized model of normal agents under input (11) can be written as:

$$\begin{cases} x_i(k+1) = x_i(k) + h \sum\limits_{j \in \mathcal{N}_i \setminus \mathcal{R}_i(k)} a_{ij}(x_j(k) - x_i(k)), i \in \mathcal{I}_1, \\ x_i(k+1) = x_i(k) + (1-\alpha)(x_{n_2+n_3+i}(k) - x_i(k)) + h \sum\limits_{j \in \mathcal{N}_i \setminus \mathcal{R}_i(k)} a_{ij}(x_j(k) - x_i(k)), i \in \mathcal{I}_2, \\ x_{n_2+n_3+i}(k+1) = x_{n_2+n_3+i}(k) + \beta(k)(x_i(k) - x_{n_2+n_3+i}(k)) + 2h \sum\limits_{j \in \mathcal{N}_i \setminus \mathcal{R}_i(k)} a_{ij}(x_j(k) - x_{n_2+n_3+i}(k)), i \in \mathcal{I}_2, \end{cases} \tag{12}$$

where $x_{n_2+n_3+i}(k) = x_i(k) + hv_i(k)$ and $\beta(k) = 2\alpha - 1 - 2h\sum_{j \in \mathcal{N}_i \setminus \mathcal{R}_i(k)} a_{ij}$. Following a similar proof to that of Theorem 1, we can get that the heterogeneous multiagent system with input (11) can reach resilient consensus with safety interval S if the communication graph $\mathcal{G}$ is $(2f+1)$-robust. ∎

Remark 4. Considering the restriction of network bandwidth, the agents sometimes cannot acquire the precise relative position information, but the information in its quantized form. Thus, the Step 3 in HP-MSR, agent i updates its state using the following predefined control input:

$$u_i(t) = \begin{cases} h_1 \sum\limits_{k=0}^{\infty} \left[\sum\limits_{j \in \mathcal{N}_i \setminus \mathcal{R}_i(t)} a_{ij} q(x_j(t) - x_i(t)) \right] \delta(t - t_k), \quad i \in \mathcal{I}_1, \\ \sum\limits_{k=0}^{\infty} \left[\sum\limits_{j \in \mathcal{N}_i \setminus \mathcal{R}_i(t)} a_{ij} q(x_j(t) - x_i(t)) \right] \delta(t - t_k) - \alpha v_i(t), \quad i \in \mathcal{I}_2, \end{cases} \tag{13}$$

where $q(\cdot)$ is a quantizer.

Logarithmic quantizer is one of the commonly used quantizers. Define the set of quantization levels as $U = \{\pm u_i, u_i = \rho^i u_0, i = \pm 1, \pm 2, \dots\} \cup \{\pm u_0\} \cup \{0\}$, where $u_0 > 0$ and accuracy parameter $0 < \rho < 1$. A logarithmic quantizer $q : R \to R$ is a map defined as: for $a > 0$, $q(a) = u_i$, $\frac{1}{1+\delta} u_i < a \leq \frac{1}{1-\delta} u_i$, for $a = 0$, $q(a) = 0$ and for $a < 0$, $q(a) = -q(-a)$, where $\delta = \frac{1-\rho}{1+\rho}$.

From the definition, it can be derived that $\forall a \in R$, $a - q(a) = \Delta a$, where $|\Delta| \leq \delta$. Thus, the heterogeneous multiagent systems with quantized relative information can be seen as a multiagent systems with time-varying weights. Suppose $h_1 < \frac{1}{(1+\delta)\sum_{j \in \mathcal{N}_i, i \in \mathcal{I}_1} a_{ij}}$ and $h(1+\delta)\sum_{j \in \mathcal{N}_i, i \in \mathcal{I}_2} a_{ij} + \frac{1}{2} < \alpha < 1$. Similar to the proofs above, it can be obtained that the heterogeneous multiagent system with control input (13) can solve the resilient consensus problem if the communication graph $\mathcal{G}$ is $(2f+1)$-robust.

4 | SIMULATIONS

In this section, we will provide three examples to demonstrate the effectiveness of the theoretical results.

Example 1. Let us consider a heterogeneous multiagent system with a 3-robust interaction graph $\mathcal{G}$ shown in Figure 1, in which 1 and 3 are normal agents with first-order dynamics, 5-7 are the normal agents with second-order dynamics, and 2 and 4 are malicious agents with dynamics $x_2(k+1) = -0.2k$ and $x_4(k+1) = 0.2k$. For simplification, we assume each edge weight is 1. According to Theorem 1, set sampling interval $h = 0.1$ and control gain $\alpha = 3$. We can see from Figure 2 that the consensus is not attained without using HP-MSR algorithm. Figure 3 shows the position trajectory of the system using the HP-MSR algorithm. We can see that normal agents reach consensus as expected.

Example 2. For the case that there are communication delays between agents, we assume, for simplification reasons, that each agent i has the same communication delays with their neighbors, that is, $\tau_{ij_1} = \tau_{ij_2}, j_1, j_2 \in \mathcal{N}_i$. Specifically, the communication delays are $\tau_{1j} = 7$, $\tau_{3j} = 2$, $\tau_{5j} = 3$, $\tau_{6j} = 7$, $\tau_{7j} = 0$. The simulation result is shown in Figure 4. We can see that the resilient consensus is achieved, but the communication delays leads to slower convergence speed.

Example 3. For the case that the normal agents update their states via impulsive input, we take sampling interval $h = 0.05$ and control gains $h_1 = 0.1$, $\alpha = 0.8$ according to Theorem 3. We can see from Figure 5 that normal agents reach consensus as expected.

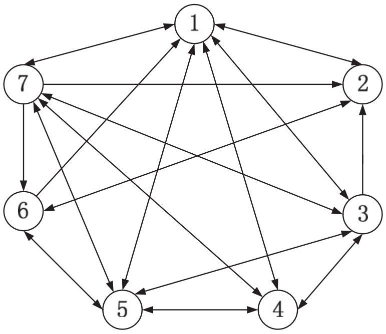

FIGURE 1 A 3-robust graph

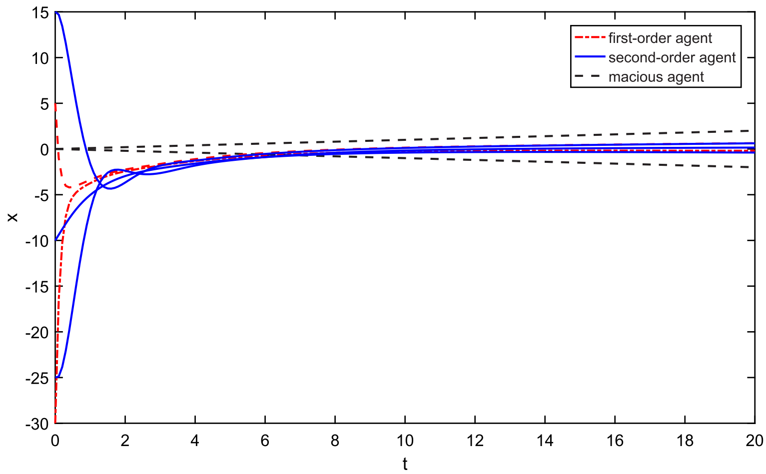

FIGURE 2 Position trajectories of agents without HP-MSR algorithm [Colour figure can be viewed at wileyonlinelibrary.com]

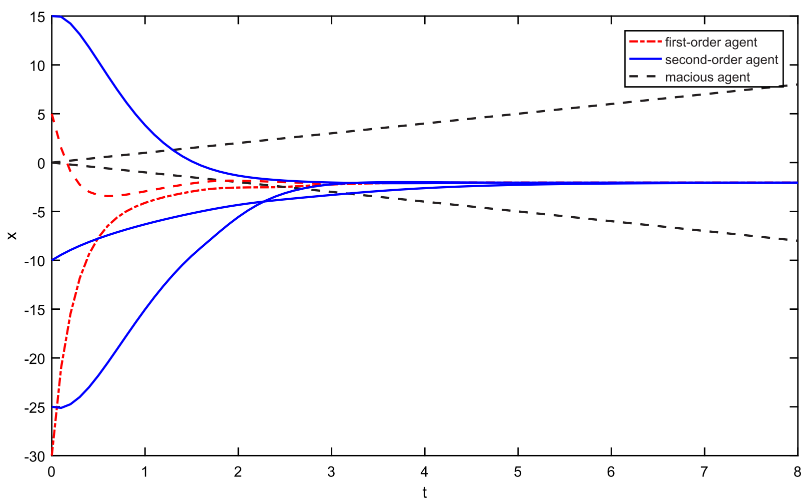

FIGURE 3 Position trajectories of agents with HP-MSR algorithm [Colour figure can be viewed at wileyonlinelibrary.com]

5 | CONCLUSIONS

In this article, the resilient consensus was considered for a group agents with heterogeneous dynamics under directed communication graph. HP-MSR algorithm was employed to mitigate the effects of malicious agents. We proved that the resilient consensus can be achieved if the communication graph is $(2f+1)$-robust. We further dealt the case with communication delays. We showed that if the time-varying delays are bounded, the same conclusion can be drawn as in the case without communication delays. Moreover, sufficient topological condition in terms of graph robustness was obtained for resilient consensus with impulsive control techniques. For future research, it is interesting to consider the resilient consensus of heterogeneous multiagent system with event-based control techniques. The other extensions of the

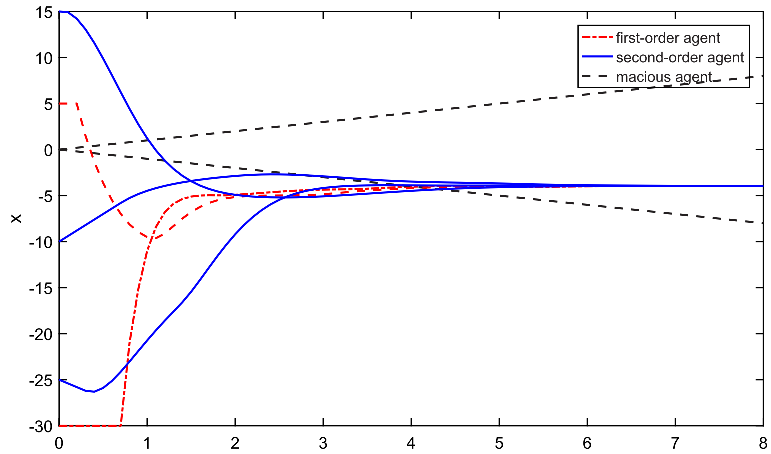

FIGURE 4 Position trajectories of agents with HP-MSR algorithm and communication delays [Colour figure can be viewed at wileyonlinelibrary.com]

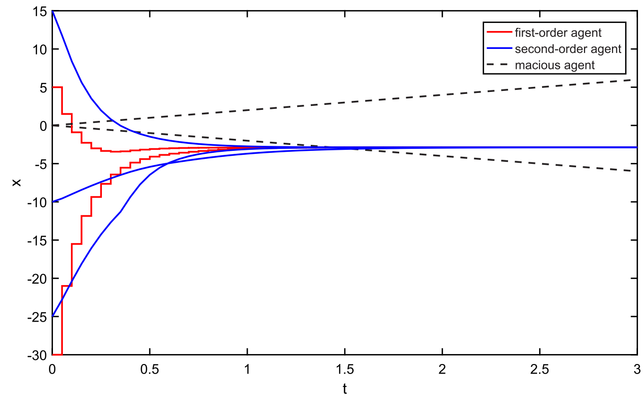

FIGURE 5 Position trajectories of agents with HP-MSR algorithm and impulsive input [Colour figure can be viewed at wileyonlinelibrary.com]

results presented in this article is to further study this problem for heterogeneous multiagent system with general linear dynamics.

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