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Fixed-time stability of positive nonlinear systems

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Abstract

This paper studies the fixed-time stability problem for positive nonlinear systems defined by cooperative vector fields. A criterion is derived to ensure fixed-time stabilization of the positive nonlinear systems using Lyapunov method. In addition, a sufficient condition is also presented for fixed-time stability of switched positive nonlinear systems under arbitrary switching. Simulation examples are provided to demonstrate the obtained results.

Keywords

Positive nonlinear systems, cooperative systems, fixed-time stability

Introduction

Many real-world systems in areas such as biology, physiology, ecology, economics, population dynamics and communications involve physical quantities that stay within the positive orthant. Such systems whose states should be always nonnegative, if started from nonnegative initial conditions, are commonly referred to positive systems. Multi-agent systems of single or double integrators are good examples of positive systems (Ma et al., 2019; Ren et al., 2007; Valcher and Misra, 2014; Zhu et al., 2018). Stability is an important property of positive systems and has been extensively investigated. There are abundant results available in the literatures for the stability of positive linear systems (Farina and Rinaldi, 2011; Liu et al., 2010; Sun, 2016; Sun et al., 2017). It is well known that nonlinearity is ubiquitous in many practical systems and the nonlinearity makes system analysis more difficult (Chang et al., 2018, 2019; Xia et al., 2018b, 2019). It is natural to extend the properties of positive linear systems to positive nonlinear systems. In Mason and Verwoerd (2009), the authors extended two fundamental properties to homogeneous cooperative positive nonlinear systems. In Feyzmahdavian et al. (2014b), a necessary and sufficient condition was obtained for exponential stability of homogeneous cooperative positive systems composed of two homogeneous nonlinear parts of degree one. In Feyzmahdavian et al. (2014a) and Dong (2015), the authors extended the work in Feyzmahdavian et al. (2014b) to homogeneous cooperative systems with any homogeneous degree. In Bokharaie et al. (2011), some results on stability of homogeneous systems were extended to subhomogeneous systems. In reality, many dynamic systems can be modeled as switched systems (Li et al., 2018; Liu et al., 2018 ; Xie et al., 2019; Zheng et al., 2018). Thus, stability of positive switched nonlinear systems also attracts a lot of attention. In Liu (2015), the author investigated the stability problem for nonlinear positive switched systems with delays. In Dong (2016), a sufficient and necessary condition was derived for exponential stability of switched homogeneous cooperative systems with degree one under the average dwell time switching.

Finite-time stability means that the system trajectories converge to the equilibrium within finite time, which has been extensively studied (Bhat and Bernstein, 2000; Gao et al., 2015; Liu et al., 2019; Qi and Gao, 2016; Shen and Wang, 2017; Xia et al., 2018a; Zheng and Wang, 2012; Zheng et al., 2014). In Qi and Gao (2016), by using a co-positive Lyapunov function method, sufficient conditions were obtained for finite-time stability of positive switched systems with time delays. In Shen and Wang (2017), finite-time L_1 control was investigated for positive Markovian jump systems. In Liu et al. (2019), finite-time stability was studied for nonlinear impulsive positive switched systems in use of average dwell time technique. For a finite-time stable system, if the convergence time of the system is independent of the initial states, the system is said to be fixed-time stable. So, when the initial state of a fixed-time stable system is unknown, the convergence time can be obtained in advance. Actually, in many practical applications, it is often hard for us to get the initial states of systems. Thus, compared with the finite-time stability, the fixed-time stability is more meaningful. Fixed-time

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stability was first discovered in Andrieu et al. (2008). In Polyakov (2012), the authors obtained a Lyapunov sufficient condition for fixed-time stability of nonlinear systems. In Polyakov et al., (2015), Implicit Lyapunov Function method was developed for fixed-time stability analysis. In Yu et al. (2019), the authors extended some results of fixed-time stability for deterministic systems to stochastic systems. However, to the best of our knowledge, there are no literature considering the fixed-time stability of positive systems, which will be the subject of this paper. Different from the positive systems in Feyzmahdavian et al. (2014a, 2014b) and Dong (2015), the system in this paper is composed of two homogeneous nonlinear parts of different degree. Thus, the system is nonhomogeneous, which undoubtedly increases the difficulty of analysis. Using Lyapunov function method, we derive a sufficient condition for fixed-time stability. In addition, we consider the fixed-time stability of switched positive nonlinear systems under arbitrary switching. The numerical simulations are given to demonstrate the obtained results.

The rest of this paper is organized as follows. First, some mathematical preliminaries are presented. Some sufficient conditions on fixed-time stability are next established. Two examples are then given to demonstrated the obtained results. Finally, a conclusion is given.

Notation

Let R and R^n denote the set of real numbers and n-dimensional real vectors, respectively, $\mathcal{I}_n = \{1, 2, \dots, n\}$. For a vector $x \in R^n$, x_i denotes the i th coordinate of x . R^n_+ denotes the set of all vectors in R^n with nonnegative entries, i.e., $R^n_+ = \{x \in R^n : x_i \geqslant 0, i \in \mathcal{I}_n\}$. For vectors $x, y \in R^n$, $x \geqslant y(x?y)$ means $x_i \geqslant y_i(x_i > y_i)$ for $i \in \mathcal{I}_n$; $x > y$ means that $x \geqslant y$ and $x \neq y$. For a matrix $A = [a_{ij}]_{n\times n}$, if $a_{ij} \geqslant 0$ for every $i \neq j$, the matrix A is said to be Metzler. For a continuous function $f : R \to R$, the upper-right Dini derivative of is defined as $D^+f(t)|_{t=t_0} = \limsup\limits_{\Delta \to 0^+} \frac{f(t_0+\Delta) - f(t_0)}{\Delta}$.

Preliminaries

Consider the system

$$\dot{x}(t) = f(x(t)) \tag{1}$$

where $x(t) \in R^n$ is the system state and $f(\cdot) : R^n \to R^n$ is a nonlinear function. For convenience, we use 0 and x_0 to denote the initial time and the initial state of system (1), respectively. $X(t, x_0)$ denotes the solution of system (1) with the initial state x_0 . We say that system (1) is positive if for $\forall x_0 \in R^n_+$, the corresponding solution $X(t, x_0) \in R^n_+, \forall t \geqslant 0$.

In the following, some definitions and a lemma to be used are listed:

Definition 1: (Bhat and Bernstein, 2000) The origin of the system (1) is globally finite-time stable if it is globally asymptotically stable and there exists a function $T(x_0) : R^n_+ \to R_+$ such that $X(t, x_0) = 0, \forall t \geqslant T(x_0)$. The function $T(x_0)$ is called the settling time function.

Definition 2. (Polyakov et al., 2015) The origin of the system (1) is globally fixed-time stable if it is globally finite-time stable and there exists a constant $T_{\max} > 0$ such that $T(x_0) \leqslant T_{\max}$ for all $x_0 \in R^n_+$.

Definition 3. (Smith, 2008) A vector field $f : R^n \to R^n$, which is continuously differentiable on $R^n \backslash \{0\}$, is said to be cooperative if the Jacobian matrix $(\partial f/\partial x)(a)$ is Metzler for all $a \in R^n_+ \backslash \{0\}$.

Lemma 1. (Smith, 2008) Let f be a cooperative vector field. For $\forall x, y \in R^n_+ \backslash \{0\}$ satisfying $x \geqslant y$ and $x_i = y_i$, it has $f_i(x) \geqslant f_i(y)$.

Definition 4: (Feyzmahdavian et al., 2014a) A vector field $f : R^n \to R^n$ is homogeneous of degree $\alpha > 0$ if $f(\lambda x) = \lambda^\alpha f(x)$ for $\forall x \in R^n$ and $\forall \lambda > 0$.

Main results

In this paper, we first consider the following nonlinear system

$$\dot{x}(t) = f(x(t)) + g(x(t)) \tag{2}$$

where $x(t) \in R^n$ is the system state, nonlinear functions $f(\cdot), g(\cdot) : R^n \to R^n$ are continuous on R^n and continuous differentiable on $R^n \backslash \{0\}$, which implies that there exists a unique solution of system (2) for any $x_0 \in R^n_+$. In addition, we assume that $f(\cdot)$ and $g(\cdot)$ satisfy the following assumption.

Assumption 1: The following properties hold:

(1) $f(\cdot)$ and $g(\cdot)$ are cooperative;
(2) $f(\cdot)$ is homogeneous of degree α and g is homogeneous of degree β.

Remark 1: It is shown in Smith (2008) that cooperative systems are monotone. Formally, for system (1), if f is cooperative on R^n, then $x_0 \leqslant y_0, x_0, y_0 \in R^n_+$, implies $X(t, x_0) \leqslant X(t, y_0)$ for all $t \geqslant 0$. Since f and g are cooperative, it is easy to have that system (2) is a cooperative system. The homogeneity of f and g implies that $f(0) = 0$ and $g(0) = 0$. Hence, it has $X(t, 0) = 0$ in system (2). Using the monotonicity of the cooperative system, it follows that the solutions of system (2) satisfy that $X(t, 0) \leqslant X(t, x_0)$ for $\forall x_0 \in R^n_+$, that is system (2) is positive. Throughout this paper, we always assume that the initial conditions of system (2) are nonnegative.

Theorem 1: Consider the system (2) under Assumption 1. If $0 < \alpha < 1$, $\beta > 1$ and $\exists \omega \gg 0$ such that $f(\omega) \ll 0$ and $g(\omega) \ll 0$, the system (2) is globally fixed-time stable.

Proof: Take the Lyapunov function

$$V(x(t)) = \max_{i \in \mathcal{I}_n} \frac{x_i(t)}{\omega_i},$$

where ω_i is the ith coordinate of ω.

Let the index $s \in \mathcal{I}_n$ be such that $V(x(t)) = \frac{x_s(t)}{\omega_s}$. Obviously, s can be the function of t. Note that $x(t) \leqslant V(x(t))\omega$ and $x_s(t) = V(x(t))\omega_s$.

From Assumption 1 and Lemma 1, we have

$$f_s(x(t)) \leqslant f_s(V(x(t))\omega) = V^\alpha(x(t)) f_s(\omega),$$

and

$$g_s(x(t)) \leqslant g_s(V(x(t))\omega) = V^{\beta}(x(t))g_s(\omega).$$

It follows from the above two inequalities that

$$\frac{\dot{x}_s(t)}{\omega_s} = \frac{f_s(x(t)) + g_s(x(t))}{\omega_s} n \leqslant \frac{f_s(\omega)}{\omega_s} V^{\alpha}(x(t)) + \frac{g_s(\omega)}{\omega_s} V^{\beta}(x(t)).$$

Since $f(\omega) \ll 0$ and $g(\omega) \ll 0$, it has

$$\frac{\dot{x}_s(t)}{\omega_s} \leqslant -\eta V^{\alpha}(x(t)) - \gamma V^{\beta}(x(t)),$$

where $\eta = -\max_{i\in\mathcal{I}_n} \frac{f_i(\omega)}{\omega_i} > 0$ and $\gamma = -\max_{i\in\mathcal{I}_n} \frac{g_i(\omega)}{\omega_i} > 0$.

Thus, we obtain that

$$D^+ V(x(t)) \leqslant -\eta V^{\alpha}(x(t)) - \gamma V^{\beta}(x(t)).$$

From the above inequality, it is obvious that $D^+ V(x(t)) \leqslant -\eta V^{\alpha}(x(t))$ *and* $D^+ V(x(t)) \leqslant -\gamma V^{\beta}(x(t))$. Hence, $V(x(0)) \geqslant 1$, from $D^+ V(x(t)) \leqslant -\gamma V^{\beta}(x(t))$, it has $V(x(t)) = 1$ for $t \geqslant \int_1^{V(x(0))} \frac{1}{\gamma V^{\beta}(x(t))} dV = \frac{1}{\gamma(\beta-1)} - \frac{V^{1-\beta}(x(0))}{\gamma(\beta-1)} \geqslant \frac{1}{\gamma(\beta-1)}$. If $V(x(0)) \leqslant 1$, from $D^+ V(x(t)) \leqslant -\eta V^{\alpha}(x(t))$, we have $V(x(t)) = 0$ for $t \geqslant \int_0^{V(x(0))} \frac{1}{\eta V^{\alpha}(t)} dV = \frac{V^{1-\alpha}(x(0))}{\eta(1-\alpha)}$.

It follows from $V(x(0)) \leqslant 1$ that $\frac{V^{1-\alpha}(x(0))}{\eta(1-\alpha)} \leqslant \frac{1}{\eta(1-\alpha)}$. Thus, we have that $V(x(t)) = 0$ for $t \geqslant \frac{1}{\eta(1-\alpha)} + \frac{1}{\gamma(\beta-1)}$, that is the system (2) is globally fixed-time stable with $T_{\max} = \frac{1}{\eta(1-\alpha)} + \frac{1}{\gamma(\beta-1)}$.

Remark 2: Since $0 < \alpha < 1$ and $\beta \geqslant 1$ in Theorem 1, system (2) is nonhomogeneous. Except this case, similar to the deduce in Theorem 1, we can get that if $\alpha, \beta \geqslant 1$, system (2) is globally asymptotically stable and if $0 < \alpha, \beta < 1$, system (2) is globally stable in finite time.

Remark 3: Note that if $\alpha = \beta$, system (2) is a homogeneous cooperative system. Thus, according to Theorem 3.3 in Feyzmahdavian et al. (2014b), the positive system (2) is asymptotically stable if and only if there exists a vector ω?0 such that $f(\omega) + g(\omega) \ll 0$. However, this condition is not sufficient for f and g of different homogeneity degree, which is demonstrated in Example 2.

Next, we will consider the fixed-time stability problem for the following switched positive nonlinear system

$$\dot{x}(t) = f_{\sigma(t)}(x(t)) + g_{\sigma(t)}(x(t)), \tag{3}$$

where $x(t) \in R^n$ is the system state and $\sigma(t) : [0, +\infty) \to \mathcal{I}_N$ is the switching signal, which is a piecewise constant and right continuous function. $\mathcal{I}_N$ is an index set and stands for the collection of subsystems. We denote the switching times by $0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots$. To avoid Zeno phenomena, we assume that there exists a constant $\tau > 0$, such that $t_{s+1} - t_s > \tau$. For every $p \in \mathcal{I}_N$, $f_p(\cdot)$ and $g_p(\cdot)$ are assumed to be continuous on R^n and continuous differentiable on $R^n \backslash \{0\}$.

We further assume that $f_p(\cdot)$ and $g_p(\cdot)$ satisfy the following condition.

Assumption 2: The following properties hold:

(1) $f_p(\cdot)$ and $g_p(\cdot)$ are cooperative;
(2) $f_p(\cdot)$ is homogeneous of degree α_p and $g_p(\cdot)$ is homogeneous of degree β_p.

Theorem 2: Consider the switched system (3) under assumption 2. If $0 < \alpha_p < 1, \beta_p > 1$ for $\forall p \in \mathcal{I}_N$ and $\exists \omega$?0 such that $f_p(\omega) \ll 0$ and $g_p(\omega) \ll 0$, the switched system (3) is globally fixed-time stable under arbitrary switching.

Proof: Take $V(x(t)) = \max_{i\in\mathcal{I}_n} \frac{x_i(t)}{\omega_i}$ as the common Lyapunov function. Consider an interval $[t_k, t_{k+1}), k \geqslant 0$, on which $\sigma(t) = p$. Similar to the deduce in Theorem 1, we have $D^+ V(t) \leqslant -\eta(V(t))^{\alpha_p} - \gamma(V(t))^{\beta_p}$, where $\eta = -\max_{p\in\mathcal{I}_N, i\in\mathcal{I}_n} \frac{f_{pi}(\omega)}{\omega_i}$ and $\gamma = -\max_{p\in\mathcal{I}_N, i\in\mathcal{I}_n} \frac{g_{pi}(\omega_p)}{\omega_i}$. Thus, we can obtain that $V(x(t)) = 0$ for $t \geqslant \frac{1}{\eta(1-\alpha_{\max})} + \frac{1}{\eta(\beta_{\min}-1)}$, where $\alpha_{\max} = \max_{p\in\mathcal{I}_N} \alpha_p$ and $\beta_{\min} = \min_{p\in\mathcal{I}_N} \beta_p$, that is the system (3) is globally fixed-time stable at the origin.

Numerical simulations

In this section, three examples are given to demonstrate the obtained results in this paper.

Example 1: Consider the system (2) with the following nonlinear functions

$$f(x) = \begin{bmatrix} (x_1^2 + x_2^2)^{\frac{1}{4}} - (3x_1^2 + x_2^2)^{\frac{1}{4}} \\ (x_1^2 + x_2^2)^{\frac{1}{4}} - (x_1^2 + 2x_2^2)^{\frac{1}{4}} \end{bmatrix},$$

$$g(x) = \begin{bmatrix} -2x_1^2 + x_2^2 \\ 2x_1^2 - 3x_2^2 \end{bmatrix}.$$

We can see f and g are homogeneous with degree $\alpha = \frac{1}{2}$ and $\beta = 2$. Note that $\partial f/\partial x = \frac{1}{2}\begin{pmatrix} * & x_2(x_1^2 + x_2^2)^{-\frac{3}{4}} - x_2(3x_1^2 + x_2^2)^{-\frac{3}{4}} \\ x_1(x_1^2 + x_2^2)^{-\frac{3}{4}} - x_1(x_1^2 + 2x_2^2)^{-\frac{3}{4}} & * \end{pmatrix}$ *and* $\partial g/\partial x = \begin{pmatrix} * & 2x_2 \\ 4x_1 & * \end{pmatrix}$. According to Definition 3, f and g are also cooperative. Since $\alpha \neq \beta$, system (2) is nonhomogeneous. Moreover, there exists a vector $s = [1, 1]^T$ such that $f(s) = [-0.4142, -0.3161]^T$ and $g(s) = [-1, -1]^T$. Thus, we get $\eta = 0.1269$ and $\gamma = 1$. Then, it follows from Theorem 1 that the positive system (2) is fixed-time stable with $T_{max} = 16.7604$. Figure 1 shows the simulation results.

Example 2: If g(x) in Example 1 is as follows

$$g(x) = 0.1\begin{bmatrix} x_2^3 \\ x_1^3 \end{bmatrix}.$$

We can see f and g are cooperative and homogeneous. Note the function g is also nondecreasing, that is $g(x) \geqslant g(y)$ for $\forall x, y \in R_+^n$ such that $x \geqslant y$. If g has the same homogeneity

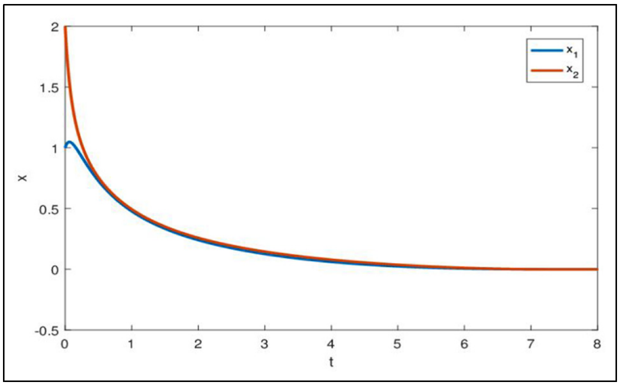

Figure 1. State trajectories of system (2).

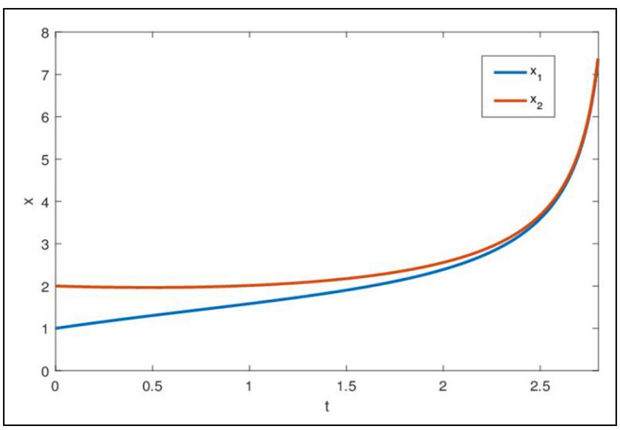

Figure 2. State trajectories of system (2).

degree with f, the authors in Feyzmahdavian et al. (2014a) pointed out that the system is stable if and only if there exists a vector $\omega \gg 0$ such that $f(\omega) + g(\omega) \ll 0$. In this example, there exists a vector $s = [1, 1]^T$ such that $f(s) + g(s) = [-0.1250, -0.0269]^T$, but f and g are of different homogeneity degree. Take the initial condition $x_0 = [1, 2]^T$ and the simulation result is shown in Figure 2. We can see that the system is unstable. Thus, the condition $f(\omega) + g(\omega) \ll 0$ is not sufficient when f and g are of different homogeneity degree.

Example 3: Consider the switched system (3) with the following nonlinear functions

$$f_1(x) = \begin{bmatrix} (x_1^2 + x_2^2)^{\frac{1}{4}} - (5x_1^2 + x_2^2)^{\frac{1}{4}} \\ (x_1^2 + x_2^2)^{\frac{1}{4}} - (x_1^2 + 2x_2^2)^{\frac{1}{4}} \end{bmatrix},$$

$$g_1(x) = \begin{bmatrix} -2x_1^2 + x_2^2 \\ 2x_1^2 - 3x_2^2 \end{bmatrix},$$

$$f_2(x) = \begin{bmatrix} (2x_1^2 + x_2^2)^{\frac{1}{3}} - (4x_1^2 + x_2^2)^{\frac{1}{3}} \\ (x_1^2 + x_2^2)^{\frac{1}{3}} - (x_1^2 + 2x_2^2)^{\frac{1}{3}} \end{bmatrix},$$

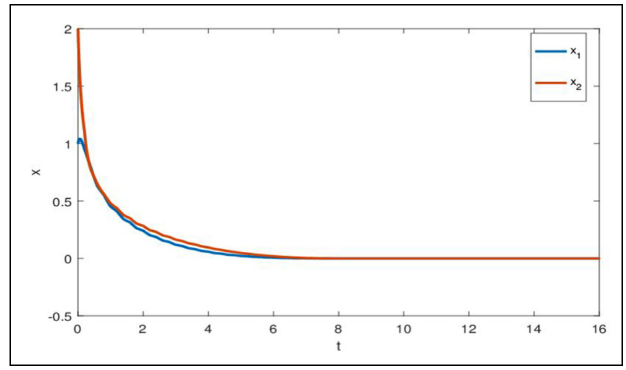

Figure 3. State trajectories of system (3).

and

$$g_2(x) = \begin{bmatrix} -2x_1^3 + x_2^3 \\ x_1^3 - 3x_2^3 \end{bmatrix}.$$

It is easy to verify that f_1, g_1, f_2 and g_2 are cooperative and homogeneous with degrees $\alpha_1 = \frac{1}{2}$, $\beta_1 = 2, \alpha_2 = \frac{2}{3}, \beta_2 = 3$, respectively. Thus, $\alpha_{\max} = \frac{2}{3}$ and $\beta_{\min} = 2$. Moreover, there exists a vector $s = [1, 1]^T$ such that $f_1(s) = [-0.2250, -0.1269]^T$, $g_1(s) = [-1, -1]^T$, $f_2(s) = [-0.3759, -0.1823]^T$ and $g_2(s) = [-1, -2]^T$. By simple calculation, we get $\eta = 0.1269$ and $\gamma = 1$. According to Theorem 2, the positive system (3) is fixed-time stable with $T_{\max} = 24.6407$. For simplicity, we assume the two subsystems are switched alternately with the same duration time. The state trajectories of the system with $x_0 = [1, 2]^T$ are depicted in Figure 3.

Conclusions

This paper derives a fixed-time stability condition for positive nonlinear systems. Different from existing works, the nonlinear system is composed of two parts of different homogeneity degree. In addition, a sufficient condition is presented for the fixed-time stability of switched positive systems. We also give some examples to demonstrate the results obtained in this paper. It will be interesting to consider the stability conditions for the positive systems proposed here with time delays.

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