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Winner-take-all competition with heterogeneous dynamic agents

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A B S T R A C T

Winner-take-all competition exists widely in nature and has been applied in many engineering fields. This paper mainly investigates a group of heterogeneous dynamic agents, which produce the winner-take-all competition. For the heterogeneous system consisting of first- and second-order dynamic agents, we propose two different kinds of protocols with and without velocity measurements, respectively. Firstly, we employ the Lasalle's invariant principle to solve the equilibrium points of the proposed system. Secondly, we prove that the proposed protocols can solve the winner-take-all problems for the heterogeneous systems. The results reveal that winner is independent from the dynamics of agents, but is determined by inputs. Finally, some examples are also gave to verify the validity of the proposed protocols.

1. Introduction

Swarm behaviour exists widely in nature, such as birds migration, ants move, the territory rivalry of wolves. Interaction of complex intra-personal and inter-personal forces operating in a swarm which determine the swarm's development, character and long-term survival. Inspired by the swarm behavior of biosystem, more and more researchers are paying attention to swarm behaviour and trying to apply it in industry systems. The classical topics of swarm behaviour mainly focus on consensus [1], formation [2], flocking [3], containment [4], winner-take-all [5], etc.

Winner-take-all competition [6] is one of the foundational Lotka-Volterra models, it is motivated by the competition observed and lateral inhibition among neurons in the brain [7]. In nature, winner-take-all phenomenon can be seen everywhere, for examples, the dominant growth of the central stem over others [8], foraging behavior and mating behavior [9], competitive decision-making behavior in the cerebral cortex [10]. Winner-take-all has also been applied in many engineering fields, especially integrated circuit [11,12]. Up to now, many models are presented by researchers to explain the winner-take-all phenomenon. Kevin et al. [13] proposed a kind of DNA-based winner-take-all multiple agents system which can scale up molecular pattern recognition. In [14], Kaski and Kohonen studied a winner-take-all competition with less than ten neurons. They derived exact formulas for the optimal parameters of winner-take-all model, such that their robustness with respect to the simplified structure are maximum. By introducing a clipped total feedback, Andrew [15] improved the robustness of the model in [14], and it also increased the number of neurons in the aforesaid model. In [16], Fukai and Tanaka used Lotka–Volterra competitive model to solve winner-take-all problem. In [17], Zhang et al. considered the stability problem for Lotka–Volterra model with delays, and obtained some criteria for nondivergence of the networks. Moreover, they also studied the global convergence problem for Lotka–Volterra model with variable delays in [18]. In [19], Li et al. proposed a cluster of first-order continuous-time agents, and proved the convergence analytically. Winner-take-all competition with discrete-time dynamics was also considered in [20].

In the past several years, swarm behaviour with first-order dynamic agents [21–26] or second-order dynamic agents [27–29] is studied by lots of researchers. However, the agents' dynamics are not necessarily same in actual systems. For instance, in view of uncertainty external and dynamic environments to the system which consist of many robots, homogeneous systems are not as practical as heterogeneous systems including robots with different abilities and structures in the real world [30]. Hence, many interests are focusing on heterogeneous dynamic agents. In [31–33], Zheng et al. considered the asymptotic/finite-time consensus problem of heterogeneous dynamic agents, respectively. In [34], Chen et al. investigated the flocking behavior and targets consensus tracking problems of heterogeneous multiple inertial agents with limited communication ranges. In [4], Zheng and Wang studied the containment problem for heterogeneous dynamic agents. As is well

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know, heterogeneity means hybrid in natural and artificial systems. In [35–37], Zheng et al. presented the hybrid multi-agent systems which composed of discrete- and continuous-time dynamic agents. Some meaningful interaction protocols were designed for the hybrid multi-agent systems. By using the system transformation method, graph theory, game theory and matrix theory, the consensus criteria were also obtained respectively.

Up to now, most of the existing results of winner-take-all were formulated by first-order dynamic agents [14–20]. However, different types of agents may interact with each other in practice. For instance, agents with various dynamics can cooperate or compete with each other. Consequently, we consider the winner-take-all problem for the system with heterogeneous dynamic agents. In order to solve the winner-take-all problem for the system with heterogeneous dynamic agents, the protocols with/without velocity information are proposed, respectively. Owing to the nonlinearity of designed protocols, it makes the difficulty of convergence analysis. By using the Lasalle's invariant principle, we obtain the equilibrium points of this heterogeneous system. Then, we prove the stability of winner-take-all solution and the instability of non-winner-take-all solutions. The main contributions of this paper concentrate on three aspects. Firstly, we present two types of nonlinear protocols which can solve the winner-take-all problem for the system with heterogeneous dynamic agents. Secondly, we use Lasalle's invariant principle and Lyapunovs indirect method to analyze the stability of the proposed nonlinear system. Last but not the least, we find that the winner is the agent with the maximal input. This fact implies that the winner is determined only by the size of the inputs, not by the dynamic of the agent itself.

An outline of this paper is shown as follows. We formulate the heterogeneous dynamic agents to be investigated, and introduce some related conceptions and key lemma in Section 2. In Section 3, we propose a kind of protocol with velocity measurements, and further prove that the heterogeneous dynamic agents can perform winner-take-all function. In Section 4, we solve the winner-take-all problem of heterogeneous dynamic agents without velocity measurements. In Section 5, some examples are given to verify the effectiveness of our results. Section 6 is a brief conclusion.

Notation: Throughout this paper, we let $\mathbb{R}$, $\mathbb{R}^n$ respectively denotes the set of real number, the $n\times n$ real vector space. $\mathcal{I}_n=\{1,2,\dots,n\}$, and $\mathcal{I}_n/\mathcal{I}_m=\{m+1,m+2,\dots,n\}$. E_n denotes the $n\times n$ identity matrix. For a given matrix or vector X, we write X^T for transpose, $||X||$ for the 2-norm of X. diag(X) for the diagonal matrix with the vector X on its diagonal and all non-diagonal elements being zero. Let $\mathbf{e}_i$ be the canonical vector with a 1 on its i-th entry and all the other elements being 0. $\mathrm{R_e}z$ is a real part of complex variable z. $\underset{x}{\mathrm{argmax}}(f(x))$ stands for the argument of the maximum, i.e. $\underset{x}{\mathrm{argmax}}(f(x))=\{x\mid\forall y: f(y)\le f(x)\}$.

2. Preliminaries

We introduce the concept of winner-take-all competition for a group of heterogeneous dynamic agents and the related theoretical results which will be used in the proof of winner-take-all competition problem in this section.

Firstly, we give the definition of winner-take-all competition.

Definition 1. The winner is the agent which keep active all the time, while the remaining agents eventually lose activity to reach zero.

Consider a heterogeneous group of agents, which contains m second-order dynamic agents and $n-m$ first-order dynamic agents. Without loss of generality, we suppose that agent $1\sim m$ are second-order dynamic agents. Thus, the heterogeneous agents' dynamics are described as follows

$$\begin{cases}\dot{p}_i=q_i, & i\in\mathcal{I}_m,\\ \dot{q}_i=u_i, & i\in\mathcal{I}_m,\\ \dot{p}_i=u_i, & i\in\mathcal{I}_n/\mathcal{I}_m,\end{cases}\tag{1}$$

where $p_i\in\mathbb{R}$, $q_i\in\mathbb{R}$ and $u_i\in\mathbb{R}$ denotes the position-like, velocity-like and protocols of the agent i, respectively. The initial status of the agent i are $p_i(0)$ and $q_i(0)$. We let $P=(p_1,p_2,\dots,p_n)^T,\ Q=(q_1,q_2,\dots,q_m)^T$.

Secondly, we introduce the Lasalles invariant principle and Lyapunovs indirect method.

Consider the autonomous nonlinear system

$$\dot{x}=f(x),\tag{2}$$

where $f:S\to\mathbb{R}^n$ is continuously differentiable with $S\subset\mathbb{R}^n$.

Lemma 1 [4]. *Let $\Psi\subset S$ be a positively invariant compact set with respect to (2). Let $V:S\to\mathbb{R}^n$ be a continuously differentiable function which satisfy $\dot{V}(x)\le 0$ in Ψ. Let F be the set of all points in Ψ which satisfy $\dot{V}(x)=0$. Let K be the largest invariant set in F. Then every solution starting in Ψ approaches K as $t\to\infty$.*

Lemma 2 [38]. *Assume that $x=x_0$ is an equilibrium point of system (2) and S is a neighborhood of this equilibrium point. Let*

$$A=\frac{\partial f}{\partial x}(x)|_{x=x_0}.$$

(1) If $\mathrm{R_e}\lambda_i<0$ for all eigenvalues of A, then we have the equilibrium point $x=x_0$ is asymptotically stable;

(2) If $\mathrm{R_e}\lambda_i>0$ for one or more of the eigenvalues of A, then we have the equilibrium point $x=x_0$ is unstable.

3. Protocol with velocity measurements

In this section, we present a kind of protocol for system (1), in which the position-like and velocity-like information can be obtained at the any time. Then we prove that (1) can perform winner-take-all competition under this protocol, and the agent with the maximal input wins the competition. The protocol is given as

$$\begin{cases}u_i=(h_i-||P||^2)p_i-k_1q_i, & i\in\mathcal{I}_m,\\ u_i=(h_i-||P||^2)p_i, & i\in\mathcal{I}_n/\mathcal{I}_m,\end{cases}\tag{3}$$

where $h_i>0$ is the input of agent i, $k_1>0$ is feedback gain.

Remark 1. According the protocol (3), we know that every agent can get its own information of position, input and velocity directly. However, $||P||^2={p_1}^2+{p_2}^2+\dots+{p_n}^2$ needs the information of all the agents of system (1) at any time t (Fig. 1). Winner-take-all competition among the agents with heterogeneous dynamics can emerge only if every agent i can get it's own information and the global statistic $||P||^2$.

Assumption A1: Assume that $k^*=\underset{i}{\mathrm{argmax}}(h_i)$, and $\forall i\ne k^*$, $i\in\mathcal{I}_n$, we all have $h_{k^*}>h_i$.

Theorem 1. *Suppose that A1 holds. Then, the system (1) can achieve winner-take-all competition with protocol (3) for any initial conditions. Moreover, the solution of winner k^* approaches $(p_e^*,q_e^*)=(\sqrt{h_{k^*}},0)$ or $(-\sqrt{h_{k^*}},0)$ and all other agents approaches $(p_e,q_e)=(0,0)$ as $t\to\infty$.*

Proof. It follows from (1) and (3) that

$$\begin{cases}\dot{p}_i=q_i, & i\in\mathcal{I}_m,\\ \dot{q}_i=(h_i-||P||^2)p_i-k_1q_i, & i\in\mathcal{I}_m,\\ \dot{p}_i=(h_i-||P||^2)p_i, & i\in\mathcal{I}_n/\mathcal{I}_m.\end{cases}\tag{4}$$

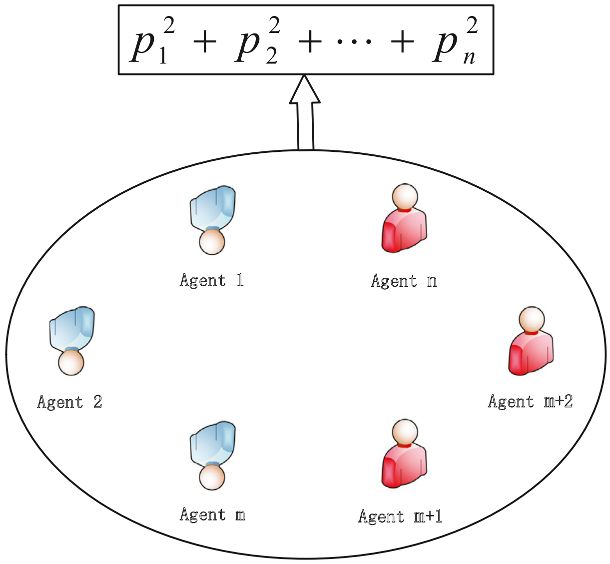

Fig. 1. Information stream with heterogeneous dynamic agents.

We rewrite (4) in matrix form as follows

$$\begin{cases} \dot{P}_1 = Q, \\ \dot{Q} = (H_1 - ||P||^2 E_m)P_1 - k_1 Q, \\ \dot{P}_2 = (H_2 - ||P||^2 E_{n-m})P_2, \end{cases} \tag{5}$$

where $P_1 = (p_1, p_2, \ldots, p_m)^T$, $P_2 = (p_{m+1}, p_{m+2}, \ldots, p_n)^T$, $Q = (q_1, q_2, \ldots, q_m)^T$, $H_1 = \mathrm{diag}(h_1, h_2, \ldots, h_m)$, and $H_2 = \mathrm{diag}(h_{m+1}, h_{m+2}, \ldots, h_n)$.

We assume that $V_1 = -\frac{1}{2}P^T HP + \frac{1}{4}||P||^4 + \frac{1}{2}||Q||^2$, where $H = \mathrm{diag}(h_1, h_1, \ldots, h_n)$. For V_1, we have

$$\begin{aligned} \dot{V}_1 &= -P^T H\dot{P} + ||P||^2 P^T \dot{P} + Q^T \dot{Q} \\ &= -P_1^T H_1 \dot{P}_1 - P_2^T H_2 \dot{P}_2 + ||P||^2 P_1^T \dot{P}_1 + ||P||^2 P_2^T \dot{P}_2 + Q^T \dot{Q} \\ &= -P_1^T H_1 Q + ||P||^2 P_1^T Q + Q^T((H_1 - ||P||^2 E_m)P_1 - k_1 Q) \\ &\quad + P_2^T(||P||^2 E_{n-m} - H_2)\dot{P}_2 \\ &= -k_1 Q^T Q - \dot{P_2}^T \dot{P}_2 \le 0. \end{aligned} \tag{6}$$

According to Lemma 1, we can know that the solutions of $\dot{V}_1 = 0$ are the equilibrium points of system (1) under protocol (3). When $\dot{V}_1 = 0$, we can get $Q = 0_{m\times 1}$ and $\dot{P}_2 = 0_{(n-m)\times 1}$. Together with (5), we have

$$\begin{cases} \dot{P}_1 = 0_{m\times 1}, \\ \dot{Q} = (H_1 - ||P||^2 E_m)P_1 = 0_{m\times 1}, \\ \dot{P}_2 = (H_2 - ||P||^2 E_{n-m})P_2 = 0_{(n-m)\times 1}. \end{cases} \tag{7}$$

The solutions of (7) are $\begin{cases} P_e = 0_{n\times 1} \\ Q_e = 0_{m\times 1} \end{cases}$ and $\begin{cases} P_e = \pm\sqrt{h_i}e_i \\ Q_e = 0_{m\times 1} \end{cases}$, $i \in \mathcal{I}_n$. As a result, the equilibrium points of the nonlinear system (5) are $\binom{P_e}{Q_e} = \binom{0_{n\times 1}}{0_{m\times 1}}$ and $\binom{\pm\sqrt{h_i}e_i}{0_{m\times 1}}$, $i \in \mathcal{I}_n$.

In what follows, we analyze the stability of all equilibrium points. According to Lemma 2, we linearize the system (5) at the equilibrium point $\binom{P_e}{Q_e} = \binom{0_{n\times 1}}{0_{m\times 1}}$ as $\dot{X} = A_1 X$, where

$$A_1 = \left(\begin{array}{cc|c} 0 & E_m & 0 \\ H_1 & -k_1 E_m & 0 \\ \hline 0 & 0 & H_2 \end{array}\right). \tag{8}$$

It is easy to known that $h_{m+1}, h_{m+2}, \ldots, h_n$ are positive eigenvalues of A_1. Therefore, we have conclude that the equilibrium point $\binom{P_e}{Q_e} = \binom{0_{n\times 1}}{0_{m\times 1}}$ is unstable.

For the equilibrium point $\binom{P_e}{Q_e} = \binom{\sqrt{h_i}e_i}{0_{m\times 1}}$, $i \in \mathcal{I}_n$, the linearization expression of nonlinear system (5) is $\dot{X} = A_2 X$, where

$$A_2 = \left(\begin{array}{cc|c} 0 & E_m & 0 \\ B_1 & -k_1 E_m & 0 \\ \hline 0 & 0 & B_2 \end{array}\right), \tag{9}$$

where $B_1 = H_1 - 2P_{e1}P_{e1}^T - ||P_e||^2 E_m$, $B_2 = H_2 - 2P_{e2}P_{e2}^T - ||P_e||^2 E_{n-m}$, P_{e1} is a vector consisting of the first m elements of vector P_e, P_{e2} is a vector consisting of the latter $n-m$ elements of vector P_e. Obviously, the eigenvalues of A_2 are all the eigenvalues of $\begin{pmatrix} 0 & E_m \\ B_1 & -k_1 E_m \end{pmatrix}$ and the eigenvalues of B_2. The eigenpolynomial of $\begin{pmatrix} 0 & E_m \\ B_1 & -k_1 E_m \end{pmatrix}$ is

$$\begin{aligned} &\left|\lambda E_{2m} - \begin{pmatrix} 0 & E_m \\ B_1 & -k_1 E_m \end{pmatrix}\right| = \begin{vmatrix} \lambda E_m & -E_m \\ -B_1 & (\lambda + k_1)E_m \end{vmatrix} \\ &= |\lambda^2 E_m + \lambda k_1 E_m - B_1| = 0. \end{aligned} \tag{10}$$

Since $\lambda^2 E_m + \lambda k_1 E_m - B_1$ is a diagnal matrix, all eigenvalues of $\begin{pmatrix} 0 & E_m \\ B_1 & -k_1 E_m \end{pmatrix}$ are the roots of equations $\lambda^2 + k_1\lambda - (B_1)_{ii} = 0, i \in \mathcal{I}_m$, where $(B_1)_{ii}$ is the ith diagonal entry of B_1. Suppose λ_{1i} and λ_{2i} are the roots of equation $\lambda^2 + k_1\lambda - (B_1)_{ii} = 0$. The equilibrium point $\binom{P_e}{Q_e} = \binom{\sqrt{h_i}e_i}{0_{m\times 1}}$ is stable only if $\lambda_{1i} < 0$ and $\lambda_{2i} < 0$. According to Vieta's theorem, we have $\lambda_{1i} \cdot \lambda_{2i} = -(B_1)_{ii}$, $\lambda_{1i} + \lambda_{2i} = -k_1$. Due to $k_1 > 0$, we obtain that $\lambda_{1i} < 0$ and $\lambda_{2i} < 0$ if and only if $(B_1)_{ii} < 0$ for $i \in \mathcal{I}_m$.

Because B_2 is a diagonal matrix, elements of main diagonal are the eigenvalues of B_2. The equilibrium point $\binom{P_e}{Q_e} = \binom{\sqrt{h_i}e_i}{0_{m\times 1}}$ is stable only if the eigenvalues of B_2 are all negative, i.e. $(B_2)_{ii} < 0$ for $i \in \mathcal{I}_n/\mathcal{I}_m$.

From the above analysis, we know that the equilibrium point $\binom{P_e}{Q_e} = \binom{\sqrt{h_i}e_i}{0_{m\times 1}}$ is stable if and only if the main diagonal entry of B_1 and B_2 are all negative. It is not difficult to find that the main diagonal entry of B_1 and B_2 are all negative if and only if the main diagonal entry of $H - 2P_eP_e^T - ||P_e||^2 E_n$ are all negative. Since

$$H - 2P_eP_e^T - ||P_e||^2 E_n$$

$$= \begin{array}{c} \\ 1 \\ 2 \\ \vdots \\ i \\ \vdots \\ n \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & \cdots & i & \cdots & n \end{array} \\ \begin{pmatrix} h_1 - h_i & 0 & \cdots & 0 & \cdots & 0 \\ 0 & h_2 - h_i & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & & \vdots \\ 0 & 0 & \cdots & -2h_i & \cdots & 0 \\ \vdots & \vdots & & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & h_n - h_i \end{pmatrix} \end{array} \tag{11}$$

its jth eigenvalue is $h_j - h_i$ for $i \neq j$, and $-2h_i$ for $i = j$. It is easy to figure out that $-2h_i < 0$ always holds. When $i \neq j$, we can figure out that $h_j - h_i < 0$ holds if and only if $i = k^*$. Therefore, we can conclude that the equilibrium point $\binom{P_e}{Q_e} = \binom{\sqrt{h_{k^*}}e_{k^*}}{0_{m\times 1}}$ is stable, and all other equilibriums are unstable.

The proof of stability at the equilibrium point $\binom{P_e}{Q_e} = \binom{-\sqrt{h_i}e_i}{0_{m\times 1}}$, $i \in \mathcal{I}_n$, can be obtained in a similar way. Therefore, we omit it here.

Hence, the evolution of heterogeneous dynamic agents system (1) with protocol (3) is that the k^*th agent asymptotically reach $(p_e^*, q_e^*) = (\sqrt{h_{k^*}}, 0)$ or $(p_e^*, q_e^*) = (-\sqrt{h_{k^*}}, 0)$, and all other agents asymptotically reach $(p_e, q_e) = (0, 0)$ as $t \to \infty$. □

Remark 2. This paper considers the winner-take-all problem for the system with heterogeneous dynamic agents. The difference between our paper and [19] is that our system includes both first- and second-order dynamic agents, while the system in [19] includes only first-order dynamic agents. For the scenario of $m=0$, all agents of this system are first-order dynamics, where our result in Theorem 1 is consistent with Theorem 1 in [19].

Remark 3. By comparing our results with the results in [19], we know that the winner is independent from the dynamics of the agent — the winner is still the largest-input agent. That is to say, the winner is determined not by dynamic structure, but by the amount of inputs.

4. Protocol without velocity measurements

In this section, a kind of protocol is proposed for the heterogeneous dynamic agents (1) without velocity-like measurements. Consider the following protocol

$$\begin{cases} u_i=(h_i-||P||^2)p_i+k_2\widehat{q}_i, & i\in\mathcal{I}_m,\\ u_i=(h_i-||P||^2)p_i, & i\in\mathcal{I}_n/\mathcal{I}_m, \end{cases} \tag{12}$$

where $\dot{\widehat{q}}_i=(h_i-||P||^2)p_i-k_3\widehat{q}_i$, $h_i>0$ is the input of agent i, k_2, $k_3>0$ is feedback gains.

Theorem 2. *Suppose that A1 holds. Then, the system (1) with protocol (12) can emerge winner-take-all competition for any initial conditions. Moreover, the winner k^* approaches $(p_e^*,q_e^*)=(\sqrt{h_{k^*}},0)$ or $(-\sqrt{h_{k^*}},0)$ and all other agents reach $(p_e,q_e)=(0,0)$ as $t\to\infty$.*

Proof. From (1) and (12), we get

$$\begin{cases} \dot{p}_i=q_i, & i\in\mathcal{I}_m,\\ \dot{q}_i=(h_i-||P||^2)p_i+k_2\hat{\widehat{q}}_i, & i\in\mathcal{I}_m,\\ \dot{p}_i=(h_i-||P||^2)p_i, & i\in\mathcal{I}_n/\mathcal{I}_m. \end{cases} \tag{13}$$

Let $P_1=(p_1,p_2,\dots,p_m)^T$, $P_2=(p_{m+1},p_{m+2},\dots,p_n)^T$, $Q=(q_1,q_2,\dots,q_m)^T$, $\widehat{Q}=(\widehat{q}_1,\widehat{q}_2,\dots,\widehat{q}_m)^T$, $H_1=\mathrm{diag}(h_1,h_2,\dots,h_m)$, $H_2=\mathrm{diag}(h_{m+1},h_{m+2},\dots,h_n)$. Then, we can express (13) as the following matrix form

$$\begin{cases} \dot{P}_1=Q,\\ \dot{Q}=(H_1-||P||^2E_m)P_1+k_2\hat{\widehat{Q}},\\ \dot{P}_2=(H_2-||P||^2E_{n-m})P_2, \end{cases} \tag{14}$$

where $\dot{\widehat{Q}}=(H_1-||P||^2E_m)P_1-k_3\widehat{Q}$.

We suppose that $V_2=-\frac{1}{2}P^THP+\frac{1}{4}||P||^4+\frac{1}{2}(Q-k_2\widehat{Q})^2+\frac{k_2}{2}\widehat{Q}^T\widehat{Q}$, where $H=\mathrm{diag}(h_1,h_1,\dots,h_n)$. Then, we have

$$\begin{aligned} \dot{V}_2 &= -P^TH\dot{P}+||P||^2P^T\dot{P}+(Q-k_2\widehat{Q})^T(\dot{Q}-k_2\dot{\widehat{Q}})+k_2\widehat{Q}^T\dot{\widehat{Q}}\\ &= k_2\widehat{Q}^T((H_1-||P||^2E_m)P_1-k_3\widehat{Q})+(Q-k_2\widehat{Q})^T(H_1P_1-||P||^2P_1)\\ &\quad -P_1^TH_1\dot{P}_1-P_2^TH_2\dot{P}_2+||P||^2P_1^T\dot{P}_1+||P||^2P_2^T\dot{P}_2\\ &= -P_1^TH_1Q+||P||^2P_1^TQ+Q^TH_1P_1-Q^T||P_1||^2P_1-k_2Q^TH_1P_1\\ &\quad +k_2Q^T||P_1||^2P_1+\ k_2Q^TH_1P_1-k_2Q^T||P_1||^2P_1\\ &\quad -k_1k_2\widehat{Q}^T\widehat{Q}+P_2^T(||P||^2E_{n-m}-H_2)\dot{P}_2\\ &= -k_2k_3\widehat{Q}^T\widehat{Q}-\dot{P}_2{}^T\dot{P}_2\le 0. \end{aligned} \tag{15}$$

On the basis of Lemma 1, we can know that the solutions of $\dot{V}_1=0$ are the equilibrium points of system (1) with protocol (12). When $\dot{V}_2=0$, we can get $\widehat{Q}=0_{m\times1}$ and $\dot{P}_2=0_{(n-m)\times1}$. Together with (14), we have

$$\begin{cases} \dot{P}_1=Q,\\ \dot{Q}=(H_1-||P||^2E_m)P_1,\\ \dot{\widehat{Q}}=(H_1-||P||^2E_m)P_1=0_{m\times1},\\ \dot{P}_2=(H_2-||P||^2E_{n-m})P_2=0_{(n-m)\times1}, \end{cases} \tag{16}$$

the solution can be solved as $\begin{cases}P_e=0_{n\times1}\\ Q_e=0_{m\times1}\end{cases}$ and $\begin{cases}P_e=\pm\sqrt{h_i}e_i\\ Q_e=0_{m\times1}\end{cases}$, $i\in\mathcal{I}_n$, i.e. the equilibrium points of this nonlinear system are $\binom{P_e}{Q_e}=\binom{0_{n\times1}}{0_{m\times1}}$ and $\binom{\pm\sqrt{h_i}e_i}{0_{m\times1}}$, $i\in\mathcal{I}_n$.

According to Lemma 2, the system dynamic (14) is linearized around the equilibrium point $\binom{P_e}{Q_e}=\binom{0_{n\times1}}{0_{m\times1}}$. The linearization expression of nonlinear system (14) is $\dot{X}=A_3X$, where

$$A_3=\left(\begin{array}{ccc|c} 0 & E_m & 0 & 0\\ (1+k_2)H_1 & 0 & -k_2k_3E_m & 0\\ H_1 & 0 & -k_3E_m & 0\\ 0 & 0 & 0 & H_2 \end{array}\right). \tag{17}$$

Obviously, $h_{m+1},h_{m+2},\dots,h_n$ are positive eigenvalues of A_3. Therefore, we have conclude that the equilibrium point $\binom{P_e}{Q_e}=\binom{0_{n\times1}}{0_{m\times1}}$ is unstable.

For the equilibrium point $\binom{P_e}{Q_e}=\binom{\sqrt{h_i}e_i}{0_{m\times1}}$, $i\in\mathcal{I}_n$, the linearization expression of nonlinear system (14) is $\dot{X}=A_4X$, where

$$A_4=\left(\begin{array}{ccc|c} 0 & E_m & 0 & 0\\ (1+k_2)B_1 & 0 & -k_2k_3E_m & 0\\ B_1 & 0 & -k_3E_m & 0\\ 0 & 0 & 0 & B_2 \end{array}\right), \tag{18}$$

where $B_1=H_1-2P_{e1}P_{e1}^T-||P_e||^2E_m$, $B_2=H_2-2P_{e2}P_{e2}^T-||P_e||^2E_{n-m}$, P_{e1} is a vector consisting of the first m elements of vector P_e, P_{e2} is a vector consisting of the latter $n-m$ elements of vector P_e. It is easy to get that the eigenvalues of A_4 are the eigenvalues of $\begin{pmatrix}0 & E_m & 0\\ (1+k_2)B_1 & 0 & -k_2k_3E_m\\ B_1 & 0 & -k_3E_m\end{pmatrix}$ and the eigenvalues of B_2. Then the eigenpolynomial of $\begin{pmatrix}0 & E_m & 0\\ (1+k_2)B_1 & 0 & -k_2k_3E_m\\ B_1 & 0 & -k_3E_m\end{pmatrix}$ is

$$\begin{aligned} &\left|\lambda E_{3m}-\begin{pmatrix}0 & E_m & 0\\ (1+k_2)B_1 & 0 & -k_2k_3E_m\\ B_1 & 0 & -k_3E_m\end{pmatrix}\right|\\ &=|\lambda^3E_m+\lambda^2k_3E_m-\lambda(1+k_2)B_1-k_3B_1|=0. \end{aligned} \tag{19}$$

Because $\lambda^3E_m+\lambda^2k_3E_m-\lambda(1+k_2)B_1-k_3B_1$ is a diagonal matrix, we can get that its ith diagonal element is also its ith eigenvalue of $\begin{pmatrix}0 & E_m & 0\\ (1+k_2)B_1 & 0 & -k_2k_3E_m\\ B_1 & 0 & -k_3E_m\end{pmatrix}$. Every eigenvalue of $\begin{pmatrix}0 & E_m & 0\\ (1+k_2)B_1 & 0 & -k_2k_3E_m\\ B_1 & 0 & -k_3E_m\end{pmatrix}$ can be expressed as $\lambda^3+k_3\lambda^2-(1+k_2)(B_1)_{ii}\lambda-k_3(B_1)_{ii}=0$. Suppose λ_{1i}, λ_{2i} and λ_{3i} are the roots of equation $\lambda^3+k_3\lambda^2-(1+k_2)(B_1)_{ii}\lambda-k_3(B_1)_{ii}=0$. The equilibrium point $\binom{P_e}{Q_e}=\binom{\sqrt{h_i}e_i}{0_{m\times1}}$ is stable only if $\lambda_{1i}<0$, $\lambda_{2i}<0$ and $\lambda_{3i}<0$. According to Vieta's theorem, we have $\lambda_{1i}\cdot\lambda_{2i}\cdot\lambda_{3i}=k_3(B_1)_{ii}$, $\lambda_{1i}\lambda_{2i}+\lambda_{1i}\lambda_{3i}+\lambda_{2i}\lambda_{2i}=-(1+k_2)(B_1)_{ii}$, and $\lambda_{1i}+\lambda_{2i}+\lambda_{3i}=-k_3$. Due to k_2, $k_3>0$, we can get that $\lambda_{1i}<0$, $\lambda_{2i}<0$ and $\lambda_{3i}<0$ if and only if $(B_1)_{ii}<0$ for $i\in\mathcal{I}_m$.

Since B_2 is a diagonal matrix, elements of main diagonal are the eigenvalues of B_2. Obviously, the equilibrium point $\binom{P_e}{Q_e}=\binom{\sqrt{h_i}e_i}{0_{m\times1}}$ is stable only if the eigenvalues of B_2 are all negative, i.e. $(B_2)_{ii}<0$ for $i\in\mathcal{I}_n/\mathcal{I}_m$.

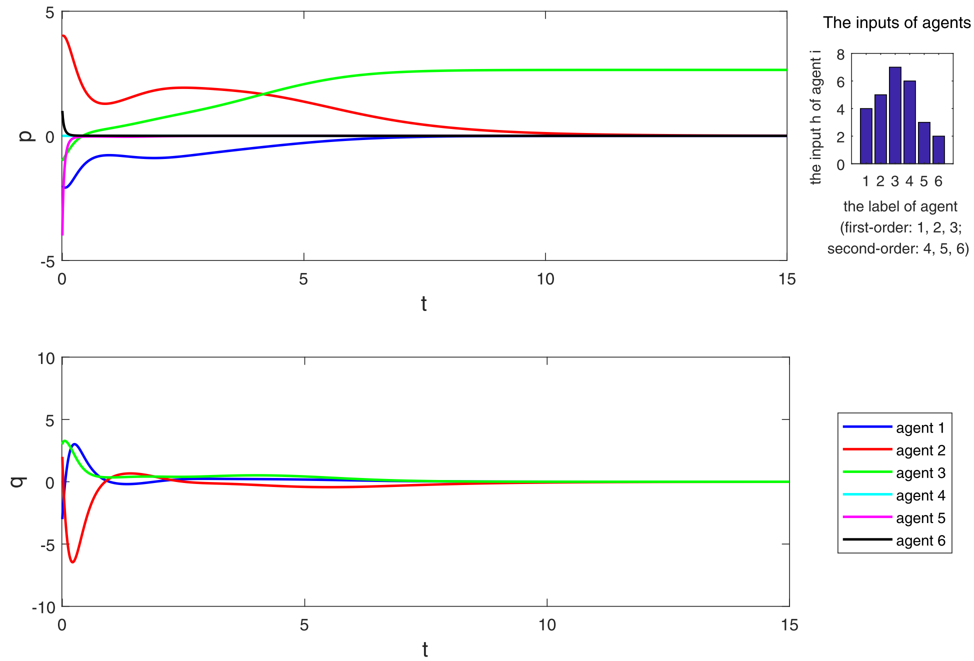

Fig. 2. The state trajectories and inputs of all agents with protocol (3).

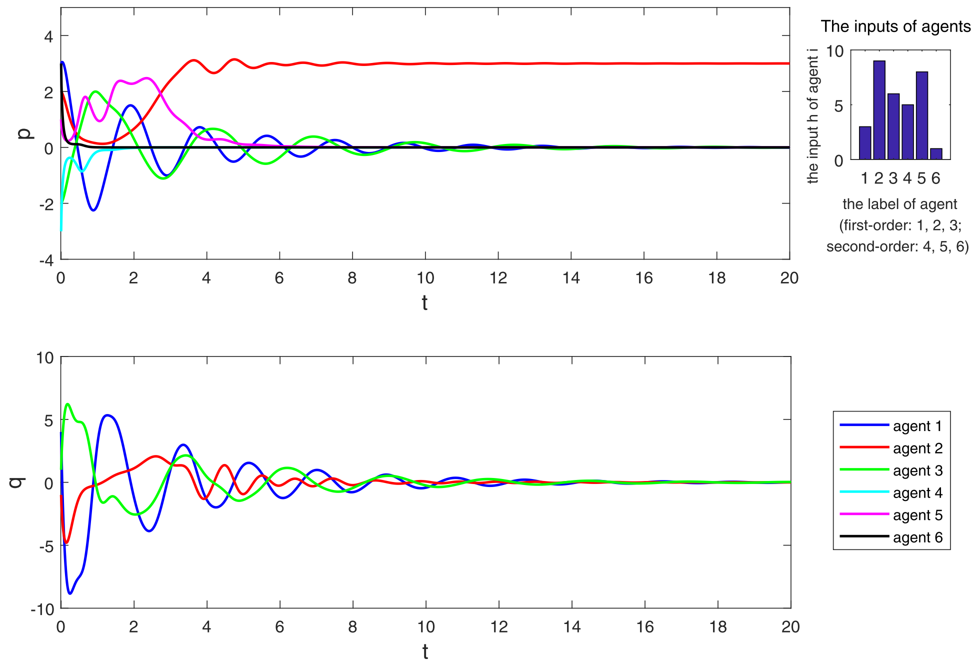

Fig. 3. The state trajectories and inputs of all agents with protocol (12).

From the above analysis, we know that the equilibrium point $\binom{P_e}{Q_e} = \binom{\sqrt{h_i}e_i}{0_{m\times 1}}$ is stable if and only if the main diagonal elements of $H - 2P_eP_e^T - ||P_e||^2E_n$ are all negative. According to the proof of Theorem 1, we can get the main diagonal elements of $H - 2P_eP_e^T - ||P_e||^2E_n$ are all negative.

The stability of the equilibrium point $\binom{P_e}{Q_e} = \binom{-\sqrt{h_i}e_i}{0_{m\times 1}}$, $i \in \mathcal{I}_n$, can be proved in a similar way. Therefore, we omit it here.

In summary, by considering the system (1) with protocol (12), the solution of the agent k^* approaches $(p_e^*, q_e^*) = (\sqrt{h_{k^*}}, 0)$ or $(p_e^*, q_e^*) = (-\sqrt{h_{k^*}}, 0)$, and all other agents approaches $(p_e, q_e) = (0, 0)$ as $t \to \infty$. □

5. Simulations

In this section, we first propose a numerical simulation in Example 1 to verify the effectiveness of theoretical result in Section 3. In Example 2, we give a numerical simulation to verify the effectiveness of our result in Section 4.

We suppose that the system with heterogeneous dynamic agents includes 6 agents, where the number of second-order dynamic agents is 3, labelled 1 through 3. The remaining first-order agents' labels are from 4 to 6.

Example 1. Suppose that $k_1 = 1.1$, $P(0) = (-2, 4, -1, 0, -4, 1)^T$, $Q(0) = (-3, 2, 3)^T$, $H(0) = (4, 5, 7, 6, 3, 2)^T$. According to the system (1) and protocol (3), we obtain the simulation results and show them in Fig. 2. From the bar chart, we can see that the agent 3 has the largest input 7. In addition, all agents' velocity-like values are eventually tend to zero. Only the agent 3 finally wins the competition and has a non-zero position-like value ($\lim_{t\to\infty} p_3 = \sqrt{h_3} = \sqrt{7}$). While the position-like values of the remaining agents eventually reach zero. All agents' velocity-like values eventually tends to zero. This result is consistent with our theoretical results of Theorem 1.

Example 2. We assume that $k_2 = 0.9$, $k_3 = 1.2$, $P(0) = (3, 2, -2, -3, 1, 3)^T$, $Q(0) = (1, -3, 3)^T$, $\widehat{Q}(0) = (4, -1, 1)^T$, $H(0) = (3, 9, 6, 5, 8, 1)^T$. Considering the system (1) and protocol (12), we carry out a numerical simulation. Fig. 3 shows the evolution of the states values along with time for all agents. We can see that the agent 2 has the largest input 9 by the bar chart. As well as, only the agent 3 has a non-zero value ($\lim_{t\to\infty} p_2 = \sqrt{h_2} = 3$) eventually, but the remaining agents' position-like values are finally suppressed to zero. The velocity-like values of all agents reach to zero as $t \to \infty$. This result is consistent with our theoretical results of Theorem 2.

6. Conclusions

This paper mainly consider the winner-take-all competition for the system with heterogeneous dynamic agents. Two kinds of protocols with and without velocity measurements are designed for heterogeneous dynamic agents, respectively. We prove that the heterogeneous dynamic agents can all achieve winner-take-all competition under the proposed protocols. We find that the dynamics of agents can not change the producing of the winner, which means that the agent with the largest input will win the competition and all others will be deactivated to zero. We find that the results of winner is independent from the dynamics of agents and is determined by inputs. In the end, some simulations are provided to demonstrate the effectiveness of our theoretical results. In the future, we may further consider the winner-take-all problem for the system with hybrid dynamic agents.

Declaration of Competing Interest

None.

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