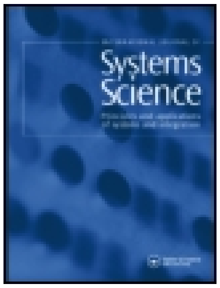

International Journal of Systems Science
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tsys20

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Version of record first published: 24 Sep 2012.

International Journal of Systems Science
2012, 1–10, iFirst

Finite-time consensus of multiple second-order dynamic agents without velocity measurements

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(*Received 25 November 2011; final version received 14 July 2012*)

This article considers the finite-time consensus of multiple second-order dynamic agents without velocity measurements. A feasible protocol under which each agent can only obtain the measurements of its position relative to its neighbours is proposed. By applying the graph theory, Lyapunov theory and the homogeneous domination method, some sufficient conditions for finite-time consensus of second-order multi-agent systems are established under the different kinds of communication topologies. Some examples are presented to illustrate the effectiveness of the theoretical results.

Keywords: multi-agent systems; second-order finite-time consensus; without velocity measurements

1. Introduction

Consensus problem of multi-agent systems has attracted great attention in many fields, such as system control theory, statistical physics, biology, communication, computer science and so on. As a fundamental of distributed coordination, consensus of multi-agent systems means that a group of agents reaches an agreement on a common value by negotiating with their neighbours asymptotically or in a finite time. Roughly speaking, the main objective of consensus problem is to design an appropriate consensus protocol such that a group of agents converges to a consensus state of interest. Up to now, by using the matrix theory, the graph theory, the frequency-domain analysis method, the Lyapunov direct method, etc., consensus problem of multi-agent systems has been studied in detail and the consensus criterions have been obtained for first-order, second-order or high-order multi-agent systems, many of which have been successfully applied in many areas including swarming (Chu, Wang, Chen, and Mu 2006), flocking (Olfati-Saber 2006) and formation control (Ji, Wang, Lin, and Wang 2009; Xiao, Wang, Chen, and Gao 2009) of social insects, unmanned air vehicles (UAVs), robotic teams, satellite clusters, and so on.

Consensus of the first-order multi-agent systems is primarily proposed and extensively explored by many researchers. Vicsek, Czirok, Jacob, Cohen, and Schochet (1995) proposed a discrete-time model of n agents all moving in the plane with the same speed and demonstrated by simulation that all agents move to one direction asymptotically. Based on the algebraic graph theory (Godsil and Royal 2001), Jadbabaie, Lin, and Morse (2003) provided a theoretical explanation of the consensus behaviour in Vecsek model, and analysed the alignment of a network of agents with switching topologies that are periodically connected. Olfati-Saber and Murray (2004) discussed consensus problem for networks of dynamic agents with switching topologies and time-delays in a continuous-time model by defining a disagreement function, and obtained some useful results for solving the average-consensus problem. With the development of issue, a lot of new consensus results have been offered with different models and protocols for first-order multi-agent systems (Ren and Beard 2005; Xiao and Wang 2006, 2008; Sun, Wang, and Xie 2008; Yu and Wang 2012). In recent years, amounting attentions have been paid to the consensus problem of second-order multi-agent systems. Xie and coworkers (Ren and Atkins 2007; Xie and Wang 2007) gave the sufficient conditions for consensus problem of second-order multi-agent systems with fixed and switching topologies. Gao and Wang (2010) investigated the consensus of second-order multi-agent systems based on sampled-data control. Other results of consensus problem have been established for high-order multi-agent systems (Jiang and Wang 2010) and heterogeneous multi-agent systems (Zheng, Zhu, and Wang 2011b).

On the one hand, most consensus protocol of second-order multi-agent systems rely on the availability of the feedback information of the full states in the

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ISSN 0020–7721 print/ISSN 1464–5319 online

http://dx.doi.org/10.1080/00207721.2012.724108
http://www.tandfonline.com

study of consensus problem. However, some information is unmeasurable because of technology limitations or environment disturbances (Gao, Zuo, Jiang, Du, and Ma 2012). For example, the agents can not obtain any velocity information in some cases. Hence, it is realistic and significative to consider the consensus problem of second-order multi-agent systems without velocity measurements. However, there are only a few works on this problem (Ren 2008; Gao, Wang, and Jia 2009; Abdessameud and Tayebi 2010; Zheng and Wang 2012a). In Ren (2008), the consensus problem of second-order multi-agent systems without velocity measurements was considered under fixed undirected topology. Gao et al. (2009) extended the results in Ren (2008) to a time-varying topology with/without time-delays. Abdessameud and Tayebi (2010) proposed the consensus protocols for second-order multi-agent systems without velocity measurements and in the presence of input saturation constraints. Zheng and Wang 2012a) considered the consensus of heterogeneous multi-agent systems without velocity measurements under undirected connected and leader-following networks.

On the other hand, the convergence speed is an active topic and can reflect the performance of the proposed consensus protocol in the analysis of consensus problem. Olfati-Saber and Murray (2004) showed that the second smallest eigenvalue of the interaction graph Laplacian, called algebraic connectivity of graph, quantifies the convergence speed of the consensus algorithm. Kim and Mesbahi (2006) considered the problem of maximising the second smallest eigenvalues of a state-dependent graph Laplacian. Xiao and Boyd (2006) considered and solved the problem of the weight design by using semi-definite convex programming, so that algebraic connectivity is increased. Although by maximising the algebraic connectivity of interaction graph, one can increase convergence speed with respect to the linear protocol, the consensus can never be reached in a finite time. However, in many situations, it is required that the consensus should be reached in a finite time, such as when the control accuracy is crucial. Based on the non-smooth stability analysis, Cortes (2006) discussed the finite-time consensus problem for multi-agent systems under some discontinuous consensus protocols. Jiang and Wang (2009) investigated the finite-time consensus for multi-agent systems with fixed and switching topologies. In Wang and Xiao (2010), the authors showed that the multi-agent systems can solve the finite-time consensus problem for both the bidirectional and unidirectional interaction cases. Wang and Hong (2010) considered the finite-time χ-consensus of multi-agent systems with variable coupling topology. Zheng, Chen, and Wang (2011a) studied the finite-time consensus of stochastic multi-agent systems with general protocol. For second-order multi-agent systems, Wang and Hong (2008) gave some protocols and showed that these protocols can reach the finite-time consensus under undirected connected graph using the homogeneous method. Sun and Guan (2012) considered the finite-time consensus of leader-following multi-agent systems with velocity measurements. Based on adding a power integrator method, Li, Du, and Lin (2011) designed a protocol and discussed the finite-time consensus of leaderless and leader-following multi-agent systems with external disturbances. Zheng and Wang (2012b) investigated the finite-time consensus of heterogeneous multi-agent systems with and without velocity measurements. Cao, Ren, and Meng (2010) studied finite-time decentralised formation tracking of second-order multi-agent systems by introducing the decentralised sliding mode estimators.

Inspired by the recent developments in multi-agent systems, we try to further investigate the finite-time consensus of second-order multi-agent systems. Different from Wang and Hong (2008), we give the consensus protocol for second-order multi-agent systems without velocity measurements and consider the finite-time consensus problem under different communication topologies. The main contribution of this article is threefold. First, we propose a continuous consensus protocol without velocity measurements for second-order multi-agent systems based on the auxiliary system approach. Second, we prove that the proposed consensus protocol can solve the finite-time consensus of second-order multi-agent systems under undirected connected graph and leader-following network by using the graph theory, Lyapunov theory and the homogeneous domination method. Finally, we discuss the finite-time consensus problem of second-order multi-agent systems under some special directed graphs. It is not only theoretically interesting but also practically important owing to the fact that many practical systems need to consider the convergence speed when the velocity information is unmeasurable.

The rest of this article is organised as follows. In Section 2, some basic definitions and preliminary results are assembled. Section 3 sets up the problem formulation. The main results are established in Sections 4 and numerical simulations to show the validity of theoretical results are presented in Section 5. Finally, the conclusions are provided in Section 6.

Notation: Throughout this article, we let $\mathbb{R}$, $\mathbb{R}_{>0}$ and $\mathbb{R}_{\geq 0}$ be the set of real number, positive real number and non-negative real number, $\mathbb{R}^n$ is the n-dimensional real vector space, $\mathcal{I}_n = \{1, 2, \ldots, n\}$. For a given matrix X (vector x), X^T (x^T) denotes its transpose, and $\|X\|$ ($\|x\|$) denotes the Euclidean norm. $\mathbf{1}_n$ is a vector with elements being all ones. A is said to be non-negative (resp. positive) if all entries a_{ij} are non-negative (resp. positive), denoted by $A \geq 0$ (resp. $A > 0$). $sig(x)^{\alpha} = sign(x)|x|^{\alpha}$, where $sign(\cdot)$ is sign function.

2. Preliminaries

The network formed by multi-agent system can always be represented by a graph. Thus, graph theory is an important tool to analyse consensus problem for multi-agent system. First, some basic definitions and results are presented in matrix theory and graph theory (Horn and Johnson 1985; Godsil and Royal 2001).

An undirected (directed) graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consists of a vertex set $\mathcal{V} = \{s_1, s_2, \ldots, s_n\}$ and an edge set $\mathcal{E} = \{e_{ij} = (s_i, s_j)\} \subset \mathcal{V} \times \mathcal{V}$. Denote the set of neighbours of s_i by $\mathcal{N}_i = \{s_j : e_{ji} = (s_j, s_i) \in \mathcal{E}\}$. A path that connects s_i and s_j in the graph $\mathcal{G}$ is a sequence of distinct vertices $s_{i_0}, s_{i_1}, s_{i_2}, \ldots s_{i_m}$, where $s_{i_0} = s_i, s_{i_m} = s_j$ and $(s_{i_r}, s_{i_{r+1}}) \in \mathcal{E}$, $0 \leq r \leq m-1$. An undirected (directed) graph is said to be connected (strong connected) if there exists a path between any two distinct vertices of the graph. Otherwise, the undirected graph can be partitioned into some subparts, where each part in the graph is connected and there is no connections between any part. Here, each part is called the connected component of undirected graph $\mathcal{G}$. For directed graph, if (s_i, s_j) is an edge of $\mathcal{G}$, s_i is called the parent of s_j and s_j is called the child of s_i. A directed tree is a directed graph, where every vertex, except one special vertex without any parent, which is called the root, has exactly one parent, and the root can be connected to any other vertices through paths. The weighted adjacency matrix $\mathcal{A} = [a_{ij}]_{n \times n}$ of a graph $\mathcal{G}$ is a non-negative matrix with rows and columns indexed by the vertices, all entries of which are non-negative, where $a_{ij} > 0$ if and only if $e_{ji} = (s_j, s_i) \in \mathcal{E}$. The degree matrix $\mathcal{D} = [d_{ij}]_{n \times n}$ is a diagonal matrix with $d_{ii} = \sum_{s_j \in \mathcal{N}_i} a_{ij}$, and the Laplacian matrix of the graph is defined as $\mathcal{L} = [l_{ij}]_{n \times n} = \mathcal{D} - \mathcal{A}$. It is easy to see that adjacency matrix $\mathcal{A}$ is symmetric if $\mathcal{G}$ is an undirected graph. The directed graph $\mathcal{G}$ is said to satisfy the detailed balance condition if there exist some scalers $\omega_i > 0$ $(i = 1, 2, \ldots, n)$ such that $\omega_i a_{ij} = \omega_j a_{ji}$ for all $i, j \in \mathcal{I}_n$ (Chu et al. 2006). For convenience of exposition, the names, agent and node, network and graph, will be used interchangeably.

Next, some necessary lemmas are given for analysis of main results.

Consider the autonomous system

$$\dot{x} = f(x), \tag{1}$$

where f: $D \to \mathbb{R}^n$ is a continuous function with $D \subset \mathbb{R}^n$.

Lemma 2.1 (Lasalle's Invariance Principle)**:** *Let $\Omega \subset D$ be a compact set that is positively invariant with respect to* (1). *Let V: $D \to \mathbb{R}$ be a continuously differentiable function such that $\dot{V}(x) \leq 0$ in Ω. Let E be the set of all points in Ω where $\dot{V}(x) = 0$. Let M be the largest invariant set in E. Then every solution starting in Ω approaches M as $t \to \infty$.* □

A function $V(x)$ is homogeneous of degree $\sigma > 0$ with dilation $(r_1, r_2, \ldots, r_n)$, $r_i > 0 (i \in \mathcal{I}_n)$, if

$$V(\varepsilon^{r_1} x_1, \varepsilon^{r_2} x_2, \ldots, \varepsilon^{r_n} x_n) = \varepsilon^{\sigma} V(x), \quad \varepsilon > 0.$$

A vector field $f(x) = (f_1(x), f_2(x), \ldots, f_n(x))$ is homogenous of degree $\sigma > 0$ with dilation $(r_1, r_2, \ldots, r_n)$, $r_i > 0 (i \in \mathcal{I}_n)$, if

$$f_i(\varepsilon^{r_1} x_1, \varepsilon^{r_2} x_2, \ldots, \varepsilon^{r_n} x_n) = \varepsilon^{\sigma + r_i} f_i(x), \quad i \in \mathcal{I}_n, \quad \varepsilon > 0.$$

Lemma 2.2 (Bhat and Bernstein 2000; Hong 2002)**:** *Suppose that the system* (1) *is homogeneous of degree σ with dilation $(r_1, r_2, \ldots, r_n)$, function $f(x)$ is continuous and $x = 0$ is its asymptotically stable equilibrium. If homogeneity degree $\sigma < 0$, the equilibrium of the system* (1) *is finite-time stable.* □

3. Problem formulation

In this section, we formulate the problem to be studied and give some basic definitions and lemmas.

Suppose that the multi-agent system consists of n agents, e.g. vehicles, robots, etc., labelled 1 through n. Each agent obeys a double integrator model of the form: $\ddot{x}_i(t) = u_i(t)$, or equivalently

$$\begin{cases} \dot{x}_i(t) = v_i(t), \\ \dot{v}_i(t) = u_i(t), \quad i \in \mathcal{I}_n, \end{cases} \tag{2}$$

where $x_i \in \mathbb{R}$, $v_i \in \mathbb{R}$ and $u_i \in \mathbb{R}$ are the position, velocity and acceleration, respectively, of agent i. The initial conditions are $x_i(0) = x_{i0}$, $v_i(0) = v_{i0}$. Let $x(0) = [x_{10}, x_{20}, \ldots, x_{n0}]$, $v(0) = [v_{10}, v_{20}, \ldots, v_{n0}]$.

In order to distinguish the finite-time consensus of first-order multi-agent systems from the finite-time consensus of second-order multi-agent systems, we define the finite-time consensus of second-order multi-agent systems as second-order finite-time consensus.

Definition 3.1 (SOFTC)**:** The multi-agent system (2) is said to reach second-order finite-time consensus (SOFTC for short) if for any initial conditions, there exists a finite-time T such that

$$\lim_{t \to T^-} \|x_i(t) - x_j(t)\| = 0,$$
$$\lim_{t \to T^-} \|v_i(t) - v_j(t)\| = 0, \quad \text{and}$$
$$x_i(t) = x_j(t), \quad v_i(t) = v_j(t), \quad \text{if } t \geq T$$

for any $i, j \in \mathcal{I}_n$. Moreover, if $T = \infty$, then the multi-agent system (2) is said to reach second-order consensus.

From Lemma 2.2, we get the next lemma. It is obvious, and the proof is omitted here.

Lemma 3.2: *Suppose that, for some given $u_i(t)$, $i \in \mathcal{I}_n$, the system* (2) *with* $(\underbrace{x_1, \ldots, x_n}_{n}, \underbrace{v_1, \ldots, v_n}_{n})$ *is homogeneous of degree σ with dilation* $(\underbrace{r_1, \ldots, r_1}_{n}, \underbrace{r_2, \ldots, r_2}_{n})$ *and can solve second-order consensus problem. If homogeneity degree $\sigma < 0$, then the system* (2) *can solve second-order finite-time consensus problem.* □

Wang and Hong (2008) proposed a finite-time consensus protocol for second-order multi-agent system (2) with velocity information as follows:

$$u_i(t) = \sum_{j=1}^{n} a_{ij}\Big[\psi_1(sig(x_j(t) - x_i(t))^{\alpha_1}) + \psi_2(sig(v_j(t) - v_i(t))^{\frac{2\alpha_1}{1+\alpha_1}})\Big],$$

where $0 < \alpha_1 < 1$, ψ_1 and ψ_2 are continuous odd functions with $z\psi_i(z) > 0$ $(\forall z \neq 0)$ and $\psi_i(z) = c_i z + o(z)$ for some positive numbers c_i $(i = 1, 2)$.

Different from Wang and Hong (2008), we present the consensus protocol without velocity measurements as follows:

$$u_i(t) = \sum_{j=1}^{n} a_{ij} sig(x_j(t) - x_i(t))^{\alpha_1} + k_1 \dot{y}_i(t), \quad i \in \mathcal{I}_n, \tag{3}$$

where $\mathcal{A} = [a_{ij}]_{n\times n}$ is the weighted adjacency matrix, $0 < \alpha_1 < 1$, $k_1 > 0$ is a feedback gain. $y_i \in \mathbb{R}$ is given by

$$\dot{y}_i(t) = -k_2 sig(y_i(t))^{\frac{2\alpha_1}{1+\alpha_1}} + k_3 \sum_{j=1}^{n} a_{ij} sig(x_j(t) - x_i(t))^{\alpha_1}, \quad i \in \mathcal{I}_n, \tag{4}$$

where $k_2 > 0$, $k_3 > 0$ and $y_i(0) = y_{i0}$ can be chosen arbitrarily. Let $y(0) = [y_{10}, y_{20}, \ldots, y_{n0}]$.

The multi-agent system (2) with consensus protocol (3–4) can be rewritten as follows:

$$\begin{cases} \dot{x}_i(t) = v_i(t), \\ \dot{v}_i(t) = \sum_{j=1}^{n} a_{ij} sig(x_j(t) - x_i(t))^{\alpha_1} + k_1 \dot{y}_i(t), \\ \dot{y}_i(t) = -k_2 sig(y_i(t))^{\frac{2\alpha_1}{1+\alpha_1}} + k_3 \sum_{j=1}^{n} a_{ij} sig(x_j(t) - x_i(t))^{\alpha_1}, \\ \quad i \in \mathcal{I}_n, \end{cases} \tag{5}$$

Owing to the fact that

$$\varepsilon^{1+\alpha_1} v_i(t) = \varepsilon^{\sigma+2} v_i(t),$$

$$\begin{aligned} &(1 + k_1 k_3) \sum_{j=1}^{n} a_{ij} sig(\varepsilon^2 x_j(t) - \varepsilon^2 x_i(t))^{\alpha_1} - k_1 k_2 sig(\varepsilon^{1+\alpha_1} y_i(t))^{\frac{2\alpha_1}{1+\alpha_1}} \\ &= \varepsilon^{\sigma+(1+\alpha_1)}\Big[(1 + k_1 k_3) \sum_{j=1}^{n} a_{ij} sig(x_j(t) - x_i(t))^{\alpha_1} - k_1 k_2 sig(y_i(t))^{\frac{2\alpha_1}{1+\alpha_1}}\Big] \end{aligned}$$

and

$$\begin{aligned} &- k_2 sig(\varepsilon^{1+\alpha_1} y_i(t))^{\frac{2\alpha_1}{1+\alpha_1}} + k_3 \sum_{j=1}^{n} a_{ij} sig(\varepsilon^2 x_j(t) - \varepsilon^2 x_i(t))^{\alpha_1} \\ &= \varepsilon^{\sigma+(1+\alpha_1)}\Big[- k_2 sig(y_i(t))^{\frac{2\alpha_1}{1+\alpha_1}} + k_3 \sum_{j=1}^{n} a_{ij} sig(x_j(t) - x_i(t))^{\alpha_1}\Big], \end{aligned}$$

we know that the multi-agent system (5) with variables $(\underbrace{x_1(t), \ldots, x_n(t)}_{n}, \underbrace{v_1(t), \ldots, v_n(t)}_{n}, \underbrace{y_1(t), \ldots, y_n(t)}_{n})$ is a homogeneous system of degree $\sigma = \alpha_1 - 1 < 0$ with dilation $(\underbrace{2, \ldots, 2}_{n}, \underbrace{1+\alpha_1, \ldots, 1+\alpha_1}_{n}, \underbrace{1+\alpha_1, \ldots, 1+\alpha_1}_{n})$. Therefore, the analysis for SOFTC problem of multi-agent system changes into second-order consensus problem.

4. Main results

In this section, we will study the second-order finite-time consensus of multi-agent system (5) under undirected and directed graphs, respectively.

4.1. *Undirected graph*

For undirected graph, we consider the communication topology $\mathcal{G}$ with two cases, one is the undirected connected graph, the other is the leader-following network.

Theorem 4.1: *Suppose that the communication topology $\mathcal{G}$ is an undirected connected graph, i.e. $a_{ij} = a_{ji}$ for all $i, j \in \mathcal{I}_n$. Then the multi-agent system* (5) *can achieve the SOFTC.*

Proof: Take a Lyapunov function for (5) as

$$V_1(t) = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} \frac{|x_i(t) - x_j(t)|^{\alpha_1+1}}{\alpha_1 + 1} + \sum_{i=1}^{n}(v_i(t) - k_1 y_i(t))^2 + \sum_{i=1}^{n} \frac{k_1}{k_3}(y_i(t))^2,$$

which is positive definite with respect to $x_i(t) - x_j(t)$ $(\forall i \neq j,\ i,\ j \in \mathcal{I}_n)$, $v_i(t)$ $(i \in \mathcal{I}_n)$ and $y_i(t)$ $(i \in \mathcal{I}_n)$.

Differentiating $V_1(t)$, gives

$$\begin{aligned}\dot V_1(t) &= \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} sig(x_j(t)-x_i(t))^{\alpha_1}(\dot x_j(t)-\dot x_i(t))\\ &\quad+\sum_{i=1}^{n} 2(v_i(t)-k_1 y_i(t))(\dot v_i(t)-k_1\dot y_i(t))\\ &\quad+\sum_{i=1}^{n}\frac{2k_1}{k_3} y_i(t)\dot y_i(t)\\ &=-\sum_{i=1}^{n} 2v_i(t)\sum_{j=1}^{n} a_{ij} sig(x_j(t)-x_i(t))^{\alpha_1}\\ &\quad+\sum_{i=1}^{n} 2(v_i(t)-k_1 y_i(t))\sum_{j=1}^{n} a_{ij} sig(x_j(t)-x_i(t))^{\alpha_1}\\ &\quad+\sum_{i=1}^{n}\frac{2k_1}{k_3} y_i(t)(-k_2 sig(y_i(t))^{\frac{2\alpha_1}{1+\alpha_1}}\\ &\quad+k_3\sum_{j=1}^{n} a_{ij} sig(x_j(t)-x_i(t))^{\alpha_1})\\ &=-\sum_{i=1}^{n}\frac{2k_1k_2}{k_3}|y_i(t)|^{\frac{3\alpha_1+1}{1+\alpha_1}}\le 0\end{aligned}$$

Denote the invariant set $S=\{(x_1,v_1,y_1,\ldots,x_n,v_n,y_n)|\dot V_1\equiv 0\}$. Note that $\dot V_1\equiv 0$ implies that $y_i=0$ $(i\in\mathcal{I}_n)$. From (4), we have

$$k_3\textstyle\sum_{j=1}^{n} a_{ij} sig(x_j(t)-x_i(t))^{\alpha_1}=0,\quad \text{for } i\in\mathcal{I}_n.$$

Because the graph $\mathcal{G}$ is undirected, we have

$$\begin{aligned}&\sum_{i=1}^{n} x_i(t)\left(\sum_{j=1}^{n} a_{ij} sig(x_j(t)-x_i(t))^{\alpha_1}\right)\\ &\quad=\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}|x_j(t)-x_i(t)|^{\alpha_1+1}=0\end{aligned}$$

Due to the fact that the graph $\mathcal{G}$ is connected, we have $x_i=x_j$ for all $i,\ j\in\mathcal{I}_n$. Thus, $v_i=v_j$ for all i, $j\in\mathcal{I}_n$. It follows Lemma 2.1 that

$$\lim_{t\to\infty}\|x_i(t)-x_j(t)\|=0,\quad\text{and}\quad \lim_{t\to\infty}\|v_i(t)-v_j(t)\|=0,\quad \text{for } i,j\in\mathcal{I}_n.$$

Next, note that the multi-agent system (5) is a homogeneous system of degree $\sigma=\alpha_1-1<0$ with dilation $(\underbrace{2,\ldots,2}_{n},\underbrace{1+\alpha_1,\ldots,1+\alpha_1}_{n},\underbrace{1+\alpha_1,\ldots,1+\alpha_1}_{n})$. Therefore, the multi-agent system (5) achieves the SOFTC by Lemma 3.2. □

In the multi-agent system, we refer to the agent as the leader if it only send the information to other agents and can't receive any information form other agents, i.e. $a_{n1}=a_{n2}=\cdots=a_{nn}=0,\ \bar a=[a_{1n},\ a_{2n},\ldots,a_{(n-1)n}]^T\ge 0$ and $\bar a\ne 0$ if the agent n is the leader. To some extension, we consider the multi-agent system has a leader and the communication topology of the followers is undirected (leader-following network for short). The leader-following network is said to be connected if at least one agent in each component of the followers is connected to the leader by a directed edge.

Theorem 4.2: *Suppose that the communication topology $\mathcal{G}$ is a connected leader-following network. Then the multi-agent system* (5) *can achieve the SOFTC.*

Proof: Without loss of generality, we assume the agents $1,2,\ldots,n-1$ are the followers and n is the leader. Thus, we have $\bar{\mathcal{A}}=[a_{ij}]_{1\le i,j\le n-1}=\bar{\mathcal{A}}^T$, $a_{n1}=a_{n2}=\cdots=a_{nn}=0$, $\bar a\ge 0$, where $\bar a=[a_{1n}, a_{2n},\ldots,a_{(n-1)n}]^T$. Then, the dynamics of agent n can be written as follows:

$$\begin{cases}\dot x_n(t)=v_n(t),\\ \dot v_n(t)=k_1\dot y_n(t),\\ \dot y_n(t)=-k_2 sig(y_n(t))^{\frac{2\alpha_1}{1+\alpha_1}}.\end{cases}\tag{6}$$

Because of $0<\frac{2\alpha_1}{1+\alpha_1}<1$, it is not difficult to prove that the auxiliary system $\dot y_n(t)=-k_2 sig(y_n(t))^{\frac{2\alpha_1}{1+\alpha_1}}$ is finite-time stable, i.e. there exists a finite-time T_1 such that $\lim_{t\to T_1^-} y_n(t)=0$, and $y_n(t)=0$ when $t\ge T_1$.

Let $\delta_i^1(t)=x_i(t)-x_n(t)$, $\delta_i^2(t)=v_i(t)-v_n(t)$, $\delta_i^3(t)=y_i(t)-y_n(t)$, $i\in\mathcal{I}_n$. If $t\ge T_1$, we have

$$\begin{cases}\dot\delta_i^1(t)=\delta_i^2(t),\\ \dot\delta_i^2(t)=\sum_{j=1}^{n-1} a_{ij} sig(\delta_j^1(t)-\delta_i^1(t))^{\alpha_1}\\ \qquad -a_{in} sig(\delta_i^1(t))^{\alpha_1}+k_1\dot\delta_i^3(t),\\ \dot\delta_i^3(t)=-k_2 sig(\delta_i^3(t))^{\frac{2\alpha_1}{1+\alpha_1}}+k_3\sum_{j=1}^{n-1} a_{ij} sig(\delta_j^1(t)-\delta_i^1(t))^{\alpha_1}\\ \qquad -k_3 a_{in} sig(\delta_i^1(t))^{\alpha_1},\\ \qquad i\in\mathcal{I}_{n-1},\end{cases}\tag{7}$$

Take a Lyapunov function for (7) as

$$\begin{aligned}V_2(t)&=\sum_{i=1}^{n-1}\sum_{j=1}^{n-1} a_{ij}\frac{|\delta_i^1(t)-\delta_j^1(t)|^{\alpha_1+1}}{\alpha_1+1}+\sum_{i=1}^{n-1} 2a_{in}\frac{|\delta_i^1(t)|^{\alpha_1+1}}{\alpha_1+1}\\ &\quad+\sum_{i=1}^{n-1}(\delta_i^2(t)-k_1\delta_i^3(t))^2+\sum_{i=1}^{n-1}\frac{k_1}{k_3}(\delta_i^3(t))^2,\end{aligned}$$

which is positive definite with respect to $\delta_i^1(t)$, $\delta_i^2(t)$ and $\delta_i^3(t)$ $(i\in\mathcal{I}_{n-1})$.

Then,

$$\dot V_2(t)=\sum_{i=1}^{n-1}\sum_{j=1}^{n-1} a_{ij} sig(\delta_j^1(t)-\delta_i^1(t))^{\alpha_1}(\dot\delta_j^1(t)-\dot\delta_i^1(t))$$

$$
\begin{aligned}
&+\sum_{i=1}^{n-1} 2a_{in} sig(\delta_i^1(t))^{\alpha_1}\dot{\delta}_i^1(t)\\
&+\sum_{i=1}^{n-1} 2(\delta_i^2(t)-k_1\delta_i^3(t))(\dot{\delta}_i^2(t)-k_1\dot{\delta}_i^3(t))\\
&+\sum_{i=1}^{n-1}\frac{2k_1}{k_3}\delta_i^3(t)\dot{\delta}_i^3(t)\\
=&-\sum_{i=1}^{n-1}2\delta_i^2(t)\sum_{j=1}^{n-1}a_{ij}sig(\delta_j^1(t)-\delta_i^1(t))^{\alpha_1}\\
&+\sum_{i=1}^{n-1}2a_{in}\delta_i^2(t)sig(\delta_i^1(t))^{\alpha_1}\\
&+\sum_{i=1}^{n-1}2(\delta_i^2(t)-k_1\delta_i^3(t))\\
&\times\left(\sum_{j=1}^{n-1}a_{ij}sig(x_j(t)-x_i(t))^{\alpha_1}-a_{in}sig(\delta_i^1(t))^{\alpha_1}\right)\\
&+\sum_{i=1}^{n-1}\frac{2k_1}{k_3}\delta_i^3(t)(-k_2 sig(\delta_i^3(t))^{\frac{2\alpha_1}{1+\alpha_1}}\\
&+k_3\sum_{j=1}^{n-1}a_{ij}sig(\delta_j^1(t)-\delta_i^1(t))^{\alpha_1}-k_3a_{in}sig(\delta_i^1(t))^{\alpha_1})\\
=&-\sum_{i=1}^{n-1}\frac{2k_1k_2}{k_3}|\delta_i^3(t)|^{\frac{3\alpha_1+1}{1+\alpha_1}}\le 0
\end{aligned}
$$

Denote the invariant set $S=\{(\delta_1^1,\delta_1^2,\delta_1^3,\ldots,\delta_{n-1}^1,\delta_{n-1}^2,\delta_{n-1}^3)|\dot{V}_2\equiv 0\}$. Note that $\dot{V}_2\equiv 0$ implies that $\delta_i^3=0\ \ (i\in\mathcal{I}_{n-1})$, thus,

$$\sum_{j=1}^{n-1}a_{ij}sig(\delta_j^1(t)-\delta_i^1(t))^{\alpha_1}-a_{in}sig(\delta_i^1(t))^{\alpha_1}=0,\quad i\in\mathcal{I}_{n-1}.$$

Then, we have

$$
\begin{aligned}
&\sum_{i=1}^{n-1}\delta_i^1(t)\left(\sum_{j=1}^{n-1}a_{ij}sig(\delta_j^1(t)-\delta_i^1(t))^{\alpha_1}-a_{in}sig(\delta_i^1(t))^{\alpha_1}\right)\\
&\quad=-\sum_{i=1}^{n-1}\sum_{j=1}^{n-1}a_{ij}|(\delta_i^1(t)-\delta_j^1(t))|^{\alpha_1+1}\\
&\qquad-\sum_{i=1}^{n-1}a_{in}|\delta_i^1(t)|^{\alpha_1+1}=0,
\end{aligned}
$$

which implies that $\delta_i^1(t)=\delta_j^1(t)=0$ for all i, $j\in\mathcal{I}_{n-1}$ by connectivity of $\mathcal{G}$. Thus, $\dot{V}_2(t)=0$ implies that $\delta_i^1(t)=\delta_i^2(t)=0,\quad i\in\mathcal{I}_{n-1}$. It follows Lemma 2.1 that $\delta_i^1(t)\to 0,\delta_i^2(t)\to 0,$ i.e. $x_i(t)\to x_n(t),\quad v_i(t)\to v_n(t)$, $i\in\mathcal{I}_{n-1}$, as $t\to\infty$.

Next, note that the multi-agent system (5) is a homogeneous system of degree $\sigma=\alpha_1-1<0$ with dilation $(\underbrace{2,\ldots,2}_{n},\underbrace{1+\alpha_1,\ldots,1+\alpha_1}_{n},\underbrace{1+\alpha_1,\ldots,1+\alpha_1}_{n})$. Therefore, the multi-agent system (5) achieves the SOFTC by Lemma 3.2. □

4.2. *Directed graph*

For directed graph, we consider the communication topology $\mathcal{G}$ with three cases – the star topology, the directed tree and the special directed tree.

Theorem 4.3: *Suppose that the communication topology $\mathcal{G}$ is a star topology, i.e. the multi-agent system* (5) *has a leader and $n-1$ followers, and each follower receives information only from the leader. Then the multi-agent system* (5) *can achieve the SOFTC.*

Proof: Analogous to Theorem 4.2's proof, we assume the agents $1,2,\ldots,n-1$ are the followers and n is the leader. Thus, we have $[a_{ij}]_{1\le i,j\le n-1}=0$ and $a_{n1}=a_{n2}=\cdots=a_{nn}=0$, $\bar{a}>0$, where $\bar{a}=[a_{1n},a_{2n},\ldots,a_{(n-1)n}]^T$. We rewrite the multi-agent system (5) as follows:

$$
\begin{cases}
\dot{x}_i(t)=v_i(t),\\
\dot{v}_i(t)=a_{in}sig(x_n(t)-x_i(t))^{\alpha_1}+k_1\dot{y}_i(t),\\
\dot{y}_i(t)=-k_2 sig(y_i(t))^{\frac{2\alpha_1}{1+\alpha_1}}+k_3a_{in}sig(x_n(t)-x_i(t))^{\alpha_1},\\
\qquad i\in\mathcal{I}_{n-1},
\end{cases}\tag{8}
$$

and

$$
\begin{cases}
\dot{x}_n(t)=v_n(t),\\
\dot{v}_n(t)=k_1\dot{y}_n(t),\\
\dot{y}_n(t)=-k_2 sig(y_n(t))^{\frac{2\alpha_1}{1+\alpha_1}}.
\end{cases}\tag{9}
$$

Because $\dot{y}_n(t)=-k_2 sig(y_n(t))^{\frac{2\alpha_1}{1+\alpha_1}}$ is finite-time stable, i.e. there exists a finite-time T_2 such that $\lim_{t\to T_2^-}y_n(t)=0$, and $y_n(t)=0$ when $t\ge T_2$.

Let $\delta_i^1(t)=x_i(t)-x_n(t),\ \delta_i^2(t)=v_i(t)-v_n(t),\ \delta_i^3(t)=y_i(t)-y_n(t),\ i\in\mathcal{I}_n$. If $t\ge T_2$, we have

$$
\begin{cases}
\dot{\delta}_i^1(t)=\delta_i^2(t),\\
\dot{\delta}_i^2(t)=-a_{in}sig(\delta_i^1(t))^{\alpha_1}+k_1\dot{\delta}_i^3(t),\\
\dot{\delta}_i^3(t)=-k_2 sig(\delta_i^3(t))^{\frac{2\alpha_1}{1+\alpha_1}}-k_3a_{in}sig(\delta_i^1(t))^{\alpha_1},\quad i\in\mathcal{I}_{n-1},
\end{cases}\tag{10}
$$

Take a Lyapunov function for (10) as

$$
\begin{aligned}
V_3(t)=&\sum_{i=1}^{n-1}2a_{in}\frac{|\delta_i^1(t)|^{\alpha_1+1}}{\alpha_1+1}+\sum_{i=1}^{n-1}(\delta_i^2(t)-k_1\delta_i^3(t))^2\\
&+\sum_{i=1}^{n-1}\frac{k_1}{k_3}(\delta_i^3(t))^2,
\end{aligned}
$$

which is positive definite with respect to $\delta_i^1(t)$, $\delta_i^2(t)$ and $\delta_i^3(t)$ $(i \in \mathcal{I}_{n-1})$. Consider the derivative of $V_3(t)$ along the trajectories of the closed-loop system (10), we have

$$\begin{aligned}
\dot{V}_3(t) &= \sum_{i=1}^{n-1} 2a_{in} sig(\delta_i^1(t))^{\alpha_1} \dot{\delta}_i^1(t) + \sum_{i=1}^{n-1} 2(\delta_i^2(t) - k_1\delta_i^3(t)) \\
&\quad \times (\dot{\delta}_i^2(t) - k_1\dot{\delta}_i^3(t)) + \sum_{i=1}^{n-1} \frac{2k_1}{k_3} \delta_i^3(t)\dot{\delta}_i^3(t) \\
&= \sum_{i=1}^{n-1} 2a_{in}\delta_i^2(t) sig(\delta_i^1(t))^{\alpha_1} + \sum_{i=1}^{n-1} 2(\delta_i^2(t) - k_1\delta_i^3(t)) \\
&\quad \times (-a_{in} sig(\delta_i^1(t))^{\alpha_1}) + \sum_{i=1}^{n-1} \frac{2k_1}{k_3} \delta_i^3(t) \\
&\quad \times \left(-k_2 sig(\delta_i^3(t))^{\frac{2\alpha_1}{1+\alpha_1}} - k_3 a_{in} sig(\delta_i^1(t))^{\alpha_1} \right) \\
&= -\sum_{i=1}^{n-1} \frac{2k_1k_2}{k_3} |\delta_i^3(t)|^{\frac{3\alpha_1+1}{1+\alpha_1}} \le 0
\end{aligned}$$

Denote the invariant set $S = \{(\delta_1^1, \delta_1^2, \delta_1^3, \ldots, \delta_{n-1}^1, \delta_{n-1}^2, \delta_{n-1}^3) | \dot{V}_3 \equiv 0\}$. Similar to the analysis of Theorem 4.2, $\dot{V}_3 \equiv 0$ implies that $\delta_i^1 = \delta_i^2 = \delta_i^3 = 0$, $i \in \mathcal{I}_{n-1}$. It follows Lemma 2.1 that $\delta_i^1(t) \to 0, \delta_i^2(t) \to 0$, i.e. $x_i(t) \to x_n(t)$, $v_i(t) \to v_n(t)$, $i \in \mathcal{I}_{n-1}$, as $t \to \infty$.

Next, note that the multi-agent system (5) is a homogeneous system of degree $\sigma = \alpha_1 - 1 < 0$ with dilation $(\underbrace{2, \ldots, 2}_{n}, \underbrace{1+\alpha_1, \ldots, 1+\alpha_1}_{n}, \underbrace{1+\alpha_1, \ldots, 1+\alpha_1}_{n})$. Therefore, the multi-agent system (5) achieves the SOFTC by Lemma 3.2. □

Theorem 4.4: *Suppose that the communication topology $\mathcal{G}$ is a directed tree. Then the multi-agent system* (5) *can achieve the SOFTC.*

Proof: Without loss of generality, we assume the agent 1 is the leader. All vertices of the directed tree $\mathcal{G}$ can be classified into the following subsets: $\mathcal{V}_0 = \{s_1\}$, $\mathcal{V}_1 = \{s_j \in \mathcal{V}$: s_j only receives information from s_1 at any time $t\}, \ldots, \mathcal{V}_q = \{s_j \in \mathcal{V}$: s_j only receives information from vertices in $\mathcal{V}_{q-1}$ at any time $t\}$. Moreover, $\bigcup_{p=0}^q \mathcal{V}_p = \mathcal{V}$.

From Theorem 4.3, there exists a finite-time T_2 such that $\lim_{t\to T_2^-} \|x_i(t) - x_1(t)\| = 0$, $\lim_{t\to T_2^-} \times \|v_i(t) - v_1(t)\| = 0$, and $x_i(t) = x_1(t)$, $v_i(t) = v_1(t)$, when $t \ge T_2$ for all $i \in \{i : s_i \in V_1\}$. Similarly, there exists a finite-time $T_3 > T_2$ such that $\lim_{t\to T_3^-} \|x_i(t) - x_1(t)\| = 0$, $\lim_{t\to T_3^-} \|v_i(t) - v_1(t)\| = 0$, and $x_i(t) = x_1(t)$, $v_i(t) = v_1(t)$, when $t \ge T_3$ for all $i \in \{i : s_i \in \mathcal{V}_2\}$. By induction, the multi-agent system (5) can achieve the SOFTC. □

Here, we consider another special directed networks which are an extension of directed tree. We assume the agent 1 is the leader and all vertices of the directed network $\mathcal{G}$ can be classified into the following subsets: $\mathcal{V}_0 = \{s_1\}$, $\mathcal{V}_1 = \{s_j \in \mathcal{V}$: s_j only receives information from s_1 at any time $t\}, \ldots, \mathcal{V}_q = \{s_j \in \mathcal{V}$: s_j only receives information from vertex in $\bigcup_{p=0}^{q-1} \mathcal{V}_p$ at any time t $\}$. Moreover, $\bigcup_{p=0}^q \mathcal{V}_p = \mathcal{V}$. Under these directed networks, similar to the Proof of Theorem 4.4, we get the following result on SOFTC. For convenience of exposition, we define these directed networks as the special directed tree.

Theorem 4.5: *Suppose that the communication topology $\mathcal{G}$ is a special directed tree. Then the multi-agent system* (5) *can achieve the SOFTC.* □

Remark 1: Suppose that the communication topology $\mathcal{G}$ is strongly connected and satisfies the detailed balance condition, i.e. there exists a vector $\omega = [\omega_1, \omega_2, \ldots, \omega_n]^T \in \mathbb{R}_{>0}^n$ such that $\omega_i a_{ij} = \omega_j a_{ji}$ for all i, $j \in \mathcal{I}_n$. Take a Lyapunov function for (5) as

$$\begin{aligned}
V(t) &= \sum_{i=1}^n \sum_{j=1}^n \omega_i a_{ij} \frac{|x_i(t) - x_j(t)|^{\alpha_1+1}}{\alpha_1 + 1} \\
&\quad + \sum_{i=1}^n (\omega_i v_i(t) - k_1 y_i(t))^2 + \sum_{i=1}^n \frac{k_1}{k_3} (y_i(t))^2.
\end{aligned}$$

Similar to the analysis of Theorem 4.1, the multi-agent system (5) can achieve the SOFTC. The result can also be extended to the leader-following network which the topology of the followers is strongly connected and satisfies the detailed balance condition.

Remark 2: In this article, we only consider the finite-time consensus of second-order multi-agent system (5) under some special directed graphs. Because the Laplician matrix is asymmetric and it is hard to select an appropriate Lyapunov function, the finite-time consensus of second-order multi-agent system (5) under general directed network is difficult to solve.

5. Simulations

In this section, we first present a numerical simulation in Example 5.1 to illustrate the effectiveness of theoretical result when the network is an undirected connected graph. In Example 5.2, we provide an illustration of theoretical result when the network is a special directed tree.

Example 5.1: Consider the communication topology $\mathcal{G}$ shown in Figure 1, where the weight of each edge is 1. We assume that $k_i = 1$ $(i = 1, 2, 3)$, $\alpha_1 = \frac{1}{5}$ and the initial states are $x(0)$=[7, 10, −3, −7, −1], $v(0)$=[1, −5, 5, 3, 2] and $y(0)$=[0, 0, 0, 0, 0]. Note that the communication topology $\mathcal{G}$ is an undirected connected graph. The simulation results using (5) are shown in Figure 2. The position and velocity trajectories of agents reach

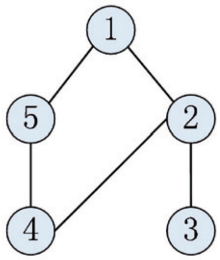

Figure 1. The network $\mathcal{G}$ in Example 5.1.

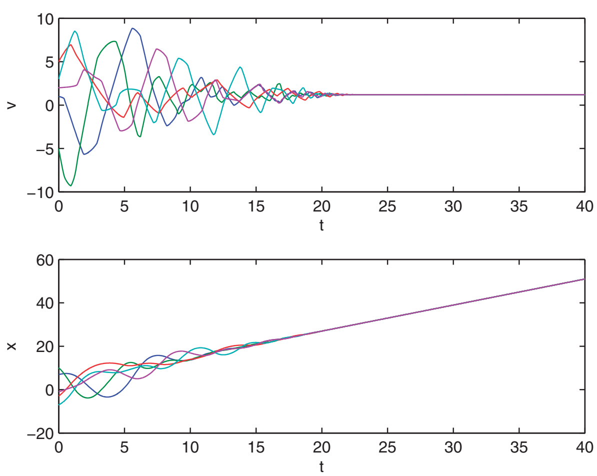

Figure 2. Simulation results with the network depicted in Figure 1.

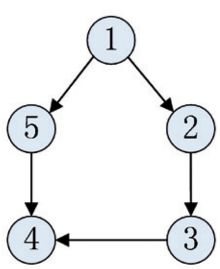

Figure 3. The network $\mathcal{G}$ in Example 5.2.

consensus at about $t = 23$ s, which accord with the results established in Theorem 4.1.

Example 5.2: Consider the communication topology $\mathcal{G}$ shown in Figure 3, where the weight of each edge is 1. All vertices of the directed network $\mathcal{G}$ can be classified into the following subsets: $\mathcal{V}_0 = \{s_1\}$, $\mathcal{V}_1 = \{s_2, s_5\}$, $\mathcal{V}_2 = \{s_3\}$ and $\mathcal{V}_3 = \{s_4\}$. We assume that $k_i = 1$ $(i = 1, 2, 3)$, $\alpha_1 = \frac{1}{5}$ and the initial states are $x(0) = [7, -3, -7, -1, 10]$, $v(0) = [1, 5, 3, 2, -5]$ and $y(0) = [0, 0, 0, 0, 0]$. The simulation results using (5) are shown

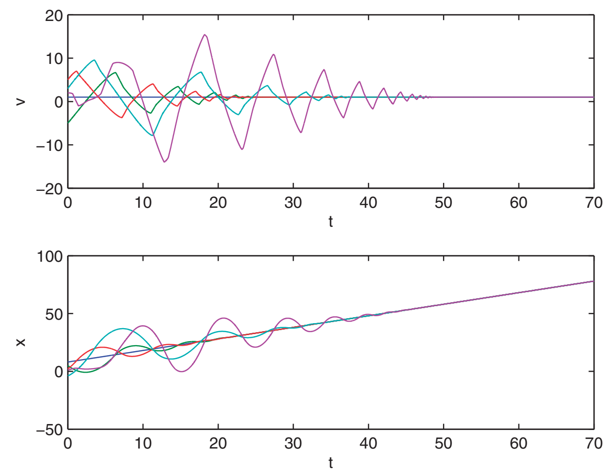

Figure 4. Simulation results with the network depicted in Figure 3.

in Figure 4. Note that the agents 2, 3, 4 and 5 reach the agent 1's state at about $t = 50$ s hierarchically, which accord with the results established in Theorem 4.5.

6. Conclusions

In this article, we consider the second-order finite-time consensus of multi-agent system without velocity measurements. Based on the auxiliary system approach, we first propose a consensus protocol for second-order multi-agent system. Then, by using the graph theory, Lyapunov theory and the homogeneous domination method, we solve the second-order finite-time consensus (SOFTC) under undirected and directed graphs, respectively. At last, some examples are given to illustrate the effectiveness of theoretical results. Compared to the traditional consensus theories, the proposed SOFTC of multi-agent system without velocity measurements in this article is not only theoretically important but also has wider applications. The future work will focus on the more complex communication topology for SOFTC of multi-agent system with/without velocity measurements, for example, SOFTC of multi-agent system under switching topologies/random networks.

Acknowledgements

This work was supported by 973 Program (Grant no. 2012CB821203), NSFC (Grant nos. 61020106005, 10972002 and 61104212) and the Fundamental Research Funds for the Central Universities (Grant nos. K50511040005 and K50510040003). The authors are very grateful to the reviewers and the editor for their valuable comments to improve the presentation of this article.

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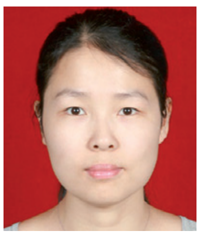

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