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A novel group consensus protocol for heterogeneous multi-agent systems

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In this paper, we consider the group consensus problem of heterogeneous multi-agent systems. Based on the feature of heterogeneous agents, a novel protocol is proposed for heterogeneous multi-agent systems. First, the state transformation method is used and an equivalent system is obtained. Then, the group consensus problem is analysed and some sufficient and/or necessary conditions are given for heterogenous multi-agent systems under undirected and directed networks, respectively. Finally, simulation examples are presented to demonstrate the effectiveness of the theoretical results.

Keywords: group consensus; heterogeneous multi-agent systems; directed network; graph theory

1. Introduction

Consensus problem studies the coordination of multiple agents and the design of consensus protocol to converge to agreement asymptotically or in a finite time. As a fundamental problem of distributed coordination, consensus problem has been investigated from biology, physics and sociology to communication, computer science and power systems, etc. There is a wide range of applications to multi-agent consensus, such as flocking of social insects, formation control and containment control of unmanned aerial vehicles, coverage control of sensor networks, and so on (Cortés, Martínez, Karatas, & Bullo, 2002; Guan, Ji, Zhang, & Wang, 2014; Ji, Lin, & Yu, 2012, 2015; Ma, Zheng, & Wang, 2014; Olfati-Saber, 2006; Su, Jia, & Chen, 2014; Wang, Xie, & Cao, 2013; Xiao, Wang, & Chen, 2013; Zheng & Wang, 2014). Up to now, by using various mathematical tools, multi-agent consensus has been studied in details, and lots of consensus criterions have been obtained (Olfati-Saber, Fax, & Murray, 2007).

Consensus problem of multi-agent systems has been studied in many viewpoints. The variations include: whether the dynamic of agents is homogeneous and heterogeneous (continuous-time or discrete-time); whether the communication network is undirected or directed (fixed or switching); whether the final convergence state is single or cluster; whether the information update of each agent is synchronous or asynchronous; whether the interconnections are affected by delays (noises) or not, etc. (Gao & Wang, 2011; Hong, Hu, & Gao, 2006; Li & Zhang, 2010; Olfati-Saber & Murray, 2004; Ren & Beard, 2005; Su, Chen, Lam, & Lin, 2014; Sun, Wang, & Xie, 2008; Xiao, Meng, & Chen, 2015; Xiao & Wang, 2008; Xie & Wang, 2007; Yu & Wang, 2010; Zhang, Hao, Zhang, & Wang, 2014; Zheng, Zhu, & Wang, 2011). In different viewpoints, sufficient and/or necessary conditions are obtained for solving the multi-agent consensus. For homogeneous multi-agent systems, Olfati-Saber and Murray (2004) studied the consensus of first-order multi-agent systems and gave some sufficient conditions for solving the consensus problem. Necessary and sufficient conditions for solving the consensus of first-order multi-agent systems under fixed and switching topologies were presented in Ren and Beard (2005). Xie and Wang (2007) gave some sufficient conditions for solving the consensus of second-order multi-agent systems. For heterogeneous multi-agent systems, it is a hot topic of research in recent years with fruitful results. The output consensus of heterogeneous linear multi-agent system was studied in Kim, Shim, and Seo (2011) and sufficient criterions were presented based on output regulation theory. Zheng et al. (2011) proposed a heterogeneous multi-agent system which is composed of first-order and second-order integrator agents. By using Lyapunov direct method, some sufficient conditions for consensus were established when communication networks are undirected connected graphs and leader-following networks. Necessary and sufficient condition for solving the consensus of heterogeneous multi-agent system with fixed directed network was also developed in Zheng and Wang (2012a). Some other results about consensus of heterogeneous multi-agent systems can be found in Zheng and Wang (2012b), Hu, Yu, Xuan, Zhang, and Xie (2014).

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What is called 'birds of the same feather flock together'. For instance, scattered animals in a system migrate by species and a community divides into many parties as incremental conflicts of opinions. At present, there has been a few work about the division of a group on coordination of multi-agent systems. Yu and Wang (2010) considered group consensus of multi-agent systems and obtained some sufficient conditions for solving the group consensus problem by using double-tree-form transformations. Based on the graph theory and the Lyapunov theory, Jing, Zheng, and Wang (2014) established some sufficient criterions for flocking problem in multiple groups of agents. Hu et al. (2014) investigated group consensus problem of the heterogeneous agents which are governed by the Euler–Lagrange system and the double-integrator system. A sufficient condition was obtained for solving the group consensus and time-delay group consensus. However, all the aforementioned analyses are on the feature of communication networks, i.e., the group is not decided by the feature of agents. In the practical systems, the agents will converge to a different group in a cooperative network. Especially for the heterogeneous multi-agent systems, the same dynamic agents will converge to the same group and the different dynamic agents will separate.

Motivated by all the above analyses, we extend the consensus of heterogeneous multi-agent system in existing results to group consensus. Different from the previous group consensus (flocking) protocols of multi-agent systems, a novel protocol is proposed based on the feature of heterogeneous agents. First, the state transformation method is adopted for heterogeneous multi-agent system with a novel protocol and an equivalent system is obtained. Then, by using the Lyapunov method and the graph theory, we prove that the heterogeneous multi-agent system can solve the group consensus if the communication network is an undirected connected graph. For directed graph, a necessary and sufficient condition is presented for solving the group consensus of heterogeneous multi-agent system if the feedback gain $k_1 > 2\sqrt{\max_{i\in\mathcal{I}_m} \frac{d_{ii}}{c_2}}$. The results in this paper can also be extended to the group consensus analysis of homogeneous multi-agent systems.

The remainder of this paper is organised as follows. In Section 2, we present preliminaries and problem formulation. In Section 3, we give the main results. In Section 4, numerical simulations are given to illustrate the effectiveness of theoretical results. Finally, some conclusions are drawn in Section 5.

The following notations will be used throughout this paper: $\mathbb{R}$ denotes the set of real number, $\mathbb{R}^{n\times n}$ be the set of $n \times n$ matrix. $\mathcal{I}_m = \{1, 2, \dots, m\}$, $\mathcal{I}_n/\mathcal{I}_m = \{m+1, m+2, \dots, n\}$. For a given vector or matrix A, A^T denotes its transpose, $\|X\|$ and $\|X\|_1$ denote the Euclidean norm and L_1 norm of a vector X. $\mathbf{1}_n$ is a vector with elements being all ones. 0 ($0_{m\times n}$) denotes an all-zero vector or matrix with compatible dimension (dimension $m \times n$). diag$\{a_1, a_2, \dots, a_n\}$ defines a diagonal matrix with diagonal elements being $a_1, a_2, \dots, a_n$.

2. Preliminaries and problem formulation

In this section, some basic concepts about the algebraic graph theory are presented first. For more details about the algebraic graph theory, one can refer to Godsil and Royal (2001). Then, we formulate the problem to be studied.

Let $\mathscr{G}(\mathscr{A}) = (\mathscr{V}, \mathscr{E}, \mathscr{A})$ be a weighted directed graph of order n with a vertex set $\mathscr{V} = \{s_1, s_2, \dots, s_n\}$, an edge set $\mathscr{E} = \{e_{ij} = (s_i, s_j)\} \subset \mathscr{V} \times \mathscr{V}$ and a non-negative matrix $\mathscr{A} = [a_{ij}]_{n\times n}$. A directed path between two distinct vertices s_i and s_j is a finite ordered sequence of distinct edges of $\mathscr{G}$ with the form $(s_i, s_{k_1}), (s_{k_1}, s_{k_2}), \dots, (s_{k_l}, s_j)$. A directed tree is a directed graph, where there exists a vertex called the root such that there exists a unique directed path from this vertex to every other vertex. A directed spanning tree is a directed tree, which consists of all the nodes and some edges in $\mathscr{G}$. If a directed graph has the property that $(s_i, s_j) \in \mathscr{E} \Leftrightarrow (s_j, s_i) \in \mathscr{E}$, the directed graph is called undirected. An undirected graph is said to be connected if there exists a path between any two distinct vertices of the graph. The degree matrix $\mathscr{D} = [d_{ij}]_{n\times n}$ is a diagonal matrix with $d_{ii} = \sum_{j:s_j\in\mathscr{N}_i} a_{ij}$ and the Laplacian matrix of the graph is defined as $\mathscr{L} = [l_{ij}]_{n\times n} = \mathscr{D} - \mathscr{A}$. Two graphs $\mathscr{G}_1$ and $\mathscr{G}_2$ are isomorphic if there is a bijection φ from $\mathscr{V}(\mathscr{G}_1)$ to $\mathscr{V}(\mathscr{G}_2)$, such that $(s_i, s_j) \in \mathscr{E}(\mathscr{G}_1)$ if and only if $(\varphi(s_i), \varphi(s_j)) \in \mathscr{E}(\mathscr{G}_2)$.

Similar to Zheng et al. (2011), we consider a heterogeneous multi-agent system which is composed of the first-order and second-order integrator agents. The number of agents is n. Without loss of generality, we assume the first m agents with second-order integrator dynamics and the rest $n - m$ ($n > m$) agents with first-order integrator dynamics. Each agent has the dynamics as follows:

$$\begin{cases} \dot{x}_i = v_i, \quad \dot{v}_i = u_i, \quad i \in \mathcal{I}_m, \\ \dot{x}_i = u_i, \quad i \in \mathcal{I}_n/\mathcal{I}_m, \end{cases} \tag{1}$$

where $x_i \in \mathbb{R}$, $v_i \in \mathbb{R}$ and $u_i \in \mathbb{R}$ are the position, velocity and control input, respectively, of agent i. The control input with a state feedback is also said to be a protocol. The initial conditions are $x_i(0) = x_{i0}$, $v_i(0) = v_{i0}$. Let $x(0) = [x_{10}, x_{20}, \dots, x_{n0}]^T$, $v(0) = [v_{10}, v_{20}, \dots, v_{m0}]^T$.

Definition 2.1: Protocol u_i is said to solve a group consensus of heterogeneous multi-agent system (1) asymptotically if for any initial conditions, we have (1), $\lim_{t\to\infty}\|x_i(t) - x_j(t)\| = 0$, for $i, j \in \mathcal{I}_m$ and (2), $\lim_{t\to\infty}\|x_i(t) - x_j(t)\| = 0$, for $i, j \in \mathcal{I}_n/\mathcal{I}_m$.

From Definition 2.1, it is not hard to see that consensus problem in Zheng et al. (2011) and Zheng and Wang (2012a) is a special case of group consensus. To solve the group consensus problem of heterogeneous multi-agent system (1) is a challenging work. It needs to find suitable distributed protocol for each agent. In Zheng et al. (2011), the consensus protocol is designed with same mode for heterogeneous agents, i.e., the state feedback of each agent is not discounted. However, it is easy to know that the state feedback is different for heterogeneous agents. In this paper, we suppose that the first-order integrate agents discount its state as c_1x_i and the second-order integrate agents discount its state as c_2x_i, c_2v_i for feedback. Thus, a novel protocol is proposed for the heterogeneous multi-agent system (1) as follows:

$$u_i = \begin{cases} \sum_{j=1}^{n} a_{ij}(c_l x_j - c_2 x_i) - k_1 c_2 v_i, & i \in \mathcal{I}_m, \quad l \in \{1, 2\} \\ k_2 \sum_{j=1}^{n} a_{ij}(c_l x_j - c_1 x_i), & i \in \mathcal{I}_n/\mathcal{I}_m, \quad l \in \{1, 2\}, \end{cases} \tag{2}$$

where $\mathscr{A} = [a_{ij}]_{n\times n}$ is the weighted adjacency matrix associated with the graph $\mathscr{G}$, $k_1 > 0$, $k_2 > 0$ are the feedback gains, $c_1 > 0$, $c_2 > 0$ are the discount rates.

3. Main results

In this section, the group consensus of heterogeneous multi-agent system (1) will be considered with undirected graph and directed graph, respectively. Some sufficient and/or necessary conditions will be obtained for solving the group consensus problem.

First, we give a transformation of state. Let $p_i = c_2x_i$, $q_i = c_2v_i$ for $i \in \mathcal{I}_m$ and $p_i = c_1x_i$ for $i \in \mathcal{I}_n/\mathcal{I}_m$. Thus, the heterogeneous multi-agent system (1) with protocol (2) can be rewritten as follows:

$$\begin{cases} \dot{p}_i(t) = q_i(t), & i \in \mathcal{I}_m, \\ \dot{q}_i(t) = c_2 \sum_{j=1}^{n} a_{ij}(p_j - p_i) - k_1 c_2 q_i, & i \in \mathcal{I}_m, \\ \dot{p}_i(t) = k_2 c_1 \sum_{j=1}^{n} a_{ij}(p_j - p_i), & i \in \mathcal{I}_n/\mathcal{I}_m. \end{cases} \tag{3}$$

It is not difficult to know that the heterogeneous multi-agent system (1) with protocol (2) can solve the group consensus problem if and only if the heterogeneous multi-agent system (3) can achieve the consensus.

Next, we consider the group consensus of heterogeneous multi-agent system (1) with protocol (2) under the undirected graph.

Theorem 3.1: *Suppose the communication network $\mathscr{G}(\mathscr{A})$ is undirected and connected, i.e., $a_{ij} = a_{ji}$ for all $i, j \in \mathcal{I}_n$. Then the heterogeneous multi-agent system* (1) *can solve the group consensus problem with protocol* (2).

Proof: Take a Lyapunov function for (3) as

$$V(t) = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\frac{(p_i(t) - p_j(t))^2}{2} + \sum_{i=1}^{m}\frac{(q_i(t))^2}{c_2}.$$

Differentiating $V(t)$, yields that

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}(p_j - p_i)(\dot{p}_j - \dot{p}_i) + \sum_{i=1}^{m}\frac{2q_i\dot{q}_i}{c_2} \\ &= \sum_{i=1}^{m} 2q_i\left(\sum_{j=1}^{n} a_{ij}(p_j - p_i) - k_1 q_i\right) \\ &\quad + \sum_{i=1}^{m}\sum_{j=1}^{m} a_{ij}(p_j - p_i)(q_j - q_i) \\ &\quad + \sum_{i=m+1}^{n}\sum_{j=1}^{m} a_{ij}(p_j - p_i)(q_j - \dot{p}_i) \\ &\quad + \sum_{i=1}^{m}\sum_{j=m+1}^{n} a_{ij}(p_j - p_i)(\dot{p}_j - q_i) \\ &\quad + \sum_{i=m+1}^{n}\sum_{j=m+1}^{n} a_{ij}(p_j - p_i)(\dot{p}_j - \dot{p}_i). \end{aligned}$$

Because the communication network $\mathscr{G}(\mathscr{A})$ is an undirected connected graph, i.e., $\mathscr{A}$ is a symmetric matrix, we have

$$\begin{aligned} &\sum_{i=1}^{m}\sum_{j=1}^{m} a_{ij}(p_j - p_i)(q_j - q_i) = -2\sum_{i=1}^{m}\sum_{j=1}^{m} a_{ij}(p_j - p_i)q_i, \\ &\sum_{i=m+1}^{n}\sum_{j=1}^{m} a_{ij}(p_j - p_i)(q_j - \dot{p}_i) = \sum_{i=1}^{m}\sum_{j=m+1}^{n} a_{ij}(p_j - p_i)(\dot{p}_j - q_i), \\ &\sum_{i=m+1}^{n}\sum_{j=m+1}^{n} a_{ij}(p_j - p_i)(\dot{p}_j - \dot{p}_i) = 2\sum_{i=m+1}^{n}\sum_{j=m+1}^{n} a_{ij}(p_j - p_i)\dot{p}_j. \end{aligned}$$

Hence,

$$\begin{aligned} \dot{V}(t) &= -2k_1\sum_{i=1}^{m} q_i^2 + 2\sum_{i=1}^{m}\sum_{j=m+1}^{n} a_{ij}(p_j - p_i)\dot{p}_j \\ &\quad + 2\sum_{i=m+1}^{n}\sum_{j=m+1}^{n} a_{ij}(p_j - p_i)\dot{p}_j \\ &= -2k_1\sum_{i=1}^{m} q_i^2 - 2\sum_{i=m+1}^{n}\dot{p}_i\sum_{j=1}^{n} a_{ij}(p_j - p_i) \\ &= -2k_1\sum_{i=1}^{m} q_i^2 - \frac{2}{k_2c_1}\sum_{i=m+1}^{n}\dot{p}_i^2 \le 0. \end{aligned}$$

Similar to the proof in Zheng et al. (2011), it follows from Lasalle's invariance principle that

$$\lim_{t\to\infty}\|p_i(t)-p_j(t)\|=0,\quad \text{for} \quad i,j\in\mathcal{I}_n.$$

Owing to $p_i=c_2x_i$ for $i\in\mathcal{I}_m$ and $p_i=c_1x_i$ for $i\in\mathcal{I}_n/\mathcal{I}_m$, we obtain

$$\lim_{t\to\infty}c_2\|x_i(t)-x_j(t)\|=0,\quad \text{for}\quad i,j\in\mathcal{I}_m,\quad \text{and}$$
$$\lim_{t\to\infty}c_1\|x_i(t)-x_j(t)\|=0,\quad \text{for}\quad i,j\in\mathcal{I}_n/\mathcal{I}_m,$$

i.e., the heterogeneous multi-agent system (1) can solve the group consensus problem with protocol (2). □

In the following, we consider the group consensus of heterogeneous multi-agent system (1) with protocol (2) under directed graph. A key lemma is given which is an extension of the result in Ren and Beard (2005).

Lemma 3.2: *Let $\mathscr{L}$ be the Laplacian matrix of directed graph $\mathscr{G}$, and $K=diag[a_1,a_2,\ldots,a_n]$ be a diagonal matrix with diagonal elements $a_1>0,a_2>0,\ldots,a_n>0$. Then, the multi-agent system $\dot{x}=-K\mathscr{L}x$ can solve the consensus problem if and only if the directed graph $\mathscr{G}$ has a directed spanning tree. The consensus state is $\mathbf{1}_n\frac{w^TK^{-1}}{\|w^TK^{-1}\|_1}x(0)$, where $w^T\mathscr{L}=0$, $w^T\mathbf{1}_n=1$ and $K^{-1}=diag[\frac{1}{a_1},\frac{1}{a_2},\ldots,\frac{1}{a_n}]$.*

Proof: From Ren and Beard (2005), we know that the multi-agent system $\dot{x}=-\mathscr{L}x$ can solve the consensus problem if and only if the directed graph $\mathscr{G}$ has a directed spanning tree, and the consensus state is $\mathbf{1}_nw^Tx(0)$, where $w^T\mathscr{L}=0$ and $w^T\mathbf{1}_n=1$. Because $K=diag[a_1,a_2,\ldots,a_n]$ is a diagonal matrix with positive diagonal elements, $\mathscr{L}'=K\mathscr{L}$ is the corresponding Laplacian matrix of directed graph $\mathscr{G}'$. It is not hard to see that $\mathscr{G}$ and $\mathscr{G}'$ are isomorphic. Thus, $\mathscr{G}'$ has a directed spanning tree if and only if $\mathscr{G}$ has a directed spanning tree, which implies that the multi-agent system $\dot{x}=-K\mathscr{L}x$ can solve the consensus problem if and only if the directed graph $\mathscr{G}$ has a directed spanning tree, and the consensus state is $\mathbf{1}_n\frac{w^TK^{-1}}{\|w^TK^{-1}\|_1}x(0)$. □

Theorem 3.3: *Suppose the communication network $\mathscr{G}(\mathscr{A})$ is a directed graph and the feedback gain $k_1>2\sqrt{\max_{i\in\mathcal{I}_m}\frac{d_{ii}}{c_2}}$. Then, the heterogeneous multi-agent system* (1) *with consensus protocol* (2) *can solve the group consensus asymptotically if and only if $\mathscr{G}$ has a directed spanning tree.*

Proof: Let $r_{i'}=k_{3i}q_i+p_i$ for $i,i'\in\mathcal{I}_m$. From (3), we have

$$\dot{p}_i=\frac{1}{k_{3i}}(r_{i'}-p_i)$$

and

$$\begin{aligned}\dot{r}_{i'}&=k_{3i}\dot{q}_i+\dot{p}_i\\&=k_{3i}\left(c_2\sum_{j=1}^n a_{ij}(p_j-p_i)-k_1c_2q_i\right)+\frac{1}{k_{3i}}(r_{i'}-p_i)\\&=k_{3i}c_2\sum_{j=1}^n a_{ij}(p_j-p_i)+\left(k_1c_2-\frac{1}{k_{3i}}\right)(p_i-r_{i'})\\&=k_{3i}c_2\sum_{j=1}^n a_{ij}(p_j-r_{i'})\\&\quad+\left(k_1c_2-\frac{1}{k_{3i}}-k_{3i}c_2d_{ii}\right)(p_i-r_{i'}),\end{aligned}$$

for $i\in\mathcal{I}_m$. Let $b_i=(k_1c_2-\frac{1}{k_{3i}}-k_{3i}c_2d_{ii})$, $i\in\mathcal{I}_m$. If $d_{ii}=0$, let $k_{3i}\geq\frac{1}{k_1c_2}$. Otherwise, let $\frac{k_1c_2-\sqrt{(k_1c_2)^2-4c_2d_{ii}}}{2c_2d_{ii}}\leq k_{3i}\leq\frac{k_1c_2-\sqrt{(k_1c_2)^2-4c_2d_{ii}}}{2c_2d_{ii}}$. Due to $k_1>2\sqrt{\max_{i\in\mathcal{I}_m}\frac{d_{ii}}{c_2}}\geq0$, it is easy to obtain $\frac{k_1c_2-\sqrt{(k_1c_2)^2-4c_2d_{ii}}}{2c_2d_{ii}}>0$, $k_{3i}>0$ and $b_i\geq0$ for $i\in\mathcal{I}_m$.

Based on the aforementioned analysis, we obtain a first-order multi-agent system with $n+m$ agents as follows:

$$\begin{cases}\dot{p}_i=\frac{1}{k_{3i}}(r_{i'}-p_i),\quad i\in\mathcal{I}_m,\\ \dot{r}_{i'}=k_{3i}c_2\sum_{j=1}^n a_{ij}(p_j-r_{i'})+b_i(p_i-r_{i'}),\quad i'\in\mathcal{I}_m,\\ \dot{p}_i=k_2c_1\sum_{j=1}^n a_{ij}(p_j-p_i),\quad i\in\mathcal{I}_n/\mathcal{I}_m,\end{cases}\tag{4}$$

where $p_i\in\mathbb{R}$ and $r_{i'}\in\mathbb{R}$ are the states of the ith and i'th agents, respectively.

Let $\bar{\mathscr{G}}$ be a communication topology of the first-order multi-agent system (4) with a vertex set $\bar{\mathscr{V}}=\mathscr{V}_1\cup\mathscr{V}_2\cup\mathscr{V}_3$, where $\mathscr{V}_1=\{s_1,\ldots,s_m\}$, $\mathscr{V}_2=\{s_{1'},\ldots,s_{m'}\}$ and $\mathscr{V}_3=\{s_{m+1},\ldots,s_n\}$. From the proof of Theorem 3.2 in Zheng and Wang (2012a), we know that the graph $\mathscr{G}$ has a directed spanning tree if and only if the fixed topology $\bar{\mathscr{G}}$ has a directed spanning tree.

According to Lemma 3.2, it is not hard to know that the first-order multi-agent system (4) reaches consensus asymptotically if and only if the graph $\bar{\mathscr{G}}$ has a directed spanning tree.

Thus, we obtain that if the feedback gain $k_1>2\sqrt{\max_{i\in\mathcal{I}_m}\frac{d_{ii}}{c_2}}$, $\lim_{t\to\infty}\|p_i(t)-p_j(t)\|=0$ holds for $i,j\in\mathcal{I}_n$ if and only if the graph $\bar{\mathscr{G}}$ has a directed spanning tree. Accordingly, if $k_1>2\sqrt{\max_{i\in\mathcal{I}_m}\frac{d_{ii}}{c_2}}$, the heterogeneous multi-agent system (1) with consensus protocol (2) can solve the group consensus asymptotically if and only if $\mathscr{G}$ has a directed spanning tree. □

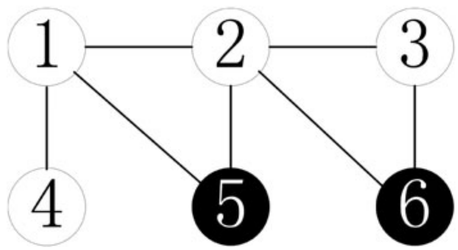

Figure 1. An undirected connected graph.

Remark 1: In fact, the results in this paper can be extended to the group consensus of heterogenous multi-agent systems with switching topologies. For homogeneous multi-agent systems, we can also analyse the group consensus problem in a cooperative network with the feature of the agents. For the simplicity of this paper, these problems are left to the interested readers as an exercise.

Remark 2: From Lemma 3.2, it is not difficult to compute the consensus state of the first-order multi-agent system (4). For convenience, let $\lim_{t\to\infty} p_i(t) = p*$. Thus, we know that $\lim_{t\to\infty} x_i(t) = \frac{p^*}{c_2}$ for $i \in \mathcal{I}_m$ and $\lim_{t\to\infty} x_i(t) = \frac{p^*}{c_1}$ for $i \in \mathcal{I}_n/\mathcal{I}_m$. If $c_1 = c_2$, it is a consensus problem of the heterogeneous multi-agent system (1). Otherwise, we can recognise which one is the first-order integrator agent (second-order integrator agent) if the relationship between c_1 and c_2 is known.

Remark 3: In Theorem 3.3, the design of feedback gain k_1 is based on the communication network. One may wonder that this is not a distributed scheme for coordination of multi-agent systems. In fact, we can choose the sufficiently large c_2 and then choose k_1 with arbitrary positive integer. Surely we can also choose sufficiently large k_1 if c_2 is fixed.

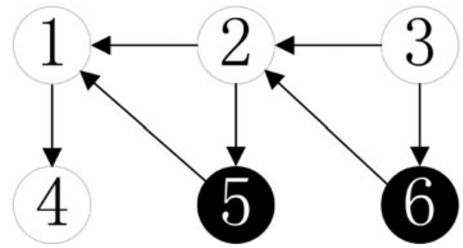

Figure 3. A directed graph which has a directed spanning tree.

4. Simulations

In this section, simulation results are worked out to demonstrate the effectiveness of our theoretic results. In the following, all graphs with 0–1 weights will be needed. Let $k_1 = 3$, $k_2 = 1$, $x(0) = [8, 5, 2, -4, 1, -5]$ and $v(0) = [1, -5, 5, 3]$. Suppose $c_1 = 1$, $c_2 = 2$.

Example 4.1: The multi-agent system is composed of six agents with an undirected connected graph shown in Figure 1. The vertices 1–4 denote the second-order integrator agents and the vertices 5–6 denote the first-order integrator agents. Using the protocol (2), the position trajectories of all the agents are shown in Figure 2. From

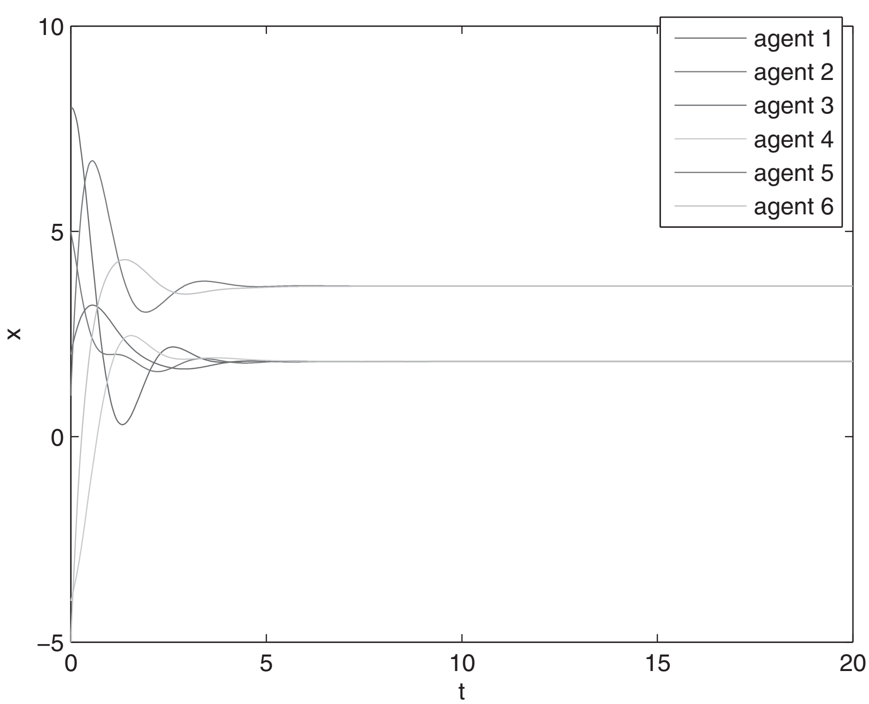

Figure 2. Trajectories of all agents with the network depicted in Figure 1.

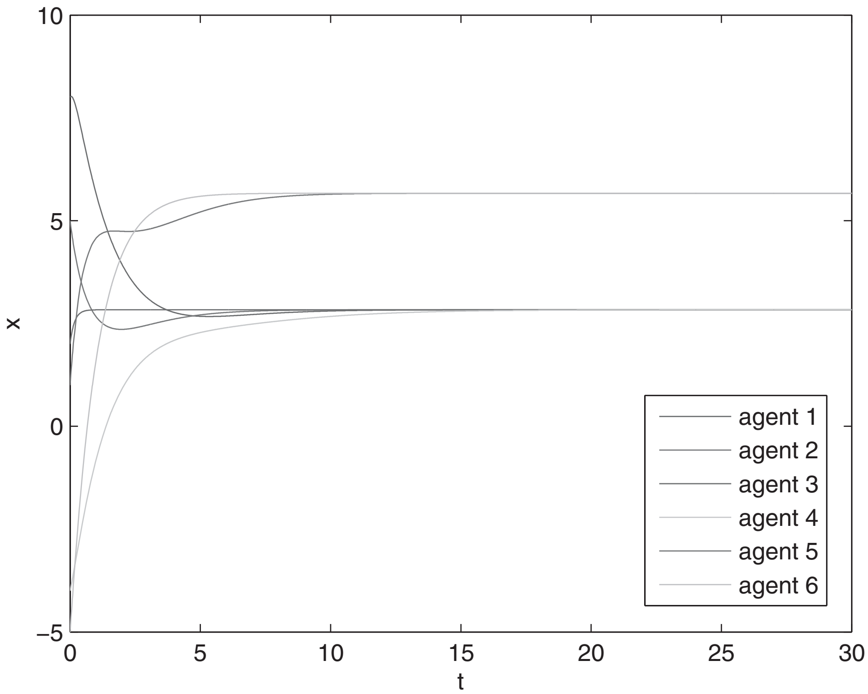

Figure 4. Trajectories of all agents with the network depicted in Figure 3.

Figure 2, we can see that the heterogeneous multi-agent system (1) reaches group consensus, which is consistent with the results in Theorem 3.1.

Example 4.2: Suppose the communication network is chosen as Figure 3. The vertices 1–4 denote the second-order integrator agents and the vertices 5–6 denote the first-order integrator agents. Note that the communication network in Figure 3 has a directed spanning tree and $k_1 > 2\sqrt{\max_{i\in\mathcal{I}_m} \frac{d_{ii}}{c_2}}$. Using consensus protocol (2), the position trajectories of all the agents reach group consensus as shown in Figure 4, which is consistent with the sufficiency of Theorem 3.3.

5. Conclusions

In this paper, the group consensus problem was analysed for heterogeneous multi-agent systems. A novel protocol was proposed based on the feature of heterogeneous agents. By using the state transformation, the graph theory and the Lyapunov theory, we gave a sufficient condition under an undirected network and a necessary and sufficient condition under a directed network for solving the group consensus. The results can also be extended to analysing the group consensus of more general multi-agent systems. In the future research, we will investigate the group containment problem and the group flocking problem of multi-agent systems.

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