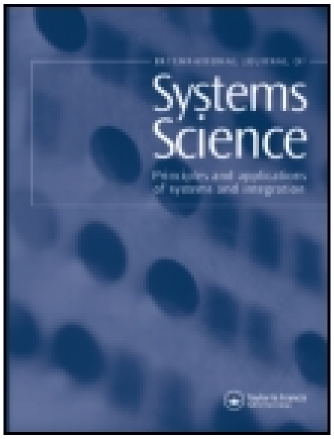

International Journal of Systems Science
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tsys20

Quantised consensus of multi-agent systems with nonlinear dynamics

Yunru Zhu[ab], Yuanshi Zheng[ab] & Long Wang[c]
[a] Key Laboratory of Electronic Equipment Structure Design of Ministry of Education, School of Mechano-electronic Engineering, Xidian University, Xian, China
[b] Center for Complex Systems, School of Mechano-electronic Engineering, Xidian University, Xi'an, China
[c] Center for Systems and Control, College of Engineering, Peking University, Beijing, China
Published online: 22 Oct 2013.

International Journal of Systems Science, 2015
Vol. 46, No. 11, 2061–2071, http://dx.doi.org/10.1080/00207721.2013.849770

Taylor & Francis
Taylor & Francis Group

Quantised consensus of multi-agent systems with nonlinear dynamics

Yunru Zhu[a,b], Yuanshi Zheng[a,b,*] and Long Wang[c]

[a]*Key Laboratory of Electronic Equipment Structure Design of Ministry of Education, School of Mechano-electronic Engineering, Xidian University, Xian, China;* [b]*Center for Complex Systems, School of Mechano-electronic Engineering, Xidian University, Xi'an, China;* [c]*Center for Systems and Control, College of Engineering, Peking University, Beijing, China*

(*Received 10 January 2013; accepted 15 September 2013*)

This paper studies the consensus problem of first-order and second-order multi-agent systems with nonlinear dynamics and quantised interactions. Continuous-time and impulsive control inputs are designed for the multi-agent systems on the logarithmic quantised relative state measurements of agents, respectively. By using nonsmooth analysis tools, we get some sufficient conditions for the consensus of multi-agent systems under the continuous-time inputs. Compared with continuous-time control inputs, impulsive distributed control inputs just use the state variables of the systems at discrete-time instances. Based on impulsive control theory, we prove that the multi-agent systems can reach consensus by choosing proper control gains and impulsive intervals. The simulation results are given to verify the effectiveness of the theoretical results.

Keywords: multi-agent system; consensus; quantised control; impulsive control

1. Introduction

In recent years, distributed coordination of multi-agent systems has attracted a great deal of attention. This is partly due to broad practical applications of multi-agent systems in many areas such as formation control (Fax & Murray, 2004; Ren & Cao, 2011), swarming (Liu, Chu, Wang, & Wang, 2005) and flocking (Tanner, Jadbabaie, & Pappas, 2007). A key problem in distributed control is the consensus problem, which aims at designing distributed algorithm to make a group of agents reach an agreement upon some quantities of interest. In Vicsek, Czirok, Jacob, Cohen, and Schochet (1995), the authors proposed a discrete-time model for the phase transition of a group of autonomous agents. Through computer simulations, it was showed that all agents eventually moved in the same direction. In Jadbabaie, Lin, and Morse (2003), a theoretical explanation was provided for the consensus behaviour of the Vicsek model through graph theory. In Olfati-Saber and Murray (2004), a theoretical framework for consensus problems of continuous-time multi-agent systems was presented. The authors provided the convergence analysis of a consensus protocol for a network of integrators with directed information flow and fixed/switching topology. In Ren and Beard (2005), the authors extended the results in Jadbabaie et al. (2003) to the case of directed graphs. The work above inspired much subsequent theoretical investigation for the consensus problems of multi-agent systems. So far, lots of works have been done for the average consensus problem of multi-agent systems under different contexts (Ji, Wang, Lin, & Wang, 2010; Sun, Wang, & Xie, 2008; Xiao & Wang, 2008; Xiao, Wang, Chen, & Gao, 2009; Yu & Wang, 2012; Zheng & Wang, 2012; Zheng, Zhu, & Wang, 2011; Zheng, Zhu, & Wang, 2014).

Due to finite memories capacity and limited communication channels in practical applications, the quantisation effects have to be considered in consensus problems. For the case that the multi-agent systems with discrete-time dynamics, there have been many results for the quantised consensus problem under various situations. In Kashyap, Basar, and Srikant (2007) and Carli, Fagnani, Frasca, and Zampieri (2010), the effect of quantised communication on the gossip consensus algorithm was studied. In Guan, Meng, Liao, and Zhang (2012), the authors considered the quantised consensus problem of second-order multi-agent systems. However, recently, quantised continuous-time systems have attracted more and more attention. This is because the dynamics of the agents is naturally described by continuous-time systems in many applications, such as robotic networks. In Dimarogonas and Johansson (2010), the logarithmically quantised consensus problem of multi-agent system was studied in the case of a tree topology. In Ceragiolia, Persis, and Frasca (2011), the authors considered the consensus problem in which the agents states were communicated through uniform quantisers. The researches above mainly considered the quantised consensus problem in the case when the agents have no inherent dynamics. However, in many practical systems, inherent dynamics often exists for the agents, such as the node in complex dynamical

*Corresponding author. Email: zhengyuanshi2005@163.com

networks. In Hou, Cheng, and Tan (2009) the authors studied the consensus problems for multi-agent systems with the uncertain nonlinear dynamics and a decentralised robust adaptive control approach was proposed to solve the consensus. In Cheng, Hou, Tan, Lin, and Zhang (2010), an adaptive leader-following controller was proposed for the consensus of multi-agent systems with the uncertain nonlinear dynamics.

Compared with general continuous-time control strategy, impulsive control strategy just uses the state variables of the system at discrete-time instances and thus impulsive controller usually has a relatively simple structure. As we known, impulsive control method has been widely used to synchronise coupled dynamical systems (Guan, Liu, Feng, & Wang, 2010; Guan, Wu, & Feng, 2012; Liu & Hill, 2011; Lu, Ho, & Cao, 2010; Wang, Yang, Wang, & Guan, 2009). In Wang et al. (2009), the authors investigated the robust stabilisation of complex switched networks with parametric uncertainties and time delays under impulsive control. In Lu et al. (2010), based on the concept named 'average impulsive interval', a unified synchronisation criterion is derived for directed impulsive dynamical networks. Recently, there are some research to use impulsive control method for the consensus problem of multi-agent systems. In Liu and Hill (2011), the authors investigate the problem of global consensus between a complex dynamical network and a known goal signal by designing an impulsive consensus control scheme. In Guan et al. (2012), an impulsive model has been proposed by taking advantages of instantaneous information, and the authors studied the consensus problems for directed networks of agents with external disturbances.

In this paper, by using the spectral properties of the graph Laplacian matrix we study the consensus problems of first-order and second-order multi-agent systems with nonlinear dynamics. Continuous-time and impulsive control inputs are established for the multi-agent systems with weighted connected topologies and logarithmic quantised information transmission. By using the graph theory, Lyapunov theory and impulsive control theory, some sufficient conditions are derived for the consensus of the multi-agent systems.

This paper is organised as follows. In Section 2, we provide some definitions and results in graph theory, nonsmooth analysis and logarithmic quantisation theory. The consensus problems of first-order and second-order multi-agent systems with nonlinear dynamics under logarithmically quantised information are discussed in Sections 3 and 4. In Section 5, the simulation results are given to show the effectiveness of the obtained results. Section 6 is a brief conclusion.

Notation: Throughout this paper, we let R^n be the n-dimensional Euclidean space, $R^{n\times m}$ be the set of $n \times m$ real matrix, Z be the set of integers, Z^+ be the set of positive integers, I_N be the N-dimensional identity matrix and 1_N be the N-dimensional vector with each entry being 1. The superscript 'T' represents the transpose. For symmetric matrices X, $Y \in R^{n\times n}$, $X \geq Y$ means $X - Y$ is semi-positive definite. $\|\cdot\|$ denotes the 2-norm both for vectors and matrices. For the set S, $|S|$ denotes its cardinality. Let $B(x, \delta)$ be the open ball of radius δ centred at x, $\mathfrak{B}(R^d)$ be the collection of all subsets of R^d, $\mu(S)$ be the Lebesgue measure of S, co be the convex hull and $\bar{co}$ be the convex closure.

2. Preliminaries

In this section, we first present some definitions and results in graph theory, nonsmooth analysis and logarithmic quantisation theory used in the sequel (Dimarogonas & Johansson, 2010; Fu & Xie, 2004; Godsil & Royal, 2001; Shevitz & Paden, 1994; Zelazo & Mesbahi, 2011).

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ be a weighted undirected graph of order N, with the sets of node $\mathcal{V} = \{1, 2, \ldots, N\}$, a set of edges $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $\mathcal{A} = (a_{ij}) \in R^{N\times N}$. If $(i, j) \in \mathcal{E}$, then i, j are adjacent. The adjacency matrix $\mathcal{A}$ is a symmetric matrix with adjacency element $a_{ij} > 0$ if i, j are adjacent, and $a_{ij} = 0$ otherwise. A path of length r in $\mathcal{G}$ is a sequence of $r + 1$ distinct vertices such that any two consecutive vertices are adjacent. An undirected graph $\mathcal{G}$ is called connected if between any two distinct vertices i,j in G, there is a path that starting with i and ending with j. The matrix $\Delta = (\Delta_{ij})$ of the graph G is a diagonal matrix with $\Delta_{ii} = \sum_{j=1}^{N} a_{ij}$. An orientation on $\mathcal{G}$ is the assignment of an arbitrary direction to each edge to make it have a head and tail. We make use of $|\mathcal{V}| \times |\mathcal{E}|$ the incidence matrix B for an arbitrary oriented graph. The columns of B are then indexed by the edge set, and the ith row entry takes the value '1' if it is the head of the corresponding edge, '−1' if it is the tail and zero otherwise. The weighting matrix W is a $|\mathcal{E}| \times |\mathcal{E}|$ diagonal matrix and the ith entry on the diagonal is the adjacency element associated with corresponding edge. The graph Laplacian matrix L of a graph G is defined as $L \triangleq BWB^T = \Delta - \mathcal{A}$.

In this paper, we will design control input on the logarithmically quantised relative measurements of agents. Q: $R \to R$ is used to denote the logarithmic quantisation function. The set of logarithmic quantisation levels is described by

$$U = \{\pm u_i, u_i = \rho^i u_0, i = \pm 1, \pm 2, \ldots\} \cup \{\pm u_0\} \cup \{0\},$$
$$0 < \rho < 1, u_0 > 0.$$

The associated quantiser is defined as follows:

$$Q(a) = \begin{cases} u_i & \text{if } \frac{1}{1+\delta}u_i < a \leq \frac{1}{1-\delta}u_i, a > 0, \\ 0 & \text{if } a = 0, \\ -Q(-a) & \text{if } a < 0, \end{cases}$$

where $\delta = \frac{1-\rho}{1+\rho}$. From the definition, for $\forall a \in R$, we have $(1 - \delta)a^2 \leq aQ(a) \leq (1 + \delta)a^2$, $\frac{1}{(1+\delta)} Q^2(a) \leq a Q(a) \leq$

$\frac{1}{(1-\delta)}Q^2(a)$ and $|Q(a)-a| \le \delta|a|$. For a vector $v = [v_1, \dots, v_d]^T \in R^d$, define $Q(v) \triangleq [Q(v_1), \dots, Q(v_d)]^T$.

Definition 2.1: Let $A = (a_{ij})_{m\times n}$, $B = (b_{ij})_{p\times q}$, the Kronecker product of A and B (denoted as $A \otimes B$) is defined as

$$A \otimes B = \begin{pmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{pmatrix} \in R^{mp\times nq}.$$

Some properties of Kronecker product are also given below:

(1) $(P\otimes Q)^T = P^T \otimes Q^T$;
(2) $(P\otimes Q)(E\otimes F) = (PE)\otimes(QF)$.

Lemma 2.2 (Boyd, Ghaoui, Feron, & Balakrishnan, 1994): *The following linear matrix inequality (LMI)*

$$\begin{pmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{pmatrix} > 0,$$

where $Q(x) = Q^T(x)$, $S(x) = S^T(x)$ is equivalent to either of the following conditions:

(1) $Q(x) > 0$, $R(x) - S^T(x)Q^{-1}(x)S(x) > 0$;
(2) $R(x) > 0$, $Q(x) - S(x)R^{-1}(x)S^T(x) > 0$.

Lemma 2.3 (Horn & Johnson, 1985): *Assume that $\mathcal{G}$ is an undirected connected graph with N vertices, then the eigenvalues of the graph Laplacian matrix L of $\mathcal{G}$ can be ordered as* $0 = \lambda_1(L) < \lambda_2(L) \le \cdots \le \lambda_N(L)$.

Definition 2.4 (Clarke, 1983): For a vector field $f(t, x)$: $R\times R^d \to R^d$, the Filippov set-valued map $K[f](t, x)$: $R\times R^d \to R^d$ is defined by

$$K[f](t,x) \triangleq \bigcap_{\delta>0}\ \bigcap_{\mu(S)=0} \bar{co}\{f(t, B(x,\delta))\backslash S\},$$

where $\bigcap_{\mu(S)=0}$ denotes the intersection over all sets of Lebesgue measure zero.

Lemma 2.5 (Paden & Sastry, 1987):

(1) *Assume that f, g: $R^m \to R^n$ are locally bounded, then*

$$K[f+g](x) \subseteq K[f](x) + K[g](x).$$

(2) *Let f: $R^m \to R^n$ be continuous, then*

$$K[f](x) = \{f(x)\}.$$

Definition 2.6 (Clarke, 1983): Consider a vector differential equation

$$\dot{x}(t) = f(t, x(t)), \tag{1}$$

where $x(t) = [x_1(t), \dots, x_d(t)]^T$. A vector function $x(\cdot)$ is called a Filippov solution of Equation (1) on $[t_0, t_1]$, where t_1 could be ∞, if $x(\cdot)$ is absolutely continuous on $[t_0, t_1]$ and for almost all $t \in [t_0, t_1]$

$$\dot{x} \in K[f](t, x).$$

Lemma 2.7 (Filippov, 1988): *Given Equation* (1), *let f be measurable and locally essentially bounded,* i.e. *bounded in any bounded neighbourhood of every point of definition excluding the sets of measure* zero. *Then for all $x_0 \in R^d$, there exists a Filippov solution to Equation* (1) *with the initial condition $x(0) = x_0$.*

Definition 2.8 (Clarke, 1983): For a locally Lipschitz function V: $R\times R^d \to R$ locally continuous, the generalised gradient of V at (t, x) is defined by

$$\partial V(t,x) \triangleq co\{\lim_{i\to\infty} \nabla V(t_i, x_i)|(t_i, x_i) \to (t, x), (t_i, x_i) \notin \Omega_V\},$$

where Ω_V is the set of measure zero where the gradient of V with respect to x or t is not defined.

The set-valued Lie derivative of $V(t, x)$ with respect to t, the trajectory of Equation (1), is defined as

$$\dot{\tilde{V}}(t,x) \triangleq \bigcap_{\xi\in\partial V(t,x)} \xi^T \begin{pmatrix} K[f](t,x) \\ 1 \end{pmatrix}.$$

In particular, if the function $V(t, x)$ has no explicit dependence on t, the generalised gradient of $V(x)$ at x becomes $\partial V(x) \triangleq co\{\lim_{i\to\infty} \nabla V(x_i)|x_i \to x, x_i \notin \Omega_V\}$, and the set-valued Lie derivative of $V(x)$ with respect to t becomes $\dot{\tilde{V}}(x) \triangleq \bigcap_{\xi\in\partial V(x)} \xi^T K[f](t,x)$.

Lemma 2.9 (Shevitz & Paden, 1994): *Given Equation* (1), *let $f(t, x)$* **be** *locally essentially bounded, and* $0 \in K[f](t, 0)$ *in a region* $Q \supset \{t/t_0 \le t < \infty\} \times \{x \in R^d/\|x\| < r\}$, *where* $r > 0$. *Also, let V: $R\times R^d \to R$ be a regular function satisfying*

$$V(t, 0) = 0$$

and

$$V_1(\| x \|) \le V(t,x) \le V_2(\| x \|) \quad for\ x \ne 0$$

in Q *for some class $\mathcal{K}$ functions V_1 and V_2. Then*

(1) $\dot{\tilde{V}}(t,x) \le 0$ *in Q implies $x(t) \equiv 0$ is a uniformly stable solution.*
(2) *If in addition, there exists a class $\mathcal{K}$ function $w(\cdot)$ in Q with the property*

$$\dot{\tilde{V}}(t,x) \le -w(x) < 0$$

then the solution $x(t) \equiv 0$ is uniformly asymptotically stable.

Lemma 2.10: *For $x = [x_1, x_2, \ldots, x_n]^T \in R^n$, $W = diag(w_1, w_2, \ldots, w_n) \ge 0 \in R^{n\times n}$, we have*

$$x^T WK[Q](x) \ge (1-\delta)x^T Wx.$$

Proof: From the definition of the Filippov set-valued map, we have for $a \ge 0 \in R$, if $Q(\cdot)$ is continuous on a, $K[Q](a) = Q(a)$, if $Q(\cdot)$ is discontinuous on a, $K[Q](a) = [(1-\delta)a, (1+\delta)a]$ and $K[Q](-a) = -K[Q](a)$. Then it is obtained that $aK[Q](a) \ge (1-\delta)a^2$ for $a \in R$. Thus, we have $x^T WK[Q](x) = \sum_{i=1}^N x_i w_i K[Q](x_i) \ge (1-\delta)\sum_{i=1}^N w_i x_i^2 = (1-\delta)x^T Wx$.

3. Consensus of first-order multi-agent systems

Given a connected graph $\mathcal{G}$ with N nodes. Suppose that each node of the graph $\mathcal{G}$ is a first-order agent with nonlinear dynamics

$$\dot{x}_i(t) = f(x_i(t)) + u_i(t), \quad t \ge t_0 \ge 0, i = 1, \ldots, N, \quad (2)$$

where $x_i(t) \in R^n$ and $f(x_i(t))$: $R^n \to R^n$ are the position state and inherent nonlinear dynamics of agent i, respectively, $u_i(t) \in R^n$ is the control input that will be designed for consensus problem.

Definition 3.1: The system (1) is said to achieve consensus if for any initial conditions

$$\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0 \;\; \forall i, j = 1, 2, \ldots, N.$$

Assumption 1: *Assume that the nonlinear function f in Equation* (2) *is continuous and there exists a constant $\theta >$ 0 such that f satisfies:*

$$\| f(x(t)) - f(y(t)) \| \le \theta \| x(t) - y(t) \|, \\ \forall x(t), y(t) \in R^n.$$

First, we give the continuous-time control input on the quantised relative measurements of agents as:

$$u_i(t) = -b\sum_{j=1}^N a_{ij} Q(x_i(t) - x_j(t)), \\ i = 1, 2, \ldots, N, \quad (3)$$

where b $>$ *0 is a control gain.*

Let $x(t) = [x_1^T(t), \ldots, x_N^T(t)]^T$, $\bar{x}(t) = \frac{1}{N}\sum_{j=1}^N x_j(t)$, $\tilde{x}(t) = x(t) - (1_N \otimes I_n)\bar{x}(t)$, $F(x(t)) = [f(x_1(t))^T, \ldots, f(x_N(t))^T]^T$, and orientating the graph G let $\hat{x}(t) = (B^T \otimes I_n)x(t)$, where B is the incidence matrix of the oriented graph. Since $1_N^T B = 0$, the error dynamical system of the multi-agent system (2) *under the control input* (3) *can be described as:*

$$\dot{\tilde{x}}(t) = \left(\left(I_N - \frac{1}{N}1_N 1_N^T\right) \otimes I_n\right) F(x(t)) \\ - b\left((BW) \otimes I_n\right) Q(\hat{x}(t)), \quad i = 1, 2, \ldots, N. \quad (4)$$

Because of the discontinuity of the quantised signals, we consider Filippov solutions to the multi-agent system (2) *under the control input* (3).

Theorem 3.2: *For a connected network of agents with Laplacian matrix L, if $b > \frac{\theta}{(1-\delta)\lambda_2(L)}$, the multi-agent system* (2) *achieves consensus under the control input* (3).

Proof: Consider the Lyapunov function

$$V(t) = \frac{1}{2}\tilde{x}^T(t)\tilde{x}(t).$$

From Equation (4), we have

$$\dot{\tilde{V}} \subseteq \tilde{x}^T(t)\Bigg(\left(\left(I_N - \frac{1}{N}1_N 1_N^T\right) \otimes I_n\right) F(x(t)) \\ - b\left((BW) \otimes I_n\right) K[Q](\hat{x}(t))\Bigg).$$

Since

$$\tilde{x}^T(t)\left(\left(I_N - \frac{1}{N}1_N 1_N^T\right) \otimes I_n\right) F(x(t)) \\ = \tilde{x}^T(t)\Bigg(F(x(t)) - \left(\left(\frac{1}{N}1_N 1_N^T\right) \otimes I_n\right) F(x(t)) \\ - 1_N \otimes f(\bar{x}(t)) + 1_N \otimes f(\bar{x}(t))\Bigg),$$

$$\begin{aligned}&\tilde{x}^T(t)(1_N \otimes f(\bar{x}(t)))\\&\quad = x^T(t)\left(\left(I_N - \frac{1}{N}1_N1_N^T\right)\otimes I_n\right)(1_N \otimes f(\bar{x}(t)))\\&\quad = x^T(t)\left(\left(1_N - \frac{1}{N}1_N1_N^T1_N\right)\otimes f(\bar{x}(t))\right) = 0,\end{aligned}$$

and

$$\begin{aligned}&\tilde{x}^T(t)\left(\left(\frac{1}{N}1_N1_N^T\right)\otimes I_n\right)F(x(t))\\&\quad = x^T(t)\left(\left(I_N - \frac{1}{N}1_N1_N^T\right)\otimes I_n\right)\\&\qquad\times\left(\left(\frac{1}{N}1_N1_N^T\right)\otimes I_n\right)F(x(t))\\&\quad = x^T(t)\left(\left(\frac{1}{N}1_N1_N^T - \frac{1}{N^2}1_N1_N^T1_N1_N^T\right)\otimes I_n\right)F(x(t))\\&\quad = 0,\end{aligned}$$

by Assumption 1, one has

$$\begin{aligned}&\tilde{x}^T(t)\left(\left(I_N - \frac{1}{N}1_N1_N^T\right)\otimes I_n\right)F(x(t)) = \tilde{x}^T(t)(F(x(t))\\&\quad - 1_N \otimes f(\bar{x}(t))) \le \theta\tilde{x}^T(t)\tilde{x}(t).\end{aligned}$$

Note that

$$\begin{aligned}&\tilde{x}^T(t)(L\otimes I_n)\tilde{x}(t)\\&\quad = x^T(t)\left(\left(I_N - \frac{1}{N}1_N1_N^T\right)\otimes I_n\right)\\&\qquad\times(L\otimes I_n)\left(\left(I_N - \frac{1}{N}1_N1_N^T\right)\otimes I_n\right)x(t)\\&\quad = x^T(t)(L\otimes I_n)x(t),\end{aligned}$$

according to Lemma 2.3 and Lemma 2.10, we get

$$\begin{aligned}&\tilde{x}^T(t)((BW)\otimes I_n)K[Q](\hat{x}(t))\\&\quad = x^T(t)\left(\left(I_N - \frac{1}{N}1_N1_N^T\right)\otimes I_n\right)\\&\qquad\times((BW)\otimes I_n)K[Q](\hat{x}(t))\\&\quad = x^T(t)((BW)\otimes I_n)K[Q](\hat{x}(t))\\&\quad = \hat{x}^T(t)(W\otimes I_n)K[Q](\hat{x}(t))\\&\quad \ge (1-\delta)\hat{x}^T(t)(W\otimes I_n)\hat{x}(t)\\&\quad = (1-\delta)x^T(t)(L\otimes I_n)x(t)\\&\quad = (1-\delta)\tilde{x}^T(t)(L\otimes I_n)\tilde{x}(t)\\&\quad \ge (1-\delta)\lambda_2(L)\tilde{x}^T(t)\tilde{x}(t).\end{aligned}$$

Then

$$\dot{\tilde{V}} \le (\theta - b(1-\delta)\lambda_2(L))\tilde{x}^T(t)\tilde{x}(t).$$

According to the Lemma 2.9, the consensus of the system (2) is achieved.

Next an impulsive control input is proposed to make multi-agent system (2) achieve consensus:

$$u_i(t) = -\sum_{k=1}^{\infty} b_k \sum_{j=1}^{N} a_{ij}\, Q(x_i(t) - x_j(t))\delta(t - t_k),\quad i = 1, 2, \dots, N, \tag{5}$$

where the impulse instant sequence $\{t_k\}_{k=1}^{\infty}$ satisfies $0 \le t_0 < t_1 < \cdots < t_k < t_{k+1} < \cdots$, $\lim_{k\to+\infty} t_k = +\infty$, $\delta(t)$ is the Dirac delta function.

Using the incidence matrix B, the error dynamical system of multi-agent system (2) under the control input (5) can be described by the following impulsive differential equations:

$$\begin{cases}\dot{\tilde{x}}(t) = \left(\left(I_N - \frac{1}{N}1_N1_N^T\right)\otimes I_n\right)F(x(t)),\\ \qquad\qquad t \ne t_k, k \in Z^+, t \ge t_0,\\ \Delta\tilde{x}(t_k) = -b_k((BW)\otimes I_n)Q(\hat{x}(t_k)).\end{cases} \tag{6}$$

where $\Delta\tilde{x}(t_k) = \tilde{x}(t_k^+) - \tilde{x}(t_k)$, $\tilde{x}(t_k^+) = \lim_{t\to t_k^+}\tilde{x}(t)$. We always assume that $\tilde{x}(t)$ is left-hand continuous at $t = t_k$. Hence, the solutions of Equation (6) are piecewise left-hand continuous functions with discontinuities at $t = t_k$, $k \in Z^+$.

Theorem 3.3: *For a connected network of agents with Laplacian matrix L, if* $0 < b_k < \frac{2}{(1+\delta)\lambda_N(L)}$ *and the impulsive intervals satisfy* $2\theta(t_k - t_{k-1}) + ln((1-\delta^2)\lambda_2(L)\lambda_N(L)b_k^2 - 2(1-\delta)\lambda_2(L)b_k + 1) < 0, k \in Z^+$, *then the multi-agent system* (2) *achieves consensus under the control input* (5)*.*

Proof: Consider the Lyapunov function

$$V(t) = \frac{1}{2}\tilde{x}^T(t)\tilde{x}(t).$$

For $t \ne t_k$, $k \in Z^+$, by Assumption 1, we have

$$\begin{aligned}\dot{V}(t) &= \tilde{x}^T(t)\left(\left(I_N - \frac{1}{N}1_N1_N^T\right)\otimes I_n\right)F(x(t))\\&\le \theta\tilde{x}^T(t)\tilde{x}(t).\end{aligned}$$

For $t = t_k$, $k \in Z^+$,

$$\begin{aligned}V(t_k^+) = \frac{1}{2}&(\tilde{x}(t_k) - b_k((BW)\otimes I_n)Q(\hat{x}(t_k)))^T(\tilde{x}(t_k)\\&- b_k((BW)\otimes I_n)Q(\hat{x}(t_k))).\end{aligned}$$

Since the matrix $W^{\frac{1}{2}}B^TBW^{\frac{1}{2}}$ has the same eigenvalues with the matrix BWB^T, we obtain

$$\begin{aligned}&Q^T(\hat{x}(t_k))\left((BW)^T\otimes I_n\right)((BW)\otimes I_n)\,Q(\hat{x}(t_k))\\&\quad = Q^T(\hat{x}(t_k))\left((WB^TBW)\otimes I_n\right)Q(\hat{x}(t_k))\end{aligned}$$

$$\leq \lambda_N(L)Q^T(\hat{x}(t_k))(W \otimes I_n)Q(\hat{x}(t_k))$$
$$\leq (1+\delta)\lambda_N(L)\hat{x}^T(t_k)(W \otimes I_n)Q(\hat{x}(t_k)).$$

Note that

$$\tilde{x}^T(t_k)((BW) \otimes I_n)Q(\hat{x}(t_k)) = \hat{x}^T(t_k)(W \otimes I_n))Q(\hat{x}(t_k))$$
$$\geq (1-\delta)\hat{x}^T(t_k)(W \otimes I_n)\hat{x}(t_k) = (1-\delta)\tilde{x}^T(t_k)$$
$$\times (L \otimes I_n)\tilde{x}(t_k),$$

and since $0 < b_k < \frac{2}{(1+\delta)\lambda_N(L)}$, we have

$$\begin{aligned} V(t_k^+) &\leq \frac{1}{2}\left((1+\delta)\lambda_N(L)b_k^2 - 2b_k\right)\hat{x}^T(t_k)(W \otimes I_n)Q(\hat{x}(t_k)) \\ &\quad + \frac{1}{2}\tilde{x}^T(t)\tilde{x}(t) \\ &\leq \frac{1}{2}\tilde{x}^T(t_k)\Big(\Big((1+\delta)(1-\delta)\lambda_N(L)b_k^2 L \\ &\quad - 2(1-\delta)b_k L + I\Big) \otimes I_n\Big)\tilde{x}(t_k) \\ &\leq \frac{1}{2}\Big((1-\delta^2)\lambda_N(L)\lambda_2(L)b_k^2 \\ &\quad - 2(1-\delta)\lambda_2(L)b_k + 1\Big)\tilde{x}^T(t_k)\tilde{x}(t_k), = \alpha_k V(t_k). \end{aligned}$$

where $0 < \alpha_k = (1-\delta^2)\lambda_N(L)\lambda_2(L)b_k^2 - 2(1-\delta)\lambda_2(L)b_k + 1 < 1$.

Therefore, for $t \in [t_{k-1}, t_k)$, $k \in Z^+$

$$V(t) \leq V(t_0)exp(2\theta(t - t_{k-1}))\prod_{i=1}^{k-1}\alpha_i exp(2\theta(t_i - t_{i-1})).$$

Thus, from the conditions in Theorem 3.3 we get $V(t) \to 0$ as $t \to \infty$. Therefore, the consensus of the system (2) is achieved.

4. Consensus of second-order multi-agent systems

Consider a second-order multi-agent system composed of N coupled nonlinear dynamic agents with dynamics

$$\begin{cases} \dot{x}_i(t) = v_i(t) + u_i^x(t), \\ \dot{v}_i(t) = f(x_i(t), v_i(t)) + u_i^v(t),\ i = 1, 2, \dots, N, \end{cases} \tag{7}$$

where $x_i(t)$, $v_i(t) \in R^n$ are the position and velocity states of agent i, respectively, f: $R^n \times R^n \to R^n$ is a continuously vector-valued function, $u_i^x(t), u_i^v(t) \in R^n$ are the control input that will be designed for the consensus problem.

Definition 4.1: The system (7) is said to achieve consensus if for any initial conditions

$$\lim_{t\to\infty}\|x_i(t) - x_j(t)\| = 0,\ \lim_{t\to\infty}\|v_i(t) - v_j(t)\| = 0 \quad \forall i, j = 1, 2, \dots, N.$$

Assumption 2: *Assume that there exist constants θ_1, $\theta_2 > 0$ such that the nonlinear function f in Equation (7) satisfies:*

$$\|f(x(t), v(t)) - f(y(t), z(t))\| \leq \theta_1\|x(t) - y(t)\| + \theta_2\|v(t) - z(t)\| \quad \forall x(t), y(t), v(t), z(t) \in R^n.$$

Let $\eta = max\{\frac{3\theta_1+\theta_2}{2}\ \frac{2+\theta_1+3\theta_2}{2}\}$, $\gamma = 1 + \theta_1 + 2\theta_2$.

First, we give the continuous-time control inputs for the system (7) on the quantised relative measurements of agents as:

$$\begin{cases} u_i^x(t) = 0, \\ u_i^v(t) = -b\sum_{j=1}^{N} a_{ij}(Q(x_i(t) - x_j(t)) + Q(v_i(t) - v_j(t))), \\ \qquad i = 1, 2, \dots, N, \end{cases} \tag{8}$$

where b > 0 *is a control gain.*

Let $v(t) = [v_1^T(t), \dots, v_N^T(t)]^T$, $\bar{v}(t) = \frac{1}{N}\sum_{j=1}^{N} v_j(t)$, $\tilde{v}(t) = v(t) - (1_N \otimes I_n)\bar{v}(t)$, $\hat{v}(t) = (B^T \otimes I_n)v(t)$, $F(x(t), v(t)) = [f(x_1(t), v_1(t))^T, \dots, f(x_N(t), v_N(t))^T]^T$. Using the incidence matrix B, the error dynamical system of the system (7) with the control input (8) can be described as:

$$\begin{cases} \dot{\tilde{x}}(t) = \tilde{v}(t), \\ \dot{\tilde{v}}(t) = \left(\left(I_N - \frac{1}{N}1_N 1_N^T\right) \otimes I_n\right) F(x(t), v(t)) \\ \qquad -b((BW) \otimes I_n)\left(Q(\hat{x}(t)) + Q\left(\hat{v}(t)\right)\right). \end{cases} \tag{9}$$

Theorem 4.2: *For a connected network of agents with Laplacian matrix L, if we choose $b > \frac{\eta}{(1-2\delta)\lambda_2(L)}$ when $\delta < \frac{1}{2}$, then the system (7) achieves consensus under the control input (8).*

Proof: Let $y(t) = [\tilde{x}^T(t), \tilde{v}^T(t)]^T$, construct the following Lyapunov function

$$V(t) = \frac{1}{2}y^T(t)(\Omega \otimes I_n)y(t),$$

where $\Omega = \begin{pmatrix} 2bL & I_N \\ I_N & I_N \end{pmatrix}$.

According to Lemma 2.3, we have

$$\tilde{x}^T(t)(L \otimes I_n)\tilde{x}(t) \geq \lambda_2(L)\tilde{x}^T(t)\tilde{x}(t).$$

Then

$$V(t) \geq \frac{1}{2}y^T(t)\left(\hat{\Omega} \otimes I_n\right)y(t),$$

where $\hat{\Omega} = \begin{pmatrix} 2b\lambda_2(L)I_N & I_N \\ I_N & I_N \end{pmatrix}$.

By Lemma 2.2, we know $\hat{\Omega} > 0$ is equivalent to $b > \frac{1}{2\lambda_2(L)}$. Since $\eta > 1$, $\delta < \frac{1}{2}$, we have $b \geq \frac{\eta}{(1-2\delta)\lambda_2(L)} > \frac{1}{2\lambda_2(L)}$, and thus $\hat{\Omega} > 0$.

From Equation (9), we have

$$\dot{V} \subseteq 2b\tilde{x}^T(t)(L \otimes I_n)\tilde{v}(t) + \tilde{v}^T(t)\tilde{v}(t) + (\tilde{x}(t) + \tilde{v}(t))^T$$

$$\times\left(\left(\left(I_N-\frac{1}{N}1_N1_N^T\right)\otimes I_n\right)F(x(t),v(t))\right.$$
$$\left.-b((BW)\otimes I_n)(K[Q](\hat{x}(t))+K[Q](\hat{v}(t)))\right).$$

Similar to the proof of Theorem 3.2, we obtain

$$\tilde{x}^T(t)((BW)\otimes I_n)K[Q](\hat{x}(t))\geq(1-\delta)\tilde{x}^T(t)(L\otimes I_n)\tilde{x}(t),$$

$$\tilde{v}^T(t)((BW)\otimes I_n)K[Q](\hat{v}(t))\geq(1-\delta)\tilde{v}^T(t)(L\otimes I_n)\tilde{v}(t),$$

and according to Assumption 2, one has

$$\begin{aligned}
&\tilde{x}^T(t)\left(\left(I_N-\frac{1}{N}1_N1_N^T\right)\otimes I_n\right)F(x(t),v(t))\\
&\quad=\tilde{x}^T(t)(F(x(t),v(t))-1_N\otimes f(\bar{x}(t),\bar{v}(t)))\\
&\quad\leq\left(\theta_1+\frac{\theta_2}{2}\right)\tilde{x}^T(t)\tilde{x}(t)+\frac{\theta_2}{2}\tilde{v}^T(t)\tilde{v}(t),\\
&\tilde{v}^T(t)\left(\left(I_N-\frac{1}{N}1_N1_N^T\right)\otimes I_n\right)F(x(t),v(t))\\
&\quad=\tilde{v}^T(t)(F(x(t),v(t))-1_N\otimes f(\bar{x}(t),\bar{v}(t)))\\
&\quad\leq\frac{\theta_1}{2}\tilde{x}^T(t)\tilde{x}(t)+\left(\theta_2+\frac{\theta_1}{2}\right)\tilde{v}^T(t)\tilde{v}(t).
\end{aligned}$$

Note that

$$\begin{aligned}
&-\tilde{x}^T(t)((BW)\otimes I_n)K[Q](\hat{v}(t))\\
&\quad=-x^T(t)\left(\left(I_N-\frac{1}{N}1_N1_N^T\right)\otimes I_n\right)((BW)\otimes I_n)\\
&\qquad\times(\hat{v}(t)+e(t))\\
&\quad=-x^T(t)((BW)\otimes I_n)(\hat{v}(t)+e(t))\\
&\quad=-x^T(t)(L\otimes I_n)v(t)-\hat{x}^T(t)(W\otimes I_n)e(t)\\
&\quad\leq-x^T(t)(L\otimes I_n)v(t)+\left\|\left(W^{\frac{1}{2}}\otimes I_n\right)\hat{x}(t)\right\|\\
&\qquad\times\left\|\left(W^{\frac{1}{2}}\otimes I_n\right)e(t)\right\|\\
&\quad\leq-x^T(t)(L\otimes I_n)v(t)+\delta\left\|\left(W^{\frac{1}{2}}\otimes I_n\right)\hat{x}(t)\right\|\\
&\qquad\times\left\|\left(W^{\frac{1}{2}}\otimes I_n\right)\hat{v}(t)\right\|\\
&\quad\leq-x^T(t)(L\otimes I_n)v(t)+\frac{\delta}{2}x^T(t)(L\otimes I_n)x(t)\\
&\qquad+\frac{\delta}{2}v^T(t)(L\otimes I_n)v(t)\\
&\quad=-\tilde{x}^T(t)(L\otimes I_n)\tilde{v}(t)+\frac{\delta}{2}\tilde{x}^T(t)(L\otimes I_n)\tilde{x}(t)\\
&\qquad+\frac{\delta}{2}\tilde{v}^T(t)(L\otimes I_n)\tilde{v}(t)
\end{aligned}$$

where $e(t)=K[Q](\hat{v}(t))-\hat{v}(t)$.

Similarly

$$-\tilde{v}^T(t)((BW)\otimes I_n)K[Q](\hat{x}(t))\leq-\tilde{v}^T(t)(L\otimes I_n)\tilde{x}(t)$$
$$+\frac{\delta}{2}\tilde{x}^T(t)(L\otimes I_n)\tilde{x}(t)+\frac{\delta}{2}\tilde{v}^T(t)(L\otimes I_n)\tilde{v}(t).$$

Hence

$$\begin{aligned}
\dot{V}&\leq\tilde{x}^T(t)\left(\left(\frac{3\theta_1+\theta_2}{2}I+b\delta L-b(1-\delta)L\right)\otimes I_n\right)\tilde{x}(t)\\
&\quad+\tilde{v}^T(t)\left(\left(\frac{2+\theta_1+3\theta_2}{2}I+b\delta L-b(1-\delta)L\right)\otimes I_n\right)\tilde{v}(t).
\end{aligned}$$

Thus, from the condition in Theorem 4.2, the consensus of second-order multi-agent system (7) is achieved according to the Lemma 2.9.

Next an impulsive control input is proposed to make system (7) achieve consensus:

$$\begin{cases}u_i^x(t)=-\sum\limits_{k=1}^{\infty}b_k\sum\limits_{j=1}^{N}a_{ij}Q(x_i(t)-x_j(t))\delta(t-t_k),\\ u_i^v(t)=-\sum\limits_{k=1}^{\infty}b_k\sum\limits_{j=1}^{N}a_{ij}Q(v_j(t)-v_i(t))\delta(t-t_k),\\ \qquad\qquad i=1,2,\ldots,N.\end{cases}\tag{10}$$

Using the incidence matrix B, the error dynamical system of the system (7) with the control input (10) can be described as:

$$\begin{cases}\dot{\tilde{x}}(t)=\tilde{v}(t),\\ \dot{\tilde{v}}(t)=\left((I_N-\frac{1}{N}1_N1_N^T)\otimes I_n\right)F(x(t),v(t)),\\ \qquad\qquad t\neq t_k,k\in Z^+,t\geq t_0,\\ \Delta\tilde{x}(t_k)=-b_k((BW)\otimes I_n)Q(\hat{x}(t_k)),\\ \Delta\tilde{v}(t_k)=-b_k((BW)\otimes I_n)Q(\hat{v}(t_k)).\end{cases}\tag{11}$$

Theorem 4.3: *For a connected network of agents with Laplacian matrix L, if $0<b_k<\frac{2}{(1+\delta)\lambda_N(L)}$ and the impulsive intervals satisfy $\gamma(t_k-t_{k-1})+ln((1-\delta^2)\lambda_2(L)\lambda_N(L)b_k^2-2(1-\delta)\lambda_2(L)b_k+1)<0$, $k\in Z^+$, then the system (7) achieves consensus under the control input (10).*

Proof: Consider the Lyapunov function

$$V(t)=\frac{1}{2}\tilde{x}^T(t)\tilde{x}(t)+\frac{1}{2}\tilde{v}^T(t)\tilde{v}(t).$$

For $t\neq t_k$, $k\in Z^+$, by Assumption 2, we have

$$\begin{aligned}
\dot{V}(t)&=\tilde{x}^T(t)\tilde{v}(t)+\tilde{v}^T(t)\left(\left(I_N-\frac{1}{N}1_N1_N^T\right)\otimes I_n\right)\\
&\quad\times F(x(t),v(t))\\
&\leq\frac{1}{2}(1+\theta_1)\tilde{x}^T(t)\tilde{x}(t)+\frac{1}{2}(1+\theta_1+2\theta_2)\tilde{v}^T(t)\tilde{v}(t)\\
&\leq\gamma V(t).
\end{aligned}$$

Similar to the proof of Theorem 3.3, for $t = t_k$, $k \in N^+$ we obtain

$$V(t_k^+) \le \big((1-\delta^2)\lambda_2(L)\lambda_N(L)b_k^2 - 2(1-\delta)\lambda_2(L)b_k + 1\big)V(t_k).$$

Thus, from the conditions in Theorem 4.3 we get $V(t) \to 0$ as $t \to \infty$. Therefore, the consensus of the system (7) is achieved.

Remark 1: Compared with uniform quantiser, the logarithmic quantiser is capable of adjusting the size of quantisation step according to the input value, but it is more computationally expensive. The multi-agent systems can reach consensus with logarithmic quantisers in certain conditions. However, due to the constraint of uniform quantisation, the multi-agent systems cannot reach exact average consensus.

5. Simulations

Example 5.1: Consider a multi-agent system with 4 nodes. The state equation of agent i is

$$\dot{x}_i(t) = f(x_i(t)) + u_i(t), \quad i = 1, 2, 3, 4,$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), x_{i3}(t))^T \in R^3$, $f(x_i(t)) = (\alpha(-x_{i1}(t) + x_{i2}(t) - h(x_{i1}(t))), x_{i1}(t) - x_{i2}(t) + x_{i3}(t), -\beta x_{i2}(t))^T$, $\alpha = 10$, $\beta = 14.87$, $h(x_{i1}(t)) = -0.68x_{i1}(t) + 0.5(-1.27 + 0.68)(|x_{i1}(t) + 1| - |x_{i1}(t) - 1|)$. If the interaction graph is given by Figure 1, the graph Laplacian matrix is

$$L = \begin{pmatrix} 3.5 & -2 & -1 & -0.5 \\ -2 & 2.5 & -0.5 & 0 \\ -1 & -0.5 & 1.5 & 0 \\ -0.5 & 0 & 0 & 0.5 \end{pmatrix}.$$

By some computation, we get $\theta = 18.3436$, $\lambda_2(L) = 0.6086$, $\lambda_4(L) = 5.1642$.

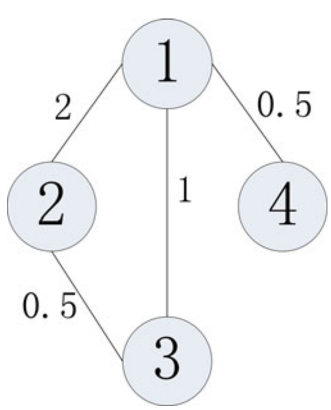

Figure 1. An undirected connected graph.

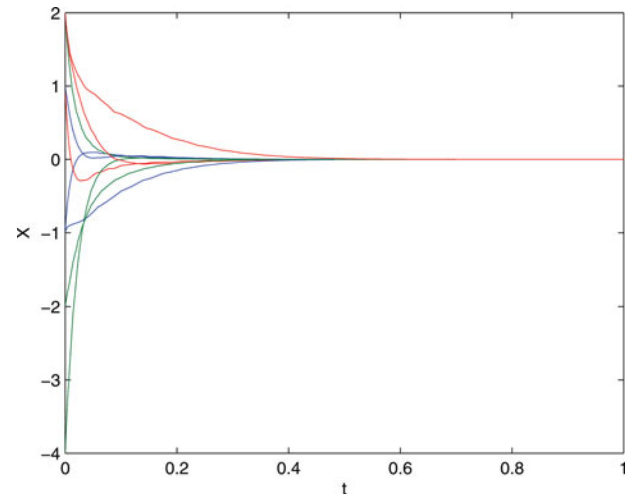

Figure 2. The position errors of first-order agent x_1 with x_i, $i = 2, 3, 4$, under the continuous-time control input (3).

First, we design the continuous-time control input as Equation (3) to system (2). By Theorem 3.2, we derive $b > 37.68$ when $\delta = 0.2$. The simulation results are shown in Figure 2 when $b = 40$. We can see that the system achieves consensus.

Then we design the impulsive control input as Equation (5) to system (2). By Theorem 3.3, we choose $b_k = 0.1$ and the impulsive interval $t_k - t_{k-1} = 0.001$ for all $k \in Z^+$ for simplicity when $\delta = 0.2$. The simulation results are shown in Figure 3. We can see as desired that the consensus is still achieved.

Example 5.2: Consider a second-order multi-agent system with interaction graph G shown in Figure 1. The state

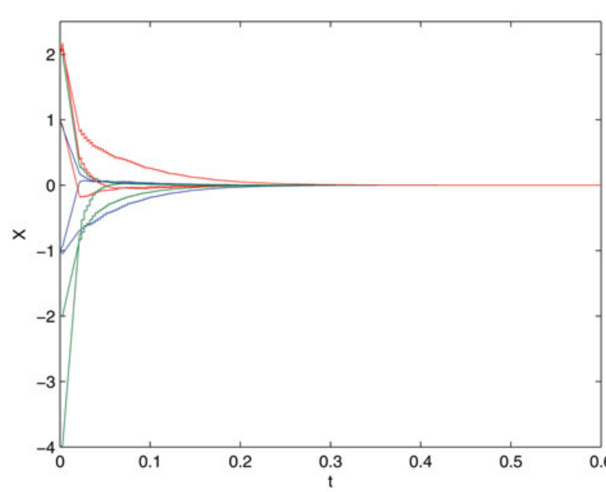

Figure 3. The position errors of first-order agent x_1 with x_i, $i = 2, 3, 4$, under the impulsive control input (5).

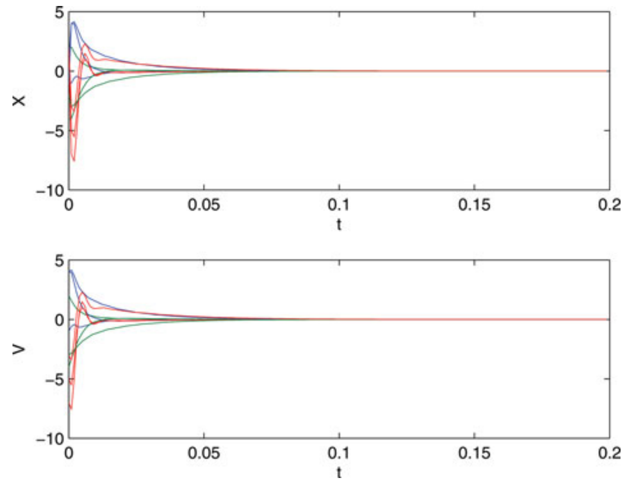

Figure 4. The position and velocity errors of second-order agent x_1 with x_i, $i = 2, 3, 4$, under the continuous-time control input (8).

equation of agent i is

$$\begin{cases} \dot{x}_i(t) = v_i(t) + u_i^x(t), \\ \dot{v}_i(t) = f(x_i(t), v_i(t)) + u_i^v(t), \end{cases}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), x_{i3}(t))^T$, $v_i(t) = (v_{i1}(t), v_{i2}(t), v_{i3}(t))^T \in R^3$, $f(x_i(t), v_i(t)) = (\alpha(-v_{i1}(t) + v_{i2}(t) - h(v_{i1}(t))), v_{i1}(t) - v_{i2}(t) + v_{i3}(t), -\beta v_{i2}(t))^T$, $\alpha = 10$, $\beta = 14.87$, $h(v_{i1}(t)) = -0.68 v_{i1}(t) + 0.5(-1.27 + 0.68)(|v_{i1}(t) + 1| - |v_{i1}(t) - 1|)$. By some computation, we get $\theta_1 = 0$, $\theta_2 = 18.3436$, $\eta = 28.0154$, $\gamma = 37.6872$.

First, we design the continuous-time control input as Equation (8) to system (7). To satisfy the condition in Theorem 4.2, we choose $b = 80$ when $\delta = 0.2$. Figure 4 shows the evolution process of the position and velocity errors. We can see that the consensus is achieved.

Then we adopt the impulsive control input as Equation (10). According to Theorem 4.3, we take $b_k = 0.1$ and the impulsive interval $t_k - t_{k-1} = 0.0014$ for all $k \in Z^+$ for simplicity when $\delta = 0.2$. The simulation results are shown in Figure 5. As desired, system (7) reaches consensus.

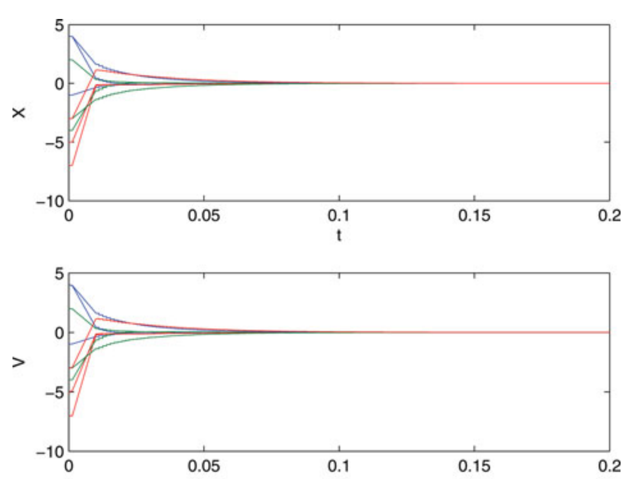

Figure 5. The position and velocity errors of second-order agent x_1 with x_i, $i = 2, 3, 4$, under the impulsive control input (10).

6. Conclusion

Using the results in graph theory, nonsmooth analysis and logarithmic quantisation theory, we studied the consensus of first-order and second-order multi-agent systems with nonlinear dynamics under logarithmically quantised information. Two kinds of control inputs have been designed for the system on the quantised relative state measurements of agents, and some sufficient conditions for the consensus are derived. Some examples are given to illustrate the effectiveness of the theoretical results in the last.

Funding

This work was supported by 973 Program [grant number 2012CB821203]; NSFC [grant numbers 61020106005, 61304160, 61375120 and 61104212]; the Fundamental Research Funds for the Central Universities [grant numbers K50511040005, K5051304031 and K5051304049].

Notes on contributors

Yunru Zhu was born in Xi'an, Shaanxi Province. She received the BS degree in Automatic from Xidian University in 2003, and the MS degree in control theory and control engineering from Huazhong University of Science and Technology in 2006. Since 2006, she has been at the School of Mechano-electronic Engineering, Xidian University, where she is now a lecturer. She is currently working towards the PhD degree in complex systems at Xidian University. Her current research interests are in the fields of coordination of multi-agent systems, consensus problems, quantised control and impulsive control.

Yuanshi Zheng was born in Jiangshan, Zhejiang Province, China on 19 February 1985. He received his bachelor, master and doctorate degrees from Ningxia University and Xidian University in 2006, 2009 and 2012, respectively. He is currently a lecturer of Xidian University. His research interests are in the fields of coordination of multi-agent systems, consensus problems, containment control and coverage control.

Long Wang was born in Xi'an, China on 13 February 1964. He received his bachelor, master, and doctorate degrees in Dynamics and Control from Tsinghua University and Peking University in 1986, 1989 and 1992, respectively. He has held research positions at the University of Toronto, Canada, and the German Aerospace Center, Munich, Germany. He is currently Cheung Kong Chair Professor of Dynamics and Control, Director of Center for Systems and Control of Peking University. He is also Guest Professor of Wuhan University and Beihang University. He serves as the editor-in-chief of the International Journal of Information and Systems Science, Chairman of Chinese Intelligent Networked Things Committee, Vice-Chairman of Chinese Intelligent Aerospace Systems Committee, Editor of the Journal of Mechanical Science and Technology, an executive member of Chinese Society of Artificial Intelligence and Chinese Society of Systems Simulation, a panel member of the Division of Information Science, National Natural Science Foundation of China and a member of IFAC (International Federation of Automatic Control) Technical Committee on Networked Systems. He is in the editorial boards of Science in China, Progress in Natural Science, Journal of Intelligent Systems, Acta Automatica Sinica, Journal of Control Theory and Applications, Control and Decision, Information and Control, Journal of System Simulation, Journal of Applied Mathematics and Computation, Journal of Intelligent and Robotic Systems, IEEE Trans. on Industrial Electronics, etc. His research interests are in the fields of complex networked systems, information dynamics, collective intelligence and bio-mimetic robotics.

References

Boyd, S., Ghaoui, L., Feron, E., & Balakrishnan, V. (1994). *Linear matrix inequalities in system and control theory*. Philadelphia, PA: SIAM.

Carli, R., Fagnani, F., Frasca, P., & Zampieri, S. (2010). Gossip consensus algorithms via quantized communication. *Automatica, 46*(1), 70–80.

Ceragiolia, F., Persis, C.D., & Frasca, P. (2011). Discontinuities and hysteresis in quantized average consensus. *Automatica, 47*(9), 1916–1928.

Cheng, L., Hou, Z., Tan, M., Lin, Y., & Zhang, W. (2010). Neural-network-based adaptive leader-following control for multi-agent systems with uncertainties. *IEEE Transactions on Neural Networks, 21*(8), 1351–1358.

Clarke, F. (1983). *Optimization and nonsmooth analysis*. New York: Wiley.

Dimarogonas, D.V., & Johansson, K.H. (2010). Stability analysis for multi-agent systems using the incidence matrix: Quantized communication and formation control. *Automatica, 46*(4), 695–700.

Fax, J.A., & Murray, R.M. (2004). Information flow and cooperative control of vehicle formations. *IEEE Transactions on Automatic Control, 49*(9), 1465–1476.

Filippov, A.F. (1988). *Differential equations with discontinuous righthand sides*. Amsterdam: Kluwer Academic.

Fu, M., & Xie, L. (2004). The sector bound approach to quantized feedback control. *IEEE Transactions on Automatic Control, 50*(11), 1689–1711.

Godsil, C., & Royal, G. (2001). *Algebraic graph theory*. New York: Springer-Verlag.

Guan, Z.H., Liu, Z.W., Feng, G., & Wang, Y.W. (2010). Synchronization of complex dynamical networks with time-varying delays via impulsive distributed control. *IEEE Transactions on Circuits and Systems I, 57*(8), 2182–2194.

Guan, Z.H., Meng, C., Liao, R.Q., & Zhang, D.X. (2012). Consensus of second-order multi-agent dynamic systems with quantized data. *Physics Letters A, 376*, 387–393.

Guan, Z., Wu, Y., & Feng, G. (2012). Consensus analysis based on impulsive systems in multiagent networks. *IEEE Transactions on Circuits and Systems I, 59*(1), 170–178.

Horn, R.A., & Johnson, C.R. (1985). *Matrix analysis*. Cambridge, UK: Cambridge University Press.

Hou, Z., Cheng, L., & Tan, M. (2009). Decentralized robust adaptive control for the multiagent system consensus problem using neural networks. *Systems, Man, and Cybernetics, Part B: Cybernetics, 39*(3), 636–647.

Jadbabaie, A., Lin, J., & Morse, A.S. (2003). Coordination of groups of mobile autonomous agents using nearest neighbor rules. *IEEE Transactions on Automatic Control, 48*(6), 988–1001.

Ji, Z., Wang, Z., Lin, H., & Wang, Z. (2010). Controllability of multi-agent systems with time-delay in state and switching topology. *International Journal of Control, 83*(2), 371–386.

Kashyap, A., Basar, T., & Srikant, R. (2007). Quantized consensus. *Automatica, 43*(7), 1192–1203.

Liu, B., Chu, T., Wang, L., & Wang, Z. (2005). Swarm dynamics of a group of mobile autonomous agents. *Chinese Physics Letters, 22*(1), 254–257.

Liu, B., & Hill, D.J. (2011). Impulsive consensus for complex dynamical networks with nonidentical nodes and coupling time-delays. *SIAM Journal on Control and Optimization, 49*(2), 315–338.

Lu, J., Ho, D.W.C., & Cao, J. (2010). A unified synchronization criterion for impulsive dynamical networks. *Automatica, 46*(7), 1215–1221.

Olfati-Saber, R., & Murray, R. (2004). Consensus problems in networks of agents with switching topology and time-delays. *IEEE Transactions on Automatic Control, 49*(9), 1520–1533.

Paden, B., & Sastry, S. (1987). A calculus for computing Filippov's differential inclusion with application to the variable structure control of robot manipulators. *IEEE Transactions on Circuits and Systems, 34*(1), 73–82.

Ren, W., & Beard, R.W. (2005). Consensus seeking in multiagent systems under dynamically changing interaction topologies. *IEEE Transactions on Automatic Control, 50*(5), 655–661.

Ren, W., & Cao, Y. (2011). *Distributed coordination of multi-agent networks*. London: Springer-Verlag.

Shevitz, D., & Paden, B. (1994). Lyapunov stability theory of nonsmooth systems. *IEEE Transactions on Automatic Control, 39*(9), 1910–1914.

Sun, Y.G., Wang, L., & Xie, G. (2008). Average consensus in networks of dynamic agents with switching topologies and multiple time-varying delays. *Systems & Control Letters, 57*(2), 175–183.

Tanner, H.G., Jadbabaie, A., & Pappas, G.J. (2007). Flocking in fixed and switching networks. *IIEEE Transactions on Automatic Control, 52*(5), 863–868.

Vicsek, T., Czirok, A., Jacob, E.B., Cohen, I., & Schochet, O. (1995). Novel type of phase transition in a system of self-driven particles. *Physical Review Letters, 75*(6), 1226–1229.

Wang, Y.W., Yang, M., Wang, H.O., & Guan, Z.H. (2009). Robust stabilization of complex switched networks with parametric uncertainties and delays via impulsive control. *IEEE Transactions on Circuits and Systems I, 56*(9), 2100–2108.

Xiao, F., & Wang, L. (2008). Asynchronous consensus in continuous-time multi-agent systems with switching topology and time-varying delays. *IEEE Transactions on Automatic Control, 53*(8), 1804–1816.

Xiao, F., Wang, L., Chen, J., & Gao, Y. (2009). Finite-time formation control for multi-agent systems. *Automatica, 45*(11), 2605–2611.

Yu, J., & Wang, L. (2012). Group consensus of multi-agent systems with directed information exchange. *International Journal of Systems Science, 43*(2), 334–348.

Zelazo, D., & Mesbahi, M. (2011). Edge agreement: Graph-theoretic performance bounds and passivity analysis. *IEEE Transactions on Automatic Control, 56*(3), 544–554.

Zheng, Y., & Wang, L. (2012). Consensus of heterogeneous systems without velocity measurements. *International Journal of Control, 85*(7), 906–914.

Zheng, Y., Zhu, Y., & Wang, L. (2011). Consensus of heterogeneous multi-agent systems. *IET Control Theory and Applications, 16*(5), 1881–1888.

Zheng, Y., Zhu, Y., & Wang, L. (2014). Finite-time consensus of multiple second-order dynamic agents without velocity measurements. *International Journal of Systems Science, 45*(3), 579–588.