Consensus of Switched Multiagent Systems

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***Abstract*—In this brief, we consider the consensus problem of a switched multiagent system composed of continuous-time (CT) and discrete-time (DT) subsystems. By combining the classical consensus protocols of CT and DT multiagent systems, we propose a linear consensus protocol for switched multiagent system. Based on the graph theory and the Lyapunov theory, we prove that the consensus of switched multiagent system is solvable under arbitrary switching with undirected connected graph, directed graph, and switching topologies, respectively. Simulation examples are also provided to demonstrate the effectiveness of the theoretical results.**

***Index Terms*—Consensus, continuous time (CT), discrete time (DT), switched multiagent systems.**

I. Introduction

In the past decade, multiagent coordination has made great progress due to the rapid developments of computer science and communication technologies. It has received a good deal of attention from multidisciplinary researchers, including system control theory, mathematics, biology, and statistical physics. This is partly due to its broad applications in many fields, such as formation control, flocking, synchronization and target tracking of robots, social insects, complex networks, sensor networks, and cyberphysical systems [1]–[7].

A. Related Work

Consensus problems are important and challenging research topics in multiagent coordination, which is to design appropriate control input based on local information that enables all agents to reach an agreement on consistent quantity of interest. Reference [8] proposed a simple model for a group of self-driven particles and demonstrated by simulation that the system will synchronize if the population density is large. By virtue of graph theory, [9] explained the consensus behavior of the Vicsek model theoretically and has shown that consensus can be achieved if the union of interaction graphs is connected frequently enough. Reference [10] discussed the consensus problem of multiagent systems with switching topologies and time delays in a continuous-time (CT) model and obtained some useful results for solving the average consensus problem. Reference [11] extended the results given in [10] and presented some more relaxable conditions for consensus with switching topologies. With the development of this issue, a lot of new results were given out with different models and consensus protocols. Reference [12] considered the multiagent consensus with an active leader and variable topology. By utilizing the pre-leader–follower decomposition, [13] studied the state consensus of discrete-time (DT) multiagent systems with switching topologies and bounded time delays. Based on linear matrix inequality (LMI) approach, Sun *et al.* [14] studied the average consensus of multiagent systems with switching topologies and time-varying delays. Reference [15] investigated the leader-following consensus of general linear multiagent systems under switching topologies. Reference [16] studied the cluster synchronization of DT nonlinear multiagent systems by using contraction theory. Zheng and Wang proposed a heterogeneous multiagent system that is composed of first- and second-order integrator agents [17] and studied the consensus problem under directed fixed and switching topologies [18]. Other research topics for consensus with switching topologies were considered, such as asynchronous consensus [19], finite-time consensus [20], stochastic consensus [21], group consensus [22], sampled-data-based consensus [23], and semiglobal consensus [24]. To date, CT/DT multiagent consensus has been wildly analyzed with time-varying topologies by using graph theory, Lyapunov theory, LMI approach, contraction theory, etc. (for more details, one can refer to survey paper [25] and the references therein).

It should be noted that all the aforementioned results were concerned with multiagent consensus under switching topologies, i.e., the multiagent system is composed of only CT subsystem or only DT subsystem. For the study of switched linear systems [26], [27], the researchers usually use the common Lyapunov functions to analyze the stability of switched systems. Different with general switched linear systems, [28] studied the stability of switched systems which are composed of a CT subsystem and a DT subsystem. Some algebraic conditions are given for solving the stability problem under arbitrary switching.

B. Our Contributions

Inspired by the stability analysis for switched systems in [28], we try to investigate the consensus problem of switched multiagent system composed of CT and DT subsystems. It is easy to find many applications of switched multiagent systems that are composed of both CT and DT subsystems. For example, in a CT multiagent system, if we sometimes use a computer to activate all the agents in a DT manner, then the switched multiagent system is composed of both CT and DT subsystems. By combining the classical consensus protocols of CT and DT

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multiagent systems, we propose a linear consensus protocol for switched multiagent system. The main aim of this brief is to obtain the graphic criterions for consensus of a switched multiagent system in different networks. One of the challenges is that the switched feature prevents the application of single Lyapunov function directly. The main contribution of this brief is threefold. First, by utilizing the graph theory and the Lyapunov theory, we obtain that the consensus can be achieved with arbitrary switching under undirected connected graph if the sampling period $0 < h < (2/\lambda_n)$, where λ_n is the maximum eigenvalue of Laplacian matrix of undirected graph. Second, the consensus under directed graph is also analyzed by using the previous results of CT and DT multiagent consensus. Third, we give a sufficient condition for the consensus of switched multiagent system with switching topologies.

The rest of this brief is organized as follows. In Section II, we present some notions in graph theory and propose the switched multiagent system. In Section III, we give the main results. In Section IV, numerical simulations are given to illustrate the effectiveness of theoretical results. Finally, some the conclusion is drawn in Section V.

Notation: The following notations will be used throughout this brief: $\mathbb{Z}$ and $\mathbb{R}$ denote the sets of integer and real number, respectively; and $\mathbb{R}^N$ denotes the N-dimensional real vector space. $\mathcal{I}_n = \{1, 2, \ldots, n\}$. For a given vector or matrix A, A^T denotes its transpose. $\mathbf{1}_n$ is a vector with elements being all ones. I_n is the $n \times n$ identity matrix. 0 ($0_{m\times n}$) denotes an all-zero vector or a matrix with compatible dimension (dimension $m \times n$). Given a complex number $\lambda \in \mathbb{C}$, $\mathrm{Re}(\lambda)$, $\mathrm{Im}(\lambda)$, and $|\lambda|$ are the real part, the imaginary part, and the modulus of λ, respectively.

II. Preliminaries

A. Graph Theory

Here, we first introduce some basic concepts and results about graph theory (for more details, please refer to [29]).

A weighted directed graph $\mathscr{G}(\mathscr{A}) = (\mathscr{V}, \mathscr{E}, \mathscr{A})$ of order n consists of a vertex set $\mathscr{V} = \{s_1, s_2, \ldots, s_n\}$, an edge set $\mathscr{E} = \{e_{ij} = (s_i, s_j)\} \subset \mathscr{V} \times \mathscr{V}$, and a nonnegative matrix $\mathscr{A} = [a_{ij}]_{n\times n}$. A directed path between two distinct vertices s_i and s_j is a finite-ordered sequence of distinct edges of $\mathscr{G}$ with the form $(s_i, s_{k_1}), (s_{k_1}, s_{k_2}), \ldots, (s_{k_l}, s_j)$. A directed tree is a directed graph, where there exists a vertex called the root such that there exists a unique directed path from this vertex to every other vertex. A directed spanning tree is a directed tree, which consists of all the nodes and some edges in $\mathscr{G}$. If a directed graph has the property that $(s_i, s_j) \in \mathscr{E} \Leftrightarrow (s_j, s_i) \in \mathscr{E}$, the directed graph is called undirected. An undirected graph is said to be connected if there exists a path between any two distinct vertices of the graph. The degree matrix $\mathscr{D} = [d_{ij}]_{n\times n}$ is a diagonal matrix with $d_{ii} = \sum_{j:s_j\in\mathscr{N}_i} a_{ij}$, and the Laplacian matrix of the graph is defined as $\mathscr{L} = [l_{ij}]_{n\times n} = \mathscr{D} - \mathscr{A}$. It is easy to see that $\mathscr{L}\mathbf{1}_n = 0$. Thus, the eigenvalues of $\mathscr{L}$ can be denoted by $0 = \lambda_1 \le \lambda_2 \le \cdots \le \lambda_n$. When $\mathscr{G}$ is a connected undirected graph, $\mathscr{L}$ is positive semidefinite and has a simple zero eigenvalue, $\xi^T\mathscr{L}\xi = (1/2)\sum_{i,j=1}^n a_{ij}(\xi_j - \xi_i)^2$, and $\min_{\xi\neq 0, \mathbf{1}_n^T\xi=0}(\xi^T\mathscr{L}\xi/\xi^T\xi) = \lambda_2$ for any $\xi = [\xi_1, \xi_2, \ldots, \xi_n]^T \in \mathbb{R}^n$.

B. System Model

Here, we propose the switched multiagent system which is composed of a CT subsystem

$$\dot{x}_i(t) = u_i(t), \quad i \in \mathcal{I}_n \tag{1}$$

and a DT subsystem

$$x_i(t+1) = x_i(t) + u_i(t), \quad i \in \mathcal{I}_n \tag{2}$$

where $x_i \in \mathbb{R}$ and $u_i \in \mathbb{R}$ are the position and the control input of agent i, respectively. The initial condition is $x_i(0) = x_{i0}$. Let $x(0) = [x_{10}, x_{20}, \ldots, x_{n0}]^T$.

Definition 1: The switched multiagent system (1), (2) is said to reach consensus if, for any initial state, there exists x^* (dependent on the initial state) such that

$$\lim_{t\to\infty} \|x_i(t) - x^*\| = 0, \quad \text{for } i \in \mathcal{I}_n.$$

The consensus protocols have been widely applied for the CT multiagent system (1) and the DT multiagent system (2). We present the linear consensus protocol for the switched multiagent system (1), (2) as

$$u_i(t) = \begin{cases} \sum\limits_{j=1}^n a_{ij}(t)\left(x_j(t) - x_i(t)\right), & \text{for CT subsystem} \\ h\sum\limits_{j=1}^n a_{ij}(t)\left(x_j(t) - x_i(t)\right), & \text{for DT subsystem} \end{cases} \tag{3}$$

where $\mathscr{A} = [a_{ij}(t)]_{n\times n}$ is the weighted adjacency matrix associated with the graph $\mathscr{G}(t)$, and $h > 0$ is the sampling period. At time instant t, the choice of subsystem is decided by the switching rule.

Thus, the switched multiagent system (1), (2) with protocol (3) can be written as

$$\dot{x}(t) = -\mathscr{L}(t)x(t) \tag{4a}$$
$$x(t+1) = (I_n - h\mathscr{L}(t))\,x(t). \tag{4b}$$

III. Main Results

Here, the consensus problem of switched multiagent system (4) will be considered for network with fixed undirected graph, directed graph, and switching topologies, respectively.

First, we consider the consensus of switched multiagent system (4) in undirected graph with fixed topology, i.e., $\mathscr{L}(t) = \mathscr{L}$ and $\mathscr{L}^T = \mathscr{L}$ for any time t.

Theorem 1: Suppose that the communication network $\mathscr{G}$ is undirected and connected. Then, the switched multiagent system (4) can solve the consensus problem under arbitrary switching if the sampling period $0 < h < (2/\lambda_n)$.

Proof: Let $c(t) = (1/n)\sum_{i=1}^n x_i(t)$. Since $a_{ij} = a_{ji}$ for all $i, j \in \mathcal{I}_n$, we have

$$\frac{dc(t)}{dt} = \frac{1}{n}\sum_{i=1}^n \frac{dx_i(t)}{dt} = -\frac{\mathbf{1}_n^T\mathscr{L}x(t)}{n} = 0 \tag{5}$$

$$\begin{aligned} c(t+1) &= \frac{1}{n}\sum_{i=1}^n x_i(t+1) = \frac{\mathbf{1}_n^T(I_n - h\mathscr{L})x(t)}{n} \\ &= \frac{1}{n}\sum_{i=1}^n x_i(t) = c(t). \end{aligned} \tag{6}$$

Therefore, $c(t)$ is time invariant, i.e., $c(t) = c(0)$. Let $\delta(t) = x(t) - \mathbf{1}_n c(t)$, we have $\mathbf{1}_n^T \delta(t) = 0$ and

$$\dot{\delta}(t) = -\mathscr{L}\delta(t) \tag{7a}$$

$$\delta(t+1) = (I_n - h\mathscr{L})\delta(t). \tag{7b}$$

We consider the common Lyapunov function $V(\delta(t)) = \delta^T(t)\delta(t)$ for CT subsystem (7a) and DT subsystem (7b). Owing to $\min_{\xi \neq 0, \mathbf{1}_n^T \xi = 0}(\xi^T \mathscr{L}\xi / \xi^T \xi) = \lambda_2$, in the period where CT subsystem (7a) is activated, we have

$$\dot{V}(\delta(t)) = -2\delta(t)^T \mathscr{L}\delta(t) \leq -2\lambda_2 \delta(t)^T \delta(t) = -2\lambda_2 V(\delta(t))$$

and in the period where DT subsystem (7b) is activated, we obtain

$$\begin{aligned} V(\delta(t+1)) - V(\delta(t)) &= \delta^T(t)(I_n - h\mathscr{L})^2 \delta(t) - \delta^T(t)\delta(t) \\ &= \delta^T(t)(-2h\mathscr{L} + h^2\mathscr{L}^2)\delta(t) \\ &\leq (-2h\lambda + h^2\lambda^2)V(\delta(t)) \end{aligned}$$

where $2h\lambda - h^2\lambda^2 = \min\{2h\lambda_2 - h^2\lambda_2^2, 2h\lambda_n - h^2\lambda_n^2\}$. Due to $0 < h < (2/\lambda_n)$, we have $-2h\lambda + h^2\lambda^2 < 0$.

For any time $t > 0$, we have $t = t_c + t_d$, where $t_c \in \mathbb{R}$ is the total duration time on CT subsystem (7a), and $t_d \in \mathbb{Z}$ is the total duration time on DT subsystem (7b). Let $k = 1 - 2h\lambda + h^2\lambda^2$. Thus, we have $\cancel{0 < k < 1}$ ~~and~~

$$\begin{aligned} V(\delta(t)) \leq e^{-2\lambda_2 t_c} k^{t_d} V(\delta(0)) &= e^{-2\lambda_2 t_c} e^{-t_d \ln\left(\frac{1}{k}\right)} V(\delta(0)) \\ &\leq e^{-2\alpha t} V(\delta(0)) \end{aligned}$$

where $\alpha = \min\{\lambda_2, \ln(1/k)/2\}$, which implies $|\delta(t)| \leq e^{-\alpha t}|\delta(0)|$, i.e., $|x(t) - \mathbf{1}_n c(0)| \leq e^{-\alpha t}|\delta(0)|$. Hence, the switched multiagent system (4) can achieve the exponential consensus under arbitrary switching. ∎

Remark 1: In fact, there are some simple bounds that do not need to compute the Laplacian spectrum for sampling period h. For example, we have $\lambda_n \leq 2\max_{i \in \mathcal{I}_n}\{d_{ii}\}$ by the Geršgorin disk theorem. Thus, the consensus problem of switched multiagent system (4) can be solved if $\mathscr{G}$ is a undirected connected graph and the sampling period $0 < h < (1/\max_{i \in \mathcal{I}_n}\{d_{ii}\})$.

Next, we consider the consensus of switched multiagent system (4) in directed graph with fixed topology, i.e., $\mathscr{L}(t) = \mathscr{L}$ for any time t. A key lemma is given, which is a summary of the work in [11] and [30].

Lemma 1: Consider a directed graph with fixed topology. Then, the CT multiagent system (4a) can solve the consensus problem if and only if the directed graph $\mathscr{G}$ has a directed spanning tree, and the DT multiagent system (4b) can solve the consensus problem if and only if the directed graph $\mathscr{G}$ has a directed spanning tree and $0 < h < \min_{i=2,\ldots,n}(2\mathrm{Re}(\lambda_i)/|\lambda_i|^2)$. The consensus state is $\mathbf{1}_n w^T x(0)$, where $w^T\mathscr{L} = 0$, and $w^T\mathbf{1}_n = 1$.

Theorem 2: Suppose that the communication network $\mathscr{G}$ is a directed graph with fixed topology. Then, the switched multiagent system (4) can solve the consensus problem under arbitrary switching if and only if the directed graph $\mathscr{G}$ has a directed spanning tree and the sampling period $0 < h < \min_{i=2,\ldots,n}(2\mathrm{Re}(\lambda_i)/|\lambda_i|^2)$.

Proof: (Sufficiency) Let $t = t_c + t_d$, where $t_c \in \mathbb{R}$ is the total duration time on CT subsystem (4a), and $t_d \in \mathbb{Z}$ is the total duration time on DT subsystem (4b). Due to $\mathscr{L}(t) = \mathscr{L}$ for any time t and $\exp(-\mathscr{L}t)(I_n - h\mathscr{L}) = (I_n - h\mathscr{L})\exp(-\mathscr{L}t)$, we have

$$x(t) = \exp(-\mathscr{L}t_c)(I_n - h\mathscr{L})^{t_d} x(0). \tag{8}$$

Because the communication network $\mathscr{G}$ has a directed spanning tree, from Lemma 1, we know that $\lim_{t_c \to \infty} \exp(-\mathscr{L}t_c) = \mathbf{1}_n w^T$ and $\lim_{t_d \to \infty}(I_n - h\mathscr{L})^{t_d} = \mathbf{1}_n w^T$ if $0 < h < \min_{i=2,\ldots,n}(2\mathrm{Re}(\lambda_i)/|\lambda_i|^2)$, where $w^T\mathscr{L} = 0$, and $w^T\mathbf{1}_n = 1$.

When $t \to \infty$, there are three cases, as follows.

1) $t_c \to \infty$ and $t_d \in \mathbb{Z}$ is a constant. Thus, from (8), we have

$$\begin{aligned} \lim_{t \to \infty} x(t) &= \lim_{t_c \to \infty} \exp(-\mathscr{L}t_c)(I_n - h\mathscr{L})^{t_d} x(0) \\ &= \mathbf{1}_n w^T (I_n - h\mathscr{L})^{t_d} x(0) = \mathbf{1}_n w^T x(0). \end{aligned}$$

2) $t_c \in \mathbb{R}$ is a constant and $t_d \to \infty$. Thus

$$\begin{aligned} \lim_{t \to \infty} x(t) &= \exp(-\mathscr{L}t_c)\mathbf{1}_n w^T x(0) \\ &= \left(I_n + (-\mathscr{L}t_c) + \frac{1}{2!}(-\mathscr{L}t_c)^2 + \cdots\right)\mathbf{1}_n w^T x(0) \\ &= \mathbf{1}_n w^T x(0). \end{aligned}$$

3) $t_c \to \infty$ and $t_d \to \infty$. Thus

$$\lim_{t \to \infty} x(t) = \mathbf{1}_n w^T \mathbf{1}_n w^T x(0) = \mathbf{1}_n w^T x(0).$$

Therefore, the switched multiagent system (4) can solve the consensus problem under arbitrary switching.

(Necessity) When the switched multiagent system (4) can solve the consensus problem under arbitrary switching, we know that the CT and DT multiagent systems can solve the consensus problem. From Lemma 1, the directed graph $\mathscr{G}$ has a directed spanning tree and $0 < h < \min_{i=2,\ldots,n}(2\mathrm{Re}(\lambda_i)/|\lambda_i|^2)$. ∎

Remark 2: Note that the switched multiagent system (4) presents a unified framework of both the CT multiagent system (1) and the DT multiagent system (2). When the duration time of CT subsystem (4a) $t_c = 0$, the distributed coordination of switched multiagent system (4) becomes the distributed coordination of DT multiagent system (2). In addition, when $t_d = 0$, the switched multiagent system (4) becomes the CT multiagent system (1).

Remark 3: If the sampling period is time variant, similar to the proof in [30], we have that the DT multiagent system can solve the consensus problem if and only if the directed graph $\mathscr{G}$ has a directed spanning tree and $0 < h(t) < \min_{i=2,\ldots,n}(2\mathrm{Re}(\lambda_i)/|\lambda_i|^2)$. Thus, it is easy to get that the results in Theorem 2 are also held.

In the following, we consider the switched multiagent system (4) in undirected graph with switching topologies $\{\mathscr{G}_s : s = \sigma(t) \in \mathscr{J}_0\}$, where $\mathscr{J}_0$ is a finite index set, and $\sigma(t)$ is a switching signal that determines the network topology.

Theorem 3: Suppose that the communication network $\mathscr{G}_s$ is undirected and connected for each $s \in \mathscr{J}_0$. Then, the switched multiagent system (4) can solve the consensus problem if the sampling period $0 < h < \min_{s \in \mathscr{J}_0}(2/\lambda_n(\mathscr{L}_s))$.

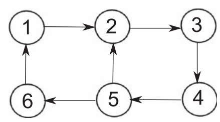

Fig. 1. Directed graph $\mathscr{G}$ which has a directed spanning tree.

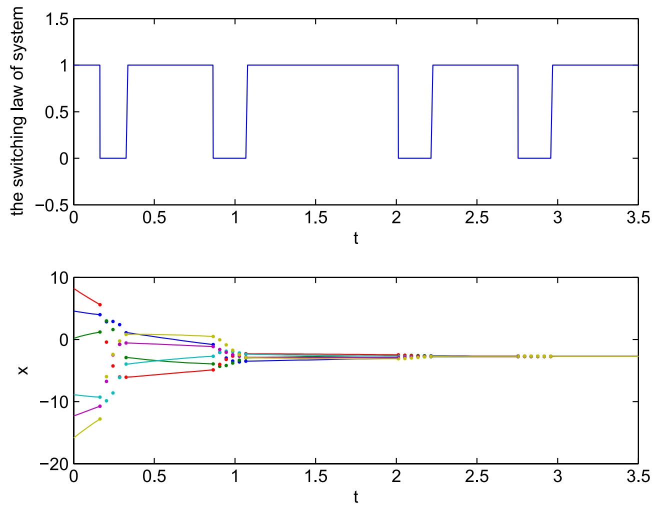

Fig. 2. (Top) Switching law of system (4) and the (bottom) state trajectories of all the agents, where $h = 0.4056$.

Proof: Because the communication network $\mathscr{G}_s$ is undirected and connected for each $s \in \mathscr{J}_0$, similar to Theorem 1, we know that $c(t) = (1/n)\sum_{i=1}^{n} x_i(t)$ is time invariant. Let $\delta(t) = x(t) - \mathbf{1}_n c(t)$ and $V(\delta(t)) = \delta^T(t)\delta(t)$, we have

$$\dot{V}(\delta(t)) = -2\delta(t)^T \mathscr{L}_s \delta(t) \leq -2\lambda_2(\mathscr{L}_s)V(\delta(t)) \leq -2k_1 V(\delta(t))$$

$$\begin{aligned} V(\delta(t+1)) - V(\delta(t)) &= \delta^T(t)(I_n - h\mathscr{L}_s)^2\delta(t) - \delta^T(t)\delta(t) \\ &\leq \left(-2h\lambda_2(\mathscr{L}_s) + h^2\lambda_2^2(\mathscr{L}_s)\right) V(\delta(t)) \leq k_2 V(\delta(t)) \end{aligned}$$

where $k_1 = \min_{s\in\mathscr{J}_0}\{\lambda_2(\mathscr{L}_s)\}$, and $k_2 = \min_{s\in\mathscr{J}_0}\{-2h\lambda_2(\mathscr{L}_s) + h^2\lambda_2^2(\mathscr{L}_s)\}$. Due to $0 < h < \min_{s\in\mathscr{J}_0}(2/\lambda_n(\mathscr{L}_s))$, we have $k_1 > 0$ and $-1 < k_2 < 0$. For any $t = t_c + t_d$

$$V(\delta(t)) \leq e^{-2k_1 t_c}(k_2 + 1)^{t_d} V(\delta(0)) \leq e^{-2\beta t} V(\delta(0))$$

where $\beta = \min\{k_1, (\ln(1/(k_2+1)))/(2)\}$. Hence, the switched multiagent system (4) can achieve the exponential consensus. ■

We consider the following nonlinear consensus protocol:

$$u_i(t) = \begin{cases} \sum_{j=1}^{n} a_{ij}(t) f(x_j(t) - x_i(t)), & \text{for CT subsystem} \\ h\sum_{j=1}^{n} a_{ij}(t) f(x_j(t) - x_i(t)), & \text{for DT subsystem} \end{cases} \tag{9}$$

where $\mathscr{A} = [a_{ij}(t)]_{n\times n}$ is the weighted adjacency matrix associated with the graph $\mathscr{G}(t)$ at time instant t, and $h > 0$ is the sampling period. Suppose that the function $f : \mathbb{R} \to \mathbb{R}$ satisfies the following assumptions.

1) $f(x) = 0$ if and only if $x = 0$.

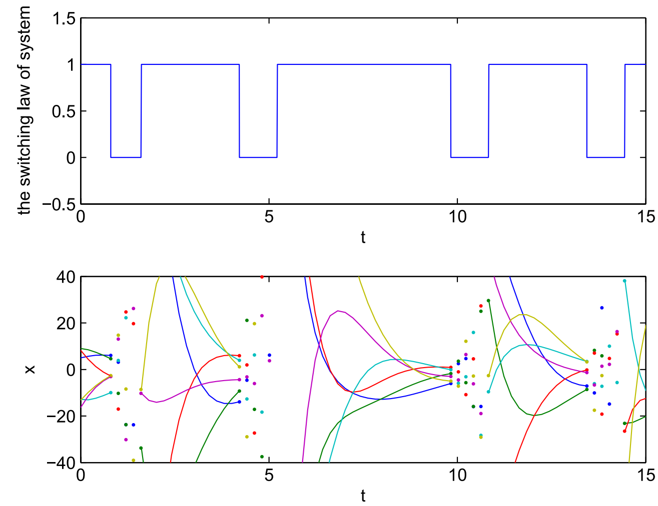

Fig. 3. (Top) Switching law of system (4) and the (bottom) state trajectories of all the agents, where $h = 2$.

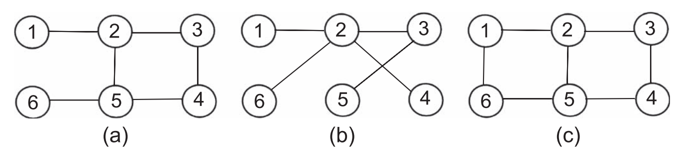

Fig. 4. Three undirected connected graphs.

2) $f(x)$ is an odd function.
3) $\gamma_1 x \leq f(x) \leq \gamma_2 x$, where $\gamma_2 > \gamma_1 > 0$, for any $x \in \mathbb{R}^+$.

Similar to the proof of Theorem 1 and Theorem 3, we can obtain the following corollary.

Corollary 1: Suppose that the communication network $\mathscr{G}_s$ is undirected and connected for each $s \in \mathscr{J}_0$. Then, the switched multiagent system (1), (2) with nonlinear consensus protocol (9) can solve the consensus problem if the sampling period $0 < h < (\gamma_1/\gamma_2^2)\min_{s\in\mathscr{J}_0}(2/\lambda_n(\mathscr{L}_s))$. ■

IV. SIMULATIONS

Here, we provided some simulations to demonstrate the effectiveness of the theoretical results in this brief, where the initial conditions of all the agents are generated randomly.

Example 1: When the communication network is chosen as in Fig. 1, it can be noted that $\mathscr{G}$ has a directed spanning tree. By calculation, the sampling period should satisfy $h < 0.8112$. We choose $h = 0.4056$. The switching law of switched multiagent system (4) is shown in the top panel in Fig. 2. The state trajectories of all the agents are shown in the bottom panel in Fig. 2. If $h = 2$, the switching law and the trajectories of all the agents are shown in Fig. 3, where the consensus does not take place.

Example 2: Suppose that the communication network is chosen as in Fig. 4. Note that $\mathscr{G}_s$ is undirected and connected for each $s \in \{a, b, c\}$. By calculation, the sampling period should satisfy $h < 0.4155$. We choose $h = 0.01$. The switching law of network depicted in Fig. 4 is shown in the top panel in

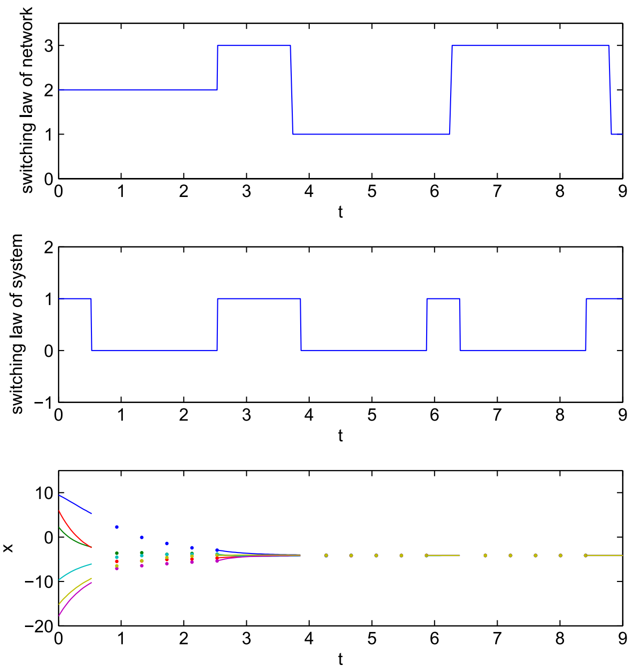

Fig. 5. (Top) Switching law of the network depicted in Fig. 4, the (middle) switching law of system (4), and the (bottom) state trajectories of all the agents, where $h = 0.01$.

Fig. 5. The switching law of switched multiagent system (4) is shown in the middle panel in Fig. 5. The state trajectories of all the agents are shown in the bottom panel in Fig. 5, which are consistent with the results in Theorem 3.

V. Conclusion

In this brief, the consensus problem of a switched multi-agent system that is composed of CT and DT subsystems has been considered. The linear protocol was presented for solving the consensus problem. If the sampling period $0 < h < \min_{i=2,\ldots,n}(2\mathrm{Re}(\lambda_i)/|\lambda_i|^2)$, we proved that the switched multiagent system can achieve the consensus under undirected connected graph and directed graph, respectively. For switching topologies, a sufficient condition was also given if the communication network $\mathscr{G}_s$ is undirected and connected for each $s \in \mathscr{J}_0$. Future work will focus on issues such as consensus of switched multiagent system with time delays and distributed filtering of a switched multiagent system.

References

[1] R. Olfati-saber, "Flocking for multi-agent dynamics systems: Algorithms and theory," *IEEE Trans. Autom. Control*, vol. 51, no. 3, pp. 401–420, Mar. 2006.

[2] C. Wang, G. Xie, and M. Cao, "Forming circle formations of anonymous mobile agents with order preservation," *IEEE Trans. Autom. Control*, vol. 58, no. 12, pp. 3248–3254, Dec. 2013.

[3] F. Xiao, L. Wang, and T. Chen, "Connectivity preservation for multi-agent rendezvous with link failure," *Automatica*, vol. 48, no. 1, pp. 25–35, Jan. 2012.

[4] L. Wang, F. Jiang, G. Xie, and Z. Ji, "Controllability of multi-agent systems based on agreement protocols," *Sci. China Ser. F, Inf. Sci.*, vol. 52, no. 11, pp. 2074–2088, 2009.

[5] Z. Ji, H. Lin, and H. Yu, "Protocols design and uncontrollable topologies construction for multi-agent networks," *IEEE Trans. Autom. Control*, vol. 60, no. 3, pp. 781–786, Mar. 2015.

[6] B. Shen, Z. Wang, and X. Liu, "A stochastic sampled-data approach to distribute H_∞ filtering in sensor networks," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 58, no. 9, pp. 2237–2246, Sep. 2011.

[7] F. Pasqualetti, F. Dorfler, and F. Bullo, "Control-theoretic methods for cyberphysical security: Geometric principles for optimal cross-layer resilient control systems," *IEEE Control Syst.*, vol. 35, no. 1, pp. 110–127, Feb. 2015.

[8] T. Vicsek, A. Czirok, E. B. Jacob, I. Cohen, and O. Schochet, "Novel type of phase transition in a system of self-driven particles," *Phys. Rev. Lett.*, vol. 75, no. 6, pp. 1226–1229, Aug. 1995.

[9] A. Jadbabaie, J. Lin, and A. S. Morse, "Coordination of groups of mobile autonomous agents using nearest neighbor rules," *IEEE Trans. Autom. Control*, vol. 48, no. 6, pp. 988–1001, Jun. 2003.

[10] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Trans. Autom. Control*, vol. 49, no. 9, pp. 1520–1533, Sep. 2004.

[11] W. Ren and R. W. Beard, "Consensus seeking in multiagent systems under dynamically changing interaction topologies," *IEEE Trans. Autom. Control*, vol. 50, no. 5, pp. 655–661, May 2005.

[12] Y. Hong, J. Hu, and L. Gao, "Tracking control for multi-agent consensus with an active leader and variable topology," *Automatica*, vol. 42, no. 7, pp. 1177–1182, Jul. 2006.

[13] L. Wang and F. Xiao, "A new approach to consensus problem in discrete-time multiagent systems with time-delays," *Sci. Chin. Ser. F, Inf. Sci.*, vol. 50, no. 4, pp. 625–635, 2007.

[14] Y. Sun, L. Wang, and G. Xie, "Average consensus in networks of dynamic agents with switching topologies and multiple time-varying delays," *Syst. Control Lett.*, vol. 57, no. 2, pp. 175–183, Feb. 2008.

[15] J. Qin, C. Yu, and H. Gao, "Coordination for linear multiagent systems with dynamic interaction topology in the leader-following framework," *IEEE Trans. Ind. Electron.*, vol. 61, no. 5, pp. 2412–2422, May 2014.

[16] G. Russo, M. di Bernardo, and J. J. E. Slotine, "Convergence and cluster synchronization of networks of discrete-time and asynchronous systems," in *Proc. 52nd IEEE Conf. Dec. Control*, Florence, Italy, 2013, pp. 5921–5926.

[17] Y. Zheng, Y. Zhu, and L. Wang, "Consensus of heterogeneous multi-agent systems," *IET Control Theory Appl.*, vol. 5, no. 16, pp. 1881–1888, 2011.

[18] Y. Zheng and L. Wang, "Distributed consensus of heterogeneous multi-agent systems with fixed and switching topologies," *Int. J. Control*, vol. 85, no. 12, pp. 1967–1976, 2012.

[19] F. Xiao and L. Wang, "Asynchronous consensus in continuous-time multi-agent systems with switching topology and time-varying delays," *IEEE Trans. Autom. Control*, vol. 53, no. 8, pp. 1804–1816, Sep. 2008.

[20] F. Xiao, L. Wang, and T. Chen, "Finite-time consensus in networks of integrator-like dynamic agents with directional link failure," *IEEE Trans. Autom. Control*, vol. 59, no. 3, pp. 756–762, Mar. 2014.

[21] T. Li and J. Zhang, "Consensus conditions of multi-agent systems with time-varying topologies and stochastic communication noises," *IEEE Trans. Autom. Control*, vol. 55, no. 9, pp. 2043–2057, Sep. 2010.

[22] J. Yu and L. Wang, "Group consensus in multi-agent systems with switching topologies and communication delays," *Syst. Control Lett.*, vol. 59, no. 6, pp. 340–348, Jun. 2010.

[23] Y. Gao and L. Wang, "Sampled-data based consensus of continuous-time multi-agent systems with time-varying topology," *IEEE Trans. Autom. Control*, vol. 56, no. 5, pp. 1226–1231, May 2011.

[24] H. Su, M. Chen, J. Lam, and Z. Lin, "Semi-global leader-following consensus of linear multi-agent systems with input saturation via low gain feedback," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 60, no. 7, pp. 1881–1889, Jul. 2013.

[25] R. Olfati-saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proc. IEEE*, vol. 95, no. 1, pp. 215–233, Jan. 2007.

[26] D. Liberzon, *Switching in Systems and Control*. Berlin, Germany: Springer-Verlag, 2003.

[27] G. Xie and L. Wang, "Controllability and stabilizability of switched linear-systems," *Syst. Control Lett.*, vol. 48, no. 2, pp. 135–155, Feb. 2003.

[28] G. Zhai, D. Liu, J. Imae, and T. Kobayashi, "Lie algebraic stability analysis for switched systems with continuous-time and discrete-time subsystems," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 53, no. 2, pp. 152–156, Feb. 2006.

[29] C. Godsil and G. Royal, *Algebraic Graph Theory*. New York, NY, USA: Springer-Verlag, 2001.

[30] M. Guo and D. V. Dimarogonas, "Nonlinear consensus via continuous, sampled, and aperiodic updates," *Int. J. Control*, vol. 86, no. 4, pp. 567–578, 2013.