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Consensus of heterogeneous multi-agent systems without velocity measurements

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This article considers the consensus problem of heterogeneous multi-agent system composed of first-order and second-order agents, in which the second-order integrator agents cannot obtain the velocity (second state) measurements for feedback. Two different consensus protocols are proposed. First, we propose a consensus protocol and discuss the consensus problem of heterogeneous multi-agent system. By applying the graph theory and the Lyapunov direct method, some sufficient conditions for consensus are established when the communication topologies are undirected connected graphs and leader-following networks. Second, due to actuator saturation, we propose another consensus protocol with input constraint and obtain the consensus criterions for heterogeneous multi-agent system. Finally, some examples are presented to illustrate the effectiveness of the obtained criterions.

Keywords: heterogeneous multi-agent systems; consensus; without velocity measurements; input constraint

1. Introduction

Recently, consensus problem of multi-agent systems has attracted researchers from various disciplines of science and engineering due to technological advances in communication and computer science, and the important practical applications of multi-agent systems in many areas, such as the formation control of robotic systems, the cooperative control of unmanned aerial vehicles, the target tracking of sensor networks and the congestion control of communication networks (Ren and Beard 2008; Ren and Cao 2011). Consensus, which is fundamental of distributed coordination, means that a group of agents reach an agreement on a common value by negotiating with their neighbours asymptotically or in a finite time. Roughly speaking, the main objective of consensus problem is to design an appropriate control input (consensus protocol) to make a group of agents converge to a consistent quantity (consensus state) of interest. In the past decade, consensus problem of multi-agent systems has been studied in detail by virtue of the matrix theory, the graph theory, the frequency-domain analysis method, the Lyapunov direct method, etc. The consensus criterions have been obtained for first-order, second-order or high-order multi-agent systems (Jabdabaie, Lin, and Morse 2003; Olfati-Saber and Murray 2004; Xie and Wang 2007; Jiang and Wang 2010 and references therein).

Consensus problem of first-order multi-agent systems is primarily proposed and investigated. Jabdabaie et al. (2003) explained the consensus behaviour reported in Vicsek et al. (1995), and analysed the alignment of a network of agents. Olfati-Saber and Murray (2004) discussed consensus problem for networks of dynamic agents with switching topologies and time delays in a continuous-time model by defining a disagreement function, and obtained some useful results for solving the average-consensus problem. Ren and Beard (2005) extended the results of Jabdabaie et al. (2003) and Olfati-Saber and Murray (2004) and presented some more relaxable conditions for consensus of states under dynamically changing interaction topologies. With the development of issue, a lot of new consensus results were given out with different models and protocols by first-order dynamics, for example, consensus problem with nonlinear protocol (Hui and Haddad 2008), consensus problem with time delays (Xiao and Wang 2006; Sun, Wang, and Xie 2008), asynchronous consensus problem (Xiao and Wang 2008), finite-time consensus problem (Jiang and Wang 2009; Wang and Xiao 2010; Zheng, Chen, and Wang 2011b), group consensus (Yu and Wang 2010), etc. For consensus problem of second-order multi-agent systems, Xie and Wang (2007) and Ren and Atkins (2007) gave sufficient conditions for consensus problem with fixed and

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switching topologies; Gao, Wang, Xie, and Wu 2009a) investigated the consensus problem based on sampled-data control. Jiang and Wang (2010) studied the consensus of high-order multi-agent systems with fixed and switching topologies.

In the study of consensus problem, most consensus protocols of second-order multi-agent systems rely on the availability of the full state for feedback. However, some information is unmeasurable due to technology limitations or environment disturbances. For example, the agents cannot obtain any velocity information in some cases. Hence, it is realistic and significant to consider the consensus problem of second-order multi-agent systems without velocity measurements. However, there are only a few works on this problem (Ren 2008; Gao et al. 2009b; Abdessameud and Tayebi 2010). In Ren (2008), the consensus problem of second-order multi-agent systems was considered in undirected graphs with fixed topology. Gao et al. (2009b) extended the results in Ren (2008) to a time-varying topology with and without time delays. In Abdessameud and Tayebi (2010), the authors proposed the consensus protocols for second-order multi-agent systems without velocity measurements and in the presence of input saturation constraints.

All the aforementioned results were concerned with the consensus of homogeneous multi-agent systems, i.e. all the agents have the same dynamics behaviours. However, the dynamics of the agents coupled with each others are different because of various restrictions or the common goals with mixed agents in the practical systems. In Zheng, Zhu, and Wang (2011a), we considered the consensus problem of heterogeneous multi-agent system composed of first-order and second-order agents, for which the consensus protocols have position and velocity information. But, the velocity information is not always easy to measure owing to technology limitations. Thus, we further consider the consensus problem of heterogeneous multi-agent system without velocity measurements in this article. The main contribution of this article is to give some consensus protocols to resolve the consensus problem for the heterogeneous multi-agent system in which the second-order integrator agents cannot obtain the velocity information. By using the graph theory and Lyapunov theory, we discuss the consensus problem of heterogeneous multi-agent system without velocity measurements under the undirected connected graphs and leader-following networks, respectively.

The rest of this article is organised as follows. In Section 2, we present some concepts in graph theory and formulate the model to be studied. In Sections 3, we give the main results. Numerical simulations to show the validity of theoretical results are presented in Section 4. Finally, this article is concluded in Section 5.

Notation: Throughout this article, we let $\mathbb{R}$, $\mathbb{R}_{>0}$ and $\mathbb{R}_{\geq 0}$ be the set of real number, positive real number and nonnegative real number, $\mathbb{R}^n$ be the n-dimensional real vector space. $\mathcal{I}_m = \{1, 2, \ldots, m\}$. For a given vector or matrix X, X^Tdenotes its transpose. $\mathbf{1}_n$ is a vector with elements being all ones. Matrix $A = [a_{ij}]$ is said to be non-negative (positive) if all entries a_{ij} are non-negative (positive), denoted by $A \geq 0$ ($A > 0$).

2. Preliminaries

2.1 *Graph theory*

In this section, some basic concepts and results about algebraic graph theory are introduced. For more details about algebraic graph theory, One can refer to Godsil and Royal (2001).

Let $G(A) = (V, E, A)$ be a weighted undirected (directed) graph of order n with a vertex set $V = \{s_1, s_2, \ldots, s_n\}$, an edge set $E = \{e_{ij} = (s_i, s_j)\} \subset V \times V$ and a non-negative symmetric matrix $A = [a_{ij}]$. $(s_j, s_i) \in E \Leftrightarrow a_{ij} > 0 \Leftrightarrow$ agent i and j can communicate with each other, namely they are adjacent. Moreover, we assume $a_{ii} = 0$. A is called the weighted matrix and a_{ij} is the weight of $e_{ij} = (s_i, s_j)$. The set of neighbours of s_i is denoted by $N_i = \{s_j\colon\ e_{ji} = (s_j, s_i) \in E\}$. A path that connects s_i and s_j in the graph G is a sequence of distinct vertices $s_{i_0}, s_{i_1}, s_{i_2}, \ldots, s_{i_m}$, where $s_{i_0} = s_i$, $s_{i_m} = s_j$ and $(s_r, s_{r+1}) \in E$, $0 \leq r \leq m-1$. An undirected (directed) graph is said to be connected (strong connected) if there exists a path between any two distinct vertices of the graph. It is easy to see that adjacency matrix A is symmetric if G is an undirected graph. The directed graph G is said to satisfy the detailed balance condition if there exist some scalers $\omega_i > 0$ $(i = 1, 2, \ldots, n)$ such that $\omega_i a_{ij} = \omega_j a_{ji}$ for all $i, j \in \mathcal{I}_n$ (Chu, Wang, Chen, and Mu 2006). In the multi-agent system, we refer to the agent as the leader if it only sends the information to other agents but cannot receive any information from other agents, i.e. $a_{n1} = a_{n2} = \cdots = a_{nn} = 0$ and $\bar{a} = [a_{1n}, a_{2n}, \ldots, a_{(n-1)n}]^T \geq 0$, $\bar{a} \neq 0$ if the agent n is the leader.

2.2 *Heterogeneous multi-agent systems*

In this section, the heterogeneous multi-agent system is proposed first. Then, the concept of consensus is given for the heterogeneous multi-agent system.

Suppose that the heterogeneous multi-agent system consists of first-order and second-order integrator agents. The number of agents is n, labelled 1 through n, where the number of second-order integrator agents is m ($m < n$). Each agent dynamics is given as follows:

$$\begin{cases} \dot{x}_i(t) = v_i(t), & i \in \mathcal{I}_m, \\ \dot{v}_i(t) = u_i(t), & i \in \mathcal{I}_m, \\ \dot{x}_i(t) = u_i(t), & i \in \{m+1, \ldots, n\}, \end{cases} \tag{1}$$

where $x_i \in \mathbb{R}$, $v_i \in \mathbb{R}$ and $u_i \in \mathbb{R}$ are the position, velocity and control input, respectively, of agent i. The initial conditions are $x_i(0) = x_{i0}$, $v_i(0) = v_{i0}$. Let $x(0) = [x_{10}, x_{20}, \ldots, x_{n0}]$, $v(0) = [v_{10}, v_{20}, \ldots, v_{m0}]$.

Definition 2.1: The heterogeneous multi-agent system (1) is said to reach consensus if for any initial conditions, we have

$$\lim_{t\to\infty} |x_i(t) - x_j(t)| = 0, \quad \text{for } i,j \in \mathcal{I}_n, \quad \text{and}$$
$$\lim_{t\to\infty} |v_i(t) - v_j(t)| = 0, \quad \text{for } i,j \in \mathcal{I}_m.$$

Each agent is regraded as a node in a graph $G(A)$. Each edge $(s_i, s_j) \in E$ corresponds to an available information link from agent i to agent j. Moreover, each agent updates its current state based on the information received from its neighbours. Different from the previous consensus protocols for the heterogeneous multi-agent system (1) in Zheng et al. (2011a), we suppose that the agent can only receive the relative position information from its neighbours.

3. Main results

3.1 *Consensus protocol I*

In this section, we first give a consensus protocol (control input) without velocity measurements for the heterogeneous multi-agent system (1) based on the auxiliary system approach of second-order integrator agent. Then, we get the consensus criteria for the heterogeneous multi-agent system (1) when communication topology is undirected and connected by using the graph theory, the Lyapunov direct method and LaSalle's invariance principle. Finally, we get the consensus criteria for the heterogeneous multi-agent system (1) when the agents have a leader and the communication topology of followers is undirected and connected (leader-following network for short).

We present the protocol without velocity measurements for the heterogeneous multi-agent system (1) as follows:

$$u_i(t) = \begin{cases} \sum_{j=1}^{n} a_{ij}(x_j - x_i) + k_1 \dot{y}_i(t), & i \in \mathcal{I}_m, \\ k_2 \sum_{j=1}^{n} a_{ij}(x_j - x_i), & i \in \{m+1, \ldots, n\}, \end{cases} \tag{2}$$

where $A = [a_{ij}]_{n\times n}$ is the weighted adjacency matrix, k_1 and k_2 are strictly positive feedback gains. $y_i \in \mathbb{R}$ is given by

$$\dot{y}_i(t) = -k_3 y_i(t) + k_4 \sum_{j=1}^{n} a_{ij}(x_j - x_i), \quad i \in \mathcal{I}_m, \tag{3}$$

where $k_3 > 0$, $k_4 > 0$ and $y_i(0)$ can be chosen arbitrarily. Let $y(0) = [y_1(0), y_2(0), \ldots, y_m(0)]$.

Theorem 3.1: *Suppose the communication network $G(A)$ is undirected and connected, i.e. $a_{ij} = a_{ji}$ for all $i,j \in \mathcal{I}_n$. Then the heterogeneous multi-agent system* (1) *can achieve the consensus with protocol* (2)–(3).

Proof: The heterogeneous multi-agent system (1) with protocol (2)–(3) can be written as follows:

$$\begin{cases} \dot{x}_i(t) = v_i(t), & i \in \mathcal{I}_m, \\ \dot{v}_i(t) = \sum_{j=1}^{n} a_{ij}(x_j - x_i) + k_1 \dot{y}_i(t), & i \in \mathcal{I}_m, \\ \dot{y}_i(t) = -k_3 y_i + k_4 \sum_{j=1}^{n} a_{ij}(x_j - x_i), & i \in \mathcal{I}_m, \\ \dot{x}_i(t) = k_2 \sum_{j=1}^{n} a_{ij}(x_j - x_i), & i \in \{m+1, \ldots, n\}. \end{cases} \tag{4}$$

Take a Lyapunov function for (4) as

$$V_1(t) = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} \frac{(x_i(t) - x_j(t))^2}{2} + \sum_{i=1}^{m} (v_i(t) - k_1 y_i(t))^2 + \sum_{i=1}^{m} \frac{k_1}{k_4} (y_i(t))^2,$$

which is positive definite with respect to $x_i(t) - x_j(t)$ ($\forall i \neq j, i,j \in \mathcal{I}_n$), $v_i(t)$ ($i \in \mathcal{I}_m$) and $y_i(t)$ ($i \in \mathcal{I}_m$). Differentiating $V_1(t)$ gives

$$\begin{aligned} \dot{V}_1(t) &= \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}(x_j - x_i)(\dot{x}_j - \dot{x}_i) \\ &\quad + \sum_{i=1}^{m} 2(v_i(t) - k_1 y_i(t))(\dot{v}_i(t) - k_1 \dot{y}_i(t)) \\ &\quad + \sum_{i=1}^{m} \frac{2k_1}{k_4} y_i(t)\dot{y}_i(t) \\ &= \sum_{i=1}^{m}\sum_{j=1}^{m} a_{ij}(x_j - x_i)(v_j - v_i) \\ &\quad + \sum_{i=m+1}^{n}\sum_{j=1}^{m} a_{ij}(x_j - x_i)(v_j - \dot{x}_i) \\ &\quad + \sum_{i=1}^{m}\sum_{j=m+1}^{n} a_{ij}(x_j - x_i)(\dot{x}_j - v_i) \end{aligned}$$

$$+\sum_{i=m+1}^{n}\sum_{j=m+1}^{n}a_{ij}(x_j-x_i)(\dot{x}_j-\dot{x}_i)$$
$$+\sum_{i=1}^{m}2(v_i(t)-k_1y_i(t))\sum_{j=1}^{n}a_{ij}(x_j-x_i)$$
$$+\sum_{i=1}^{m}\frac{2k_1}{k_4}y_i(t)(-k_3y_i+k_4\sum_{j=1}^{n}a_{ij}(x_j-x_i)).$$

As $A=[a_{ij}]_{n\times n}$ is a symmetric matrix, we have

$$\sum_{i=1}^{m}\sum_{j=1}^{m}a_{ij}(x_j-x_i)(v_j-v_i)=-2\sum_{i=1}^{m}\sum_{j=1}^{m}a_{ij}(x_j-x_i)v_i,$$
$$\sum_{i=m+1}^{n}\sum_{j=1}^{m}a_{ij}(x_j-x_i)(v_j-\dot{x}_i)=\sum_{i=1}^{m}\sum_{j=m+1}^{n}a_{ij}(x_j-x_i)(\dot{x}_j-v_i)$$

and

$$\sum_{i=m+1}^{n}\sum_{j=m+1}^{n}a_{ij}(x_j-x_i)(\dot{x}_j-\dot{x}_i)=2\sum_{i=m+1}^{n}\sum_{j=m+1}^{n}a_{ij}(x_j-x_i)\dot{x}_j.$$

Hence,

$$\begin{aligned}\dot{V}_1(t)&=-2\sum_{i=1}^{m}\sum_{j=1}^{n}a_{ij}(x_j-x_i)v_i+2\sum_{i=1}^{m}\sum_{j=m+1}^{n}a_{ij}(x_j-x_i)\dot{x}_j\\&\quad+2\sum_{i=m+1}^{n}\sum_{j=m+1}^{n}a_{ij}(x_j-x_i)\dot{x}_j+\sum_{i=1}^{m}2(v_i(t)-k_1y_i(t))\\&\quad\times\sum_{j=1}^{n}a_{ij}(x_j-x_i)+\sum_{i=1}^{m}\frac{2k_1}{k_4}y_i(t)\\&\quad\times\left(-k_3y_i+k_4\sum_{j=1}^{n}a_{ij}(x_j-x_i)\right)\\&=2\sum_{i=1}^{n}\sum_{j=m+1}^{n}a_{ij}(x_j-x_i)\dot{x}_j-\sum_{i=1}^{m}\frac{2k_1k_3}{k_4}y_i^2\\&=-2\sum_{i=m+1}^{n}\dot{x}_i\sum_{j=1}^{n}a_{ij}(x_j-x_i)-\sum_{i=1}^{m}\frac{2k_1k_3}{k_4}y_i^2\\&=-\frac{2}{k_2}\sum_{i=m+1}^{n}\dot{x}_i^2-\sum_{i=1}^{m}\frac{2k_1k_3}{k_4}y_i^2\le 0.\end{aligned}$$

Then, we employ LaSalle's invariance principle. Denote the invariant set $S=\{(x_1,v_1,y_1,\dots,x_m,v_m,y_m,x_{m+1},\dots,x_n)|\dot{V}_1\equiv 0\}$. Note that $\dot{V}_1\equiv 0$ implies that $y_i=0$ $(i\in\mathcal{I}_m)$ and $\dot{x}_i=0$ $(i\in\{m+1,\dots,n\})$, which in turn implies that $\sum_{j=1}^{n}a_{ij}(x_j-x_i)=0$ for all $i\in\mathcal{I}_n$. Then we obtain

$$\sum_{i=1}^{n}x_i\sum_{j=1}^{n}a_{ij}(x_j-x_i)=0.$$

Since the undirected graph $G(A)$ is connected, we have

$$\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}(x_j-x_i)^2=0,$$

which implies that $x_i=x_j$ for all i, $j\in\mathcal{I}_n$. which in turn implies that $v_i=\dot{x}_i=\dot{x}_j=k_2\sum_{k=1}^{n}a_{jk}(x_k-x_j)=0$ for all $i\in\mathcal{I}_m$, $j\in\{m+1,\dots,n\}$. Therefore, we have $v_i=v_j=0$ for all $i,j\in\mathcal{I}_m$. It follows from LaSalle's invariance principle that

$$\lim_{t\to\infty}|x_i(t)-x_j(t)|=0,\quad \text{for } i,j\in\mathcal{I}_n,\quad\text{and}$$
$$\lim_{t\to\infty}|v_i(t)-v_j(t)|=0,\quad \text{for } i,j\in\mathcal{I}_m.$$

Theorem 3.1 is proved. □

Remark 1: Note that the heterogeneous multi-agent system (1) is a second-order multi-agent system when $m=n$, and the second-order multi-agent system (1) with protocol (2)–(3) has been studied under undirected connected graph by Ren (2008). Moreover, the system (1) is a first-order multi-agent system when $m=0$ and has been considered by Olfati-Saber and Murray (2004). Thus, the heterogeneous multi-agent system (1) with protocol (2)–(3) presents a unified viewpoint to solve the consensus problem of the work in Ren (2008) and Olfati-Saber and Murray (2004).

The network studied in Theorem 3.1 is the undirected connected graph. As an extension, we consider the leader-following network.

Theorem 3.2: *Suppose that the heterogeneous multi-agent system* (1) *has a leader and $n-1$ followers, and the network among the followers is undirected and connected. Then the heterogeneous multi-agent system* (1) *can achieve the consensus with protocol* (2)–(3) *if the leader is a first-order integrator agent.*

Proof: Without loss of generality, we assume the agents $1,2,\dots,n-1$ are the followers and n is the leader. Thus, we have $\bar{a}=[a_{ij}]_{1\le i,j\le n-1}=\bar{A}^T$, $a_{n1}=a_{n2}=\cdots=a_{nn}=0$, $\bar{a}\ge 0$, where $\bar{a}=[a_{1n},a_{2n},\dots,a_{(n-1)n}]^T\ne 0$. We rewrite the protocol (2) as follows:

$$u_i(t)=\begin{cases}\sum_{j=1}^{n-1}a_{ij}(x_j-x_i)+a_{in}(x_n-x_i)+k_1\dot{y}_i(t),\\ \quad i\in\mathcal{I}_m,\\ k_2\sum_{j=1}^{n-1}a_{ij}(x_j-x_i)+k_2a_{in}(x_n-x_i),\\ \quad i\in\{m+1,\dots,n-1\},\\ 0,\\ \quad i=n,\end{cases}\tag{5}$$

where

$$\dot{y}_i(t) = -k_3 y_i(t) + k_4 \sum_{j=1}^{n-1} a_{ij}(x_j - x_i) + k_4 a_{in}(x_n - x_i), \quad i \in \mathcal{I}_m. \tag{6}$$

Let $z_i(t) = x_i(t) - x_n(t)$, $i \in \mathcal{I}_{n-1}$. Without loss of generality, we assume $(n-m) > 1$. Then we have

$$\begin{cases} \dot{z}_i(t) = v_i(t), & i \in \mathcal{I}_m, \\ \dot{v}_i(t) = \sum_{j=1}^{n-1} a_{ij}(z_j - z_i) - a_{in} z_i + k_1 \dot{y}_i(t), & i \in \mathcal{I}_m, \\ \dot{y}_i(t) = -k_3 y_i(t) + k_4 \sum_{j=1}^{n-1} a_{ij}(z_j - z_i) - k_4 a_{in} z_i, & i \in \mathcal{I}_m \\ \dot{z}_i(t) = k_2 \sum_{j=1}^{n-1} a_{ij}(z_j - z_i) - k_2 a_{in} z_i, & \\ \quad i \in \{m+1, \ldots, n-1\}. & \end{cases} \tag{7}$$

Take a Lyapunov function for (7) as

$$V_2(t) = \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} a_{ij} \frac{(z_i(t) - z_j(t))^2}{2} + \sum_{i=1}^{n-1} a_{in}(z_i(t))^2 + \sum_{i=1}^{m} (v_i(t) - k_1 y_i(t))^2 + \sum_{i=1}^{m} \frac{k_1}{k_4} (y_i(t))^2.$$

Differentiating $V_2(t)$ yields that

$$\begin{aligned} \dot{V}_2(t) &= \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} a_{ij}(z_j - z_i)(\dot{z}_j - \dot{z}_i) + \sum_{i=1}^{n-1} a_{in} 2 z_i(t) \dot{z}_i(t) \\ &\quad + \sum_{i=1}^{m} 2(v_i(t) - k_1 y_i(t))(\dot{v}_i(t) - k_1 \dot{y}_i(t)) \\ &\quad + \sum_{i=1}^{m} \frac{2k_1}{k_4} y_i(t) \dot{y}_i(t) \\ &= -\frac{2}{k_2} \sum_{i=m+1}^{n-1} \dot{z}_i^2 - \sum_{i=1}^{m} \frac{2k_1 k_3}{k_4} y_i^2 \le 0. \end{aligned}$$

Denote the invariant set $S = \{(z_1, z_2, \ldots, z_{n-1}, v_1, y_1, \ldots, v_m, y_m) | \dot{V}_2 \equiv 0\}$. Note that $\dot{V}_2 \equiv 0$ implies that $y_i = 0$ $(i \in \mathcal{I}_m)$ and $\dot{z}_i = 0$ $(i \in \{m+1, \ldots, n-1\})$, which in turn implies that $\sum_{j=1}^{n-1} a_{ij}(z_j - z_i) - a_{in} z_i = 0$ for all $i \in \mathcal{I}_{n-1}$. Then we obtain

$$\sum_{i=1}^{n-1} z_i \left(\sum_{j=1}^{n-1} a_{ij}(z_j - z_i) - a_{in} z_i \right) = 0.$$

Since $\bar{A} = [a_{ij}]_{1 \le i,j \le n-1} = \bar{A}^T$, we have $z_i(t) = 0$ for all $i \in \mathcal{I}_{n-1}$, i.e. $x_i(t) = x_n(t)$ for all $i \in \mathcal{I}_{n-1}$. Therefore, we have $v_i = v_j$ for all i, $j \in \mathcal{I}_m$. It follows from LaSalle's invariance principle that

$$\lim_{t \to \infty} |x_i(t) - x_n(t)| = 0, \quad \text{for } i \in \mathcal{I}_n, \quad \text{and}$$
$$\lim_{t \to \infty} |v_i(t) - v_j(t)| = 0, \quad \text{for } i, j \in \mathcal{I}_m.$$

Theorem 3.2 is proved. □

Remark 2: Suppose that the communication topology $G(A)$ is strongly connected and satisfies the detailed balance condition, i.e. there exists a vector $\omega = [\omega_1, \omega_2, \ldots, \omega_n]^T \in \mathbb{R}^n_{>0}$ such that $\omega_i a_{ij} = \omega_j a_{ji}$ for all $i, j \in \mathcal{I}_n$. Take a Lyapunov function for (4) as

$$\bar{V}_1(t) = \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i a_{ij} \frac{(x_i(t) - x_j(t))^2}{2} + \sum_{i=1}^{m} (\omega_i v_i(t) - k_1 y_i(t))^2 + \sum_{i=1}^{m} \frac{k_1}{k_4} (y_i(t))^2.$$

Similar to the analysis of Theorem 3.1, the multi-agent system (1) can achieve the consensus. The result can also be extended to the leader-following network in which the topology of the followers is strongly connected and satisfies the detailed balance condition.

3.2 *Consensus protocol II*

Although the consensus problem of heterogeneous multi-agent system (1) is solved in Section 3.1, the proposed consensus protocol presents some limitations due to actuator saturation. To overcome this problem, we propose another consensus protocol without velocity measurements for the heterogeneous multi-agent system (1) with input constraints in this section. Similar to the analysis in Section 3.1, we get the consensus criterions for the heterogeneous multi-agent system (1) under the undirected connected graphs and the leader-following networks, respectively.

We propose the protocol without velocity measurements for the heterogeneous multi-agent system (1) with input constraints as follows:

$$u_i(t) = \begin{cases} \sum_{j=1}^{n} a_{ij} \tanh(x_j - x_i) + k_1 \dot{y}_i(t), & i \in \mathcal{I}_m, \\ k_2 \sum_{j=1}^{n} a_{ij} \tanh(x_j - x_i), & i \in \{m+1, \ldots, n\}, \end{cases} \tag{8}$$

where $A = [a_{ij}]_{n \times n}$ is the weighted adjacency matrix, $k_1 > 0$, $k_2 > 0$ are the feedback gains. $y_i \in \mathbb{R}$ is given by

$$\dot{y}_i(t) = -k_3 \tanh(y_i(t)) + k_4 \sum_{j=1}^{n} a_{ij} \tanh(x_j - x_i), \quad i \in \mathcal{I}_m, \tag{9}$$

where $k_3 > 0$, $k_4 > 0$ and $y_i(0)$ can be chosen arbitrarily. Let $y(0) = [y_1(0), y_2(0), \ldots, y_m(0)]$. Note that

$$|u_i| \leq \max\left\{(1 + k_1k_4)\sum_{j=1}^{n} a_{ij} + k_1k_3, k_2\sum_{j=1}^{n} a_{ij}\right\},$$

which is independent of the initial states of the agents.

Theorem 3.3: *Suppose the communication network $G(A)$ is undirected and connected, i.e. $a_{ij} = a_{ji}$ for all $i, j \in \mathcal{I}_n$. Then the heterogeneous multi-agent system* (1) *can achieve the consensus with protocol* (8)–(9).

Proof: Analogous to the analysis of Theorem 3.1, the heterogeneous multi-agent system (1) with protocol (8)–(9) can be written as follows:

$$\begin{cases} \dot{x}_i(t) = v_i(t), & i \in \mathcal{I}_m, \\ \dot{v}_i(t) = \sum_{j=1}^{n} a_{ij}\tanh(x_j - x_i) + k_1\dot{y}_i(t), & i \in \mathcal{I}_m, \\ \dot{y}_i(t) = -k_3\tanh(y_i(t)) \\ \qquad + k_4\sum_{j=1}^{n} a_{ij}\tanh(x_j - x_i), & i \in \mathcal{I}_m, \\ \dot{x}_i(t) = k_2\sum_{j=1}^{n} a_{ij}\tanh(x_j - x_i), & i \in \{m+1, \ldots, n\}. \end{cases} \tag{10}$$

Take a Lyapunov function for (10) as

$$\begin{aligned} V_3(t) &= \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\log(\cosh(x_i(t) - x_j(t))) \\ &\quad + \sum_{i=1}^{m}(v_i(t) - k_1y_i(t))^2 + \sum_{i=1}^{m}\frac{k_1}{k_4}(y_i(t))^2, \end{aligned}$$

which is positive definite with respect to $x_i(t) - x_j(t)$ ($\forall i \neq j, i, j \in \mathcal{I}_n$), $v_i(t)$ ($i \in \mathcal{I}_m$) and $y_i(t)$ ($i \in \mathcal{I}_m$). Similar to the analysis of $\dot{V}_1(t)$, we have

$$\begin{aligned} \dot{V}_3(t) &= \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\tanh(x_j - x_i)(\dot{x}_j - \dot{x}_i) \\ &\quad + \sum_{i=1}^{m} 2(v_i(t) - k_1y_i(t))(\dot{v}_i(t) - k_1\dot{y}_i(t)) \\ &\quad + \sum_{i=1}^{m}\frac{2k_1}{k_4}y_i(t)\dot{y}_i(t) \\ &= -\frac{2}{k_2}\sum_{i=m+1}^{n}\dot{x}_i^2 - \sum_{i=1}^{m}\frac{2k_1k_3}{k_4}y_i\tanh(y_i) \leq 0. \end{aligned}$$

Denote the invariant set $S = \{(x_1, v_1, y_1, \ldots, x_m, v_m, y_m, x_{m+1}, \ldots, x_n) | \dot{V}_3 \equiv 0\}$. Note that $\dot{V}_3 \equiv 0$ implies that $y_i = 0$ ($i \in \mathcal{I}_m$) and $\dot{x}_i = 0$ ($i \in \{m+1, \ldots, n\}$), which in turn implies that $x_i = x_j$ for all $i, j \in \mathcal{I}_n$. Therefore, we have $v_i = v_j$ for all i, $j \in \mathcal{I}_m$. It follows LaSalle's invariance principle that

$$\begin{aligned} &\lim_{t\to\infty}|x_i(t) - x_j(t)| = 0, \quad \text{for } i, j \in \mathcal{I}_n, \quad \text{and} \\ &\lim_{t\to\infty}|v_i(t) - v_j(t)| = 0, \quad \text{for } i, j \in \mathcal{I}_m. \end{aligned}$$

Theorem 3.3 is proved. □

Theorem 3.4: *Suppose that the heterogeneous multi-agent system* (1) *has a leader and $n-1$ followers, and the network among the followers is undirected and connected. Then the heterogeneous multi-agent system* (1) *can achieve the consensus with protocol* (8)–(9) *if the leader is a first-order integrator agent.*

Proof: Analogous to the proof of Theorem 3.2 and Theorem 3.3, it is easy to establish this theorem. □

Remark 3: From the proof of consensus problem of heterogeneous multi-agent system with input constraints, we propose the nonlinear consensus protocol without velocity measurements as follows:

$$u_i(t) = \begin{cases} \sum_{j=1}^{n} a_{ij}f_1(x_j - x_i) + k_1\dot{y}_i(t), & i \in \mathcal{I}_m, \\ k_2\sum_{j=1}^{n} a_{ij}f_1(x_j - x_i), & i \in \{m+1, \ldots, n\}, \end{cases} \tag{11}$$

where $A = [a_{ij}]_{n\times n}$ is the weighted adjacency matrix, $k_1 > 0$, $k_2 > 0$ are the feedback gains. $y_i \in \mathbb{R}$ is given by

$$\dot{y}_i(t) = -k_3f_2(y_i(t)) + k_4\sum_{j=1}^{n} a_{ij}f_1(x_j - x_i), \quad i \in \mathcal{I}_m, \tag{12}$$

where $k_3 > 0$, $k_4 > 0$ and $y_i(0)$ can be chosen arbitrarily. Suppose that the function f_i: $\mathbb{R} \to \mathbb{R}$, ($i = 1, 2$) satisfies the following assumptions:

(1) $f_i(\cdot)$ is continuous;
(2) $f_i(0) = 0$ and $xf_i(x) > 0$ for $x \neq 0$;
(3) $f_i(\cdot)$ is an odd function.

Then, take a Lyapunov function as follows:

$$\begin{aligned} \hat{V}(t) &= \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\int_0^{(x_i(t)-x_j(t))} f_1(s)\mathrm{d}s \\ &\quad + \sum_{i=1}^{m}(v_i(t) - k_1y_i(t))^2 + \sum_{i=1}^{m}\frac{k_1}{k_4}(y_i(t))^2. \end{aligned}$$

Similar to the proof of Theorems 3.1–3.4, the heterogeneous multi-agent system (1) can solve the consensus problem with the nonlinear consensus protocol (11)–(12).

4. Simulations

In this section, we provide simulations to demonstrate the effectiveness of the theoretical results in this article.

Figure 1 shows an undirected connected graph with 6 vertices, where the weight of each edge is 1. Suppose that the vertices 1–4 denote the second-order integrator agents and the vertices 5–6 denote the first-order integrator agents. We further assume that $k_i = 1$, $(i = 1, 2, 3, 4)$ and the initial states are $x(0) = [8, 5, 2, -4, 1, -5]$, $v(0) = [1, -5\ 5\ 3]$ and $y(0) = [0, 0, 0, 0]$. Then, we give Examples 4.1 and 4.2 to illustrate the effectiveness of Theorems 3.1 and 3.3, respectively.

Example 4.1: Figure 2 shows the state trajectories of agents with consensus protocol (2)–(3) when the communication topology is depicted in Figure 1. From Figure 2, we know that the heterogeneous multi-agent system (1) with consensus protocol (2)–(3) can solve consensus problem under the undirected connected graph.

Example 4.2: Figure 3 shows the state trajectories of agents with consensus protocol (8)–(9) when the communication topology is depicted in Figure 1. From Figure 3, we know that the heterogeneous multi-agent system (1) with consensus protocol (8)–(9) can solve consensus problem under the undirected connected graph.

Figure 4 shows a leader-following network with 6 vertices, where the weight of each edge is 1. Suppose that the vertices 1–4 denote the second-order integrator agents and the vertices 5–6 denote the first-order integrator agents which the agent 6 is the leader. We further assume that $k_i = 1$, $(i = 1, 2, 3, 4)$ and the initial states are $x(0) = [8, 5, 2, -4, 1, -5]$, $v(0) = [1, -5\ 5\ 3]$ and $y(0) = [0, 0, 0, 0]$. Then, we give Examples 4.3 and 4.4 to illustrate the effectiveness of Theorems 3.2 and 3.4, respectively.

Example 4.3: Figure 5 shows the state trajectories of agents with consensus protocol (2)–(3) when the communication topology is depicted in Figure 4. From Figure 5, we know that the heterogeneous multi-agent system (1) with consensus protocol (2)–(3) can solve consensus problem under the leader-following network in which the leader is a first-order integrator agent.

Example 4.4: Figure 6 shows the state trajectories of agents with consensus protocol (8)–(9) when the communication topology is depicted in Figure 4.

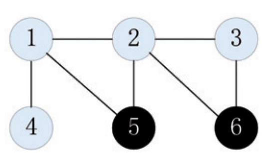

Figure 1. An undirected connected graph.

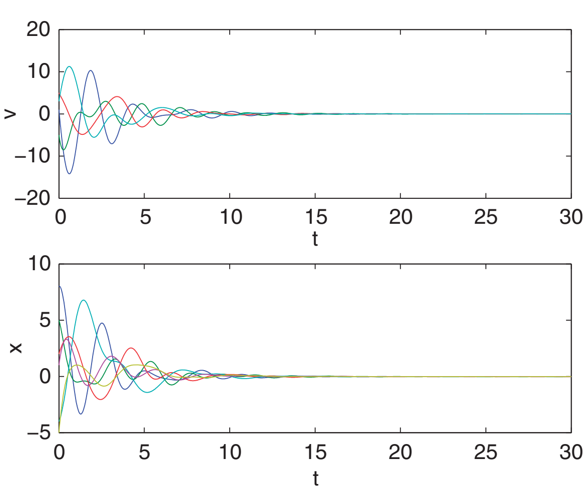

Figure 2. Simulation results with the network depicted in Figure 1 and consensus protocol (2)–(3).

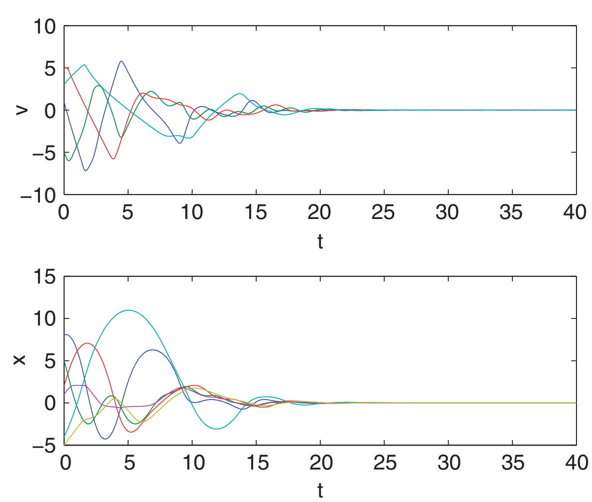

Figure 3. Simulation results with the network depicted in Figure 1 and consensus protocol (8)–(9).

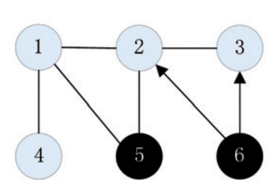

Figure 4. A leader-following network.

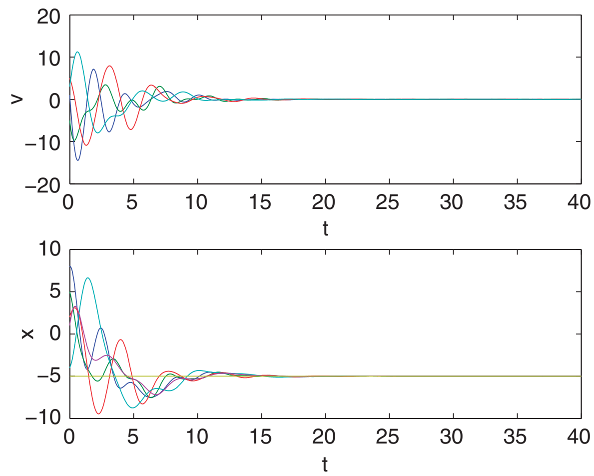

Figure 5. Simulation results with the network depicted in Figure 4 and consensus protocol (2)–(3).

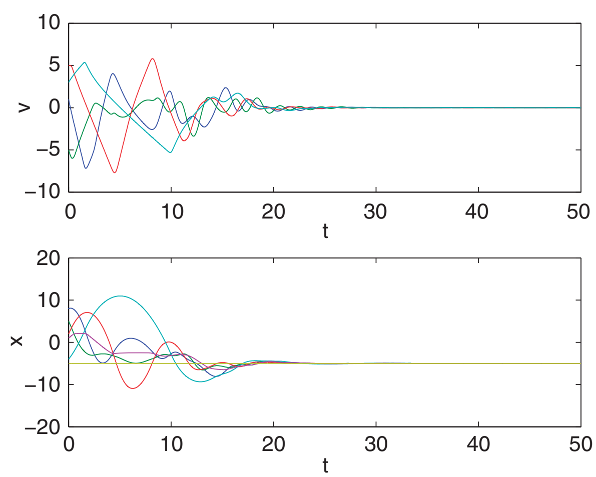

Figure 6. Simulation results with the network depicted in Figure 4 and consensus protocol (8)–(9).

From Figure 6, we know that the heterogeneous multi-agent system (1) with consensus protocol (8)–(9) can solve consensus problem under the leader-following network in which the leader is a first-order integrator agent.

5. Conclusion

In this article, we consider the consensus problem of heterogeneous multi-agent system which the second-order integrator agents cannot obtain the velocity measurements for feedback. Based on the auxiliary system approach of second-order integrator agent, we first propose a consensus protocol and obtain the consensus criterions for heterogeneous multi-agent system when the communication topologies are undirected connected graphs and leader-following networks. Although the consensus problem is solved, the proposed consensus protocol presents some limitations due to actuator saturation. To overcome this problem, we propose another consensus protocol with input constraint and obtain the same consensus criterions for heterogeneous multi-agent system. Some examples are given to illustrate the effectiveness of theoretical results at the end.

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