# Ch9 IIR Digital Filter Design

9.1 Determine the peak ripple values $\delta_p$ and $\delta_s$ for the following set of peak passband ripple $\alpha_p$ and minimum stopband attenuation $\alpha_s$.

$$\alpha_p = 0.24\text{dB}, \quad \alpha_s = 49\text{dB}$$

9.2 Let $H(z)$ be the transfer function of a lowpass digital filter with a passband edge at $\omega_p$, stopband edge at $\omega_s$, passband ripple $\delta_p$, and stopband ripple $\delta_s$. Consider a cascade of two identical filters with the same transfer function $H(z)$. What are the passband and stopband ripples of the cascade at $\omega_p$ and $\omega_s$, respectively? Generalize the results for a cascade of $M$ identical sections.

9.3 Let $H_{LP}(z)$ be the transfer function of a real-coefficient lowpass digital filter with a passband edge at $\omega_p$, stopband edge at $\omega_s$, passband ripple $\delta_p$, and stopband ripple $\delta_s$. Sketch the magnitude response of the highpass transfer function $H_{LP}(-z)$ for $-\infty < \omega < \infty$, and determine its passband and stopband edges in terms of $\omega_p$ and $\omega_s$.

9.4 Consider the transfer function $G(z) = H_{LP}(e^{j\omega_0} z)$, where $H_{LP}(z)$ is the lowpass transfer function of Problem 9.3. Sketch its magnitude response for $-\infty < \omega < \infty$, and determine its passband and stopband edges in terms of $\omega_p$ and $\omega_s$.

9.5 The impulse invariance method is another approach to the design of a causal IIR digital filter $G(z)$ based on the transformation of a prototype causal analog transfer function $H_a(s)$. If $h_a(t)$ is the impulse response of $H_a(s)$, in the impulse invariance method, we require that the unit sample response $g[n]$ of $G(z)$ be given by the sampled version of $h_a(t)$ sampled at uniform intervals of $T$ seconds; that is (Reference a: L. B. Jackson. A correction to impulse invariance. *IEEE signal Processing Letters*, 7:273-275. October 2000. Reference b: W. F. G. Mecklenbraüker. Remarks on and correction to the impulse invariance method for the design of IIR digital filters. *Signal Processing*, 80:1687-1690. August 2000.)

$$g[n] = \begin{cases} \dfrac{h_a(0+)}{2}, & n = 0, \\ h_a(nT), & n \geq 0. \end{cases}$$

a) Show that $G(z)$ and $H_a(s)$ are related through

$$G(z) = \mathcal{Z}\{g[n]\} = \frac{1}{T}\sum_{k=-\infty}^{\infty} H_a(s + j\frac{2k\pi}{T})\Big|_{s=(1/T)\ln z}$$

b) Show that the transformation

$$s = \frac{1}{T}\ln z$$

has the desirable properties enumerated in Section 9.3(Hint: the mapping from s-Domain to the z-Domain).

c) Develop the condition under which the frequency response $G(e^{j\omega})$ of $G(z)$ will be a scaled replica of the frequency response $H_a(j\Omega)$ of $H_a(s)$.

d) Show that the normalized digital angular frequency $\omega$ is related to the analog angular frequency $\Omega$ as

$$\omega = \Omega T$$

- (Optional) Show that the digital transfer function $G(z)$ obtained from an arbitrary rational analog transfer function $H_a(s)$ with sample poles via the impulse invariance method is given by

$$G(z) = \sum_{\substack{\text{all poles of} \\ H_a(s)}} \text{Residues}\left[\frac{H_a(s)}{1 - e^{sT}z^{-1}}\right]$$

9.6 The following causal IIR digital transfer function was designed using the bilinear transformation with *T*=0.4 sec. Determine its respective parent causal analog transfer functions.

$$G(z) = \frac{18z^3 + 22z^2 + 12z + 8}{(3z+1)(12z^2 - 4z + 8)}$$

9.7 An IIR digital lowpass filter is to be designed by transforming an analog lowpass filter with a passband edge frequency $F_p$ at 0.88kHz using the impulse invariance method with *T*=0.25ms.

What is the normalized passband edge angular frequency $\omega_p$ of the digital filter if there is no aliasing? What would be the normalized passband edge angular frequency $\omega_p$ of the digital filter if it is designed using the bilinear transformation with $T$=0.25ms.

9.8 Another bilinear transformation that can be used to design digital filters from an analog filter is given by:

$$s = \frac{2}{T}\left(\frac{1+z^{-1}}{1-z^{-1}}\right)$$

a) Develop the mapping of a point $\sigma_0 + j\Omega_0$ in the s-plan to a point $re^{-j\omega}$ in the z-plane.

b) Does this mapping have all the desirable properties indicated in the book? Justify your answer.

c) What is the relation of the above bilinear transformation to eh bilinear transformation of

$$s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$

d) Express the normalized digital angular frequency $\omega$ as a function of the normalized analog angular frequency $\Omega$, and plot the mapping.

e) If $H_a(s)$ is a causal analog lowpass transfer function, what is the type of the digital transfer function $G(z)$ that is obtained by the above bilinear transformation?

# Ch10 FIR Digital Filter Design

10.1 The desired specifications for a lowpass FIR filter are as follows: passband edge $\omega_p = 0.3\pi$, stopband edge $\omega_s = 0.5\pi$, and minimum stopband attenuation $\alpha_s = 40\text{dB}$. Determine the cutoff frequency and figure out the possible filter order 2$M$+1 using Table below:

| Type of Window | Main Lobe Width $\Delta_{ML}$ | Relative Sidelobe Level $A_{sl}$ | Minimum Stopband Attenuation | Transition Bandwidth $\Delta\omega$ |
|---|---|---|---|---|
| Rectangular | $4\pi/(2M+1)$ | 13.3 dB | 20.9 dB | $0.92\pi/M$ |
| Bartlett | $4\pi/(M+1)$ | 26.5 dB | See text | See text |
| Hann | $8\pi/(2M+1)$ | 31.5 dB | 43.9 dB | $3.11\pi/M$ |
| Hamming | $8\pi/(2M+1)$ | 42.7 dB | 54.5 dB | $3.32\pi/M$ |
| Blackman | $12\pi/(2M+1)$ | 58.1 dB | 75.3 dB | $5.56\pi/M$ |

10.2 For the lowpass filter specifications given below

$$\omega_p = 0.42\pi, \omega_s = 0.58\pi, \delta_p = 0.002, \delta_s = 0.008;$$

a) Design an FIR filter with the smallest length meeting the specifications using the approach based on fixed window functions

b) (Optional) Plot its magnitude response using MATLAB.

10.3 In the frequency sampling approach of FIR Filter design, the specified frequency response $H_d(e^{j\omega})$ is first uniformly sampled at $M$ equally spaced points $\omega_k = 2\pi k / M$, $0 \le k \le M-1$, providing *M frequency samples* $H[k] = H_d(e^{j\omega_k})$ .These $M$ frequency samples constitute an *M*-point DFT $H[k]$, whose *M*-point inverse-DFT thus yield the specified frequency response is uniquely characterized by the *M* frequency samples and, hence, can be fully recovered from these samples.

a) Show that the transfer function $H(z)$ of the FIR filter can be expressed as

$$H(\mathrm{z}) = \frac{1 - z^{-M}}{M} \sum_{k=0}^{M-1} \frac{H[k]}{1 - W_M^{-k} z^{-1}}$$

b) Develop a realization of the FIR filter based on the above expression.

c) Show that the frequency response $H(e^{j\omega})$ of the FIR filter designed via the frequency sampling-based approach has exactly the specified frequency samples $H(e^{j\omega_k}) = H[k]$ at $\omega_k = 2\pi k / M$, $0 \le k \le M-1$.

10.4 Plot the magnitude response of a linear-phase FIR lowpass filter by truncating the impulse response of the ideal lowpass filter

$$h_{LP}[n] = \frac{\sin(\omega_c n)}{\pi n}$$

to length $N = 2M + 1$ and shifting to the right by *M* samples for two different values of *M*.

a) Show that the truncated filter exhibits oscillatory behavior on both sides of the cutoff frequency;

b) Analysis the the oscillatory behavior in detail and consider the possible solution to reduce the phenomenon.