Chapter 9

IIR Digital Filter Design

Part A

Preliminary Considerations

1. Preliminary Considerations

- Digital Filter Specifications
- Selection of the Filter Type
- Basic Approaches to Digital Filter Design
- Estimation of the Filter Order
- Scaling the Digital Filter

1.1 Digital Filter Specifications

Objective :

- Determination of a realizable transfer function $G(z)$ approximating a given frequency response specification is an important step in the development of a digital filter
  - Digital filter design is the process of deriving the transfer function $G(z)$
  - If an IIR filter is desired, $G(z)$ should be a stable rational function

- Usually, either the magnitude and/or the phase (delay) response is specified for the design of digital filter for most applications.
- We restrict our attention in this chapter to the magnitude approximation problem only.

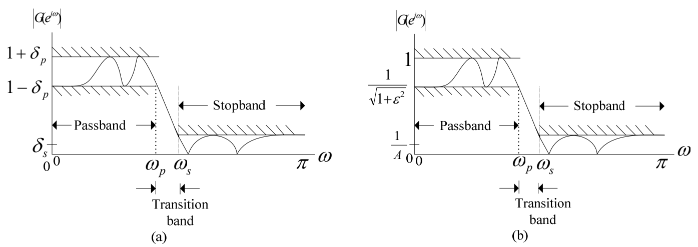
Digital Filter Design Steps

① Convert the digital filter specifications into an analog prototype lowpass filter specifications

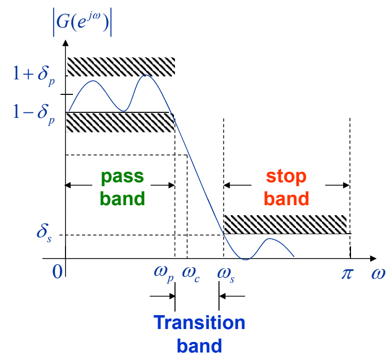
② Determine the analog lowpass filter transfer function $H_a(s)$ (Consider: IIR/FIR?)

③ Transform $H_a(s)$ into the desired digital transfer function $G(z)$

Digital Filter Specifications



Digital Filter Specifications



- Passband:

  $1-\delta_p \le |G(e^{j\omega})| \le 1+\delta_p, \quad |\omega| \le \omega_p$
- Stopband:

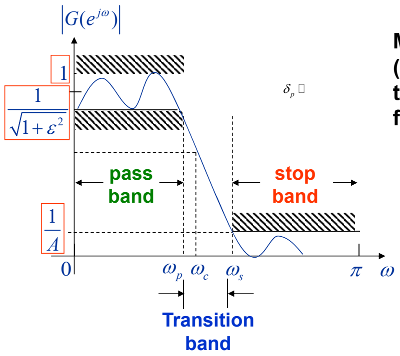
  $|G(e^{j\omega})| \le \delta_s, \quad \omega_s \le \omega \le \pi$
- Peak Passband ripple:

  $\alpha_p = -20\log_{10}(1-\delta_p)$ dB
- Minimum Stopband attenuation

  $\alpha_s = -20\log_{10}(\delta_s)$ dB

1.1 Digital Filter Specifications

Normalized Specifications



Maximum value of the magnitude (gain) is assumed to be unity or the minimum value of the loss function is 0 dB

- Maximum Passband ripple:

$$\alpha_{\max} \cong 20\log_{10}(\sqrt{1+\varepsilon^2})$$

for $\delta_p << 1$

$$\alpha_{\max} \cong -20\log_{10}(1-2\delta_p)$$

$$\cong 2\alpha_p \ \ \text{dB}$$

1.1 Digital Filter Specifications

- Frequency specifications are normalized using the sampling rate:

$$\omega_p = \frac{\Omega_p}{F_T} = \frac{2\pi F_p}{F_T} = 2\pi F_p T$$

$$\omega_s = \frac{\Omega_s}{F_T} = \frac{2\pi F_s}{F_T} = 2\pi F_s T$$

- $\omega=\pi$ corresponds to half the sampling rate, $F_T/2$

Q: What is the condition for non-overlapping?

1.2 Selection of the Filter Type

- **FIR filters**:
  - √ Linear phase response
  - √ Stability with quantized coefficients
  - ✗ Higher order required than using IIR filters

1.2 Selection of the Filter Type

- **IIR filters:**
  - √ Better attenuation properties
  - √ Closed form approximation formulas
  - ✗ Nonlinear phase response
  - ✗ Instability with finite wordlength computation
  - √ Lower order

N_{FIR}/N_{IIR} is typically of the order of tens (or more)

IIR Filter Design

- An analog filter transfer function $H_a(s)$ is transformed into the desired digital filter transfer function $G(z)$
  - Analog approximation techniques are highly advanced
  - Usually yield closed-form solutions

IIR Filter Design

- Extensive tables are available for analog filter design or the methods are easy to program
- Digital filters often replace (or simulate) analog filters

$$H_a(s) = \frac{P_a(s)}{D_a(s)} \Longrightarrow G(z) = \frac{P(z)}{D(z)}$$

IIR Filter Design

- The basic idea behind the conversion of an analog prototype transfer function $H_a(s)$ to a digital filter transfer function $G(z)$ is to apply a mapping from the s-domain to the z-domain so that the *essential properties of the analog frequency response* are preserved.
- Bilinear transformation—most widely used transformation

IIR Filter Design

- Requirements for the transform are:
- The imaginary axis ($j\Omega$) of the s-plane is mapped onto the unit circle in the z-plane
- Stable $H_a(s)$ must be transformed into a stable $G(z)$

FIR Filter Design

- No analog prototype filters are available
- FIR filter design is based on a direct approximation of the specified magnitude response
- A linear phase response is usually required

FIR Filter Design

- FIR transfer function:

$$H(z) = \sum_{n=0}^{N} h[n] z^{-n}$$

- The corresponding frequency response:

$$H(e^{j\omega}) = \sum_{n=0}^{N} h[n] e^{-j\omega n}$$

- Linear phase requirement:

$$h[n] = \pm h[N-n]$$

- **IIR Design --** Filter order is solved from the approximation formulas
- **FIR Design --** Several formulas proposed for estimating the minimum length of the impulse response

Kaiser:

$$N \cong \frac{-20\log_{10}\left(\sqrt{\delta_p \delta_s}\right) - 13}{14.6(\omega_s - \omega_p)/2\pi}$$

$$N \cong \frac{-20\log_{10}\left(\sqrt{\delta_p \delta_s}\right) - 13}{14.6(\omega_s - \omega_p)/2\pi}$$

- N is inversely proportional to the normalized transition width and does not depend on the location of the transition band
- N depends also on the product of δ_p and δ_s

- $G(z)$ has to be scaled in magnitude so that the maximum gain in the passband is unity

$$G_t(z) = kG(z)$$

- Notice that the scaling coefficient K does not affect the shape of the magnitude response, i.e., it does not affect the locations of poles and zeros in the z-plane

1.5 Scaling the Digital Filter

- **Lowpass filter:** Unity gain at zero frequency $\omega = 0$ (or $z=1$)

$$KG(e^{j\omega})\Big|_{\omega=0} = KG(z)\Big|_{z=1} \Rightarrow K = 1/G(1)$$

- **Highpass filter:** Unity gain at $\omega = \pi$ (or $z=-1$)

$$KG(e^{j\omega})\Big|_{\omega=\pi} = KG(z)\Big|_{z=-1} \Rightarrow K = 1/G(-1)$$