Ch4 Digital Processing of Continuous-Time Signals

4.1 Show that if the spectrum $G_a(j\Omega)$ of $g_a(t)$ (Band-limited to Ω_m) also contained an impulse at Ω_m, the sampling rate Ω_T must be greater than $2\Omega_m$ to recover fully $g_a(t)$ from the sampled version.

4.2 The Nyquist frequency of a continuous-time signal $g_a(t)$ is Ω_m. Determine the Nyquist frequency of each of the following continuous-time signals derived from $g_a(t)$:

a) $y_1(t) = g_a^{\,2}(t)$;

b) $y_2(t) = g_a(3t)$;

c) $y_3(t) = \int_{-\infty}^{\infty} g_a(t-\tau)g_a(\tau)d\tau$;

d) $y_4(t) = \dfrac{dg_a(t)}{dt}$.

4.3 (Optional) A finite-energy continuous-time signal $g_a(t)$ is sampled at a rate satisfying the Nyquist condition, generating a discrete-time sequence $g[n]$. Develop the relation between the total energy $\mathcal{E}_{g_a(t)}$ of the continuous-time signal $g_a(t)$ and the total energy $\mathcal{E}_{g[n]}$ of the discrete-time signal $g[n]$.

4.4 A continuous-time signal $x_a(t)$ is composed of a linear combination of sinusoidal signals of frequencies 300Hz, 500Hz, 1.2 kHz, 2.15 kHz and 3.5 kHz. The signal $x_a(t)$ is sampled at a 2.0-kHz rate, and the sampled sequence is passed through an ideal lowpass filter with a cutoff frequency of 900Hz, generating a continuous-time signal $y_a(t)$. What are the frequency components present in the reconstructed signal $y_a(t)$.

4.5 The continuous-time signal

$$x_a(t) = 4\sin(20\pi t) - 5\cos(24\pi t) + 3\sin(120\pi t) + 2\cos(176\pi t)$$

is sampled at a 50 Hz rate, generating the sequence $x[n]$. Determine the exact expression of $x[n]$.

4.6 Consider the system of Fig.1, where the input continuous-time signal $x_a(t)$ has a band-limited spectrum $X_a(j\Omega)$, as sketched in Fig.2(a), and is being sampled at the Nyquist rate. The discrete-time processor is an ideal lowpass filter with a frequency response $H(e^{j\omega})$, as shown in Fig.2(b), and has a cutoff frequency $\omega_c = \Omega_m T/3$, where T is the sampling period. Sketch as accurately as possible the spectrum of $Y_a(j\Omega)$ of the output continuous-time signal $y_a(t)$.

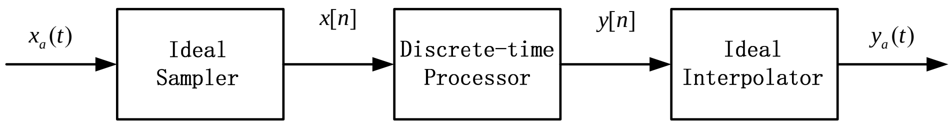

Fig. 1

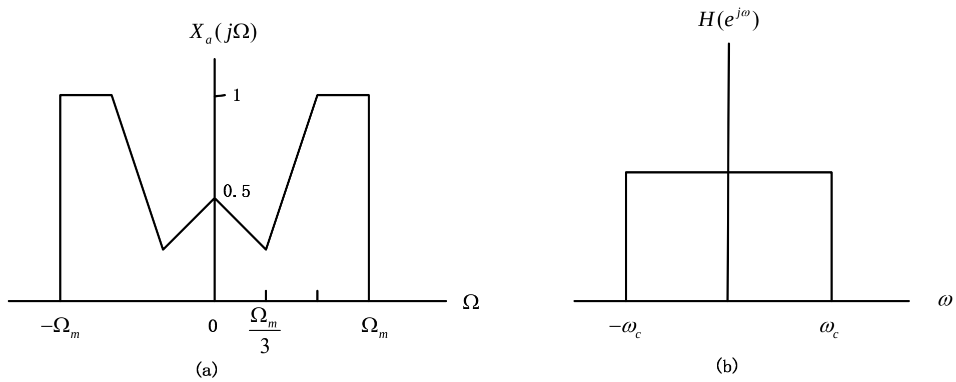

Fig. 2

4.7 A continuous-time signal $x_a(t)$ has a band-limited spectrum $X_a(j\Omega)$, as indicated in Fig.3. Determine the smallest frequency F_T that can be employed to sample $x_a(t)$ so that it can be recovered from its sampled version $x[n]$ for each of the following sets of values of the banedges Ω_1 and Ω_2. Sketch the Fourier transform of the sampled version $x[n]$ obtained by sampling $x_a(t)$ at the smallest sampling rate F_T and the frequency response of the ideal reconstruction filter needed to fully recover $x_a(t)$ for each case.

a) $\Omega_1 = 100\pi$, $\Omega_2 = 150\pi$;

b) $\Omega_1 = 160\pi$, $\Omega_2 = 250\pi$;

c) $\Omega_1 = 110\pi$, $\Omega_2 = 180\pi$;

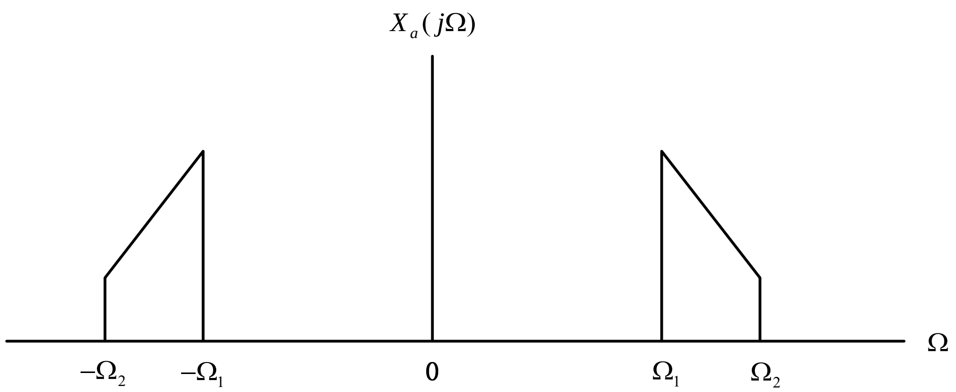

Fig. 3

4.8 An ideal lowpass filter is given as below:

$$H_r(j\Omega) = \begin{cases} T, & |\Omega| \le \Omega_c \\ 0, & |\Omega| > \Omega_c \end{cases}$$

The impulse response $h_r(t)$ is derived as:

$$h_r(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H_r(j\Omega)e^{j\Omega t}d\Omega = \frac{T}{2\pi}\int_{-\Omega_c}^{\Omega_c} e^{j\Omega t}d\Omega$$
$$= \frac{\sin(\Omega_c t)}{\Omega_T t/2}, \quad -\infty < t < \infty$$

Show that $h_r(t)$ takes the value $h_r(nT) = \delta[n]$ for all n if the cutoff frequency $\Omega_c = \Omega_T / 2$, where Ω_T is the sampling frequency.

- (Optional) An alternative to the zero-order hold circuit used for signal reconstruction at the output of a D/A converter is the *first-order hold circuit*, which approximates $y_a(t)$ according to the following relation:

$$y_f(t) = y_p(nT) + \frac{y_p(nT) - y_p(nT - T)}{T}(t - nT), \quad nT \le t \le (n+1)T$$

As indicated by the above equation, the first-order hold circuit approximates $y_a(t)$ by straight-line segments. The slope of the segment between $t = nT$ and $t = (n+1)T$ is determined from the values $y_p(nT)$ and $y_p(nT-T)$. Determine the impulse response $h_f(t)$ and the frequency response $H_f(j\Omega)$ of the first-order hold circuit, and compare its performance with that of the zero-order circuit.

- (Optional) A more improved signal reconstruction at the output of a D/A converter is provided by a linear interpolation circuit, which approximates $y_a(t)$ by connecting successive sample points of $y_p(t)$ with straight-line segments. The input-output relation of this circuit is given by

$$y_f(t) = y_p(nT-T) + \frac{y_p(nT) - y_p(nT-T)}{T}(t-nT), \quad nT \le t \le (n+1)T$$

Determine the impulse response $h_f(t)$ and the frequency response $H_f(j\Omega)$ of the linear interpolation circuit, and compare its performance with that of the first-order circuit.