16. 低通原型和滤波器原理电路综合

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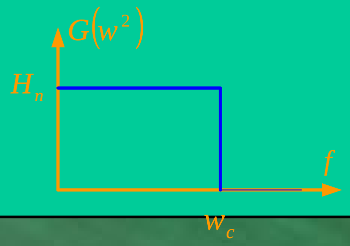
理想响应



逼近函数

$$G(-s^2)=S_{21}(s)S_{21}(-s)$$

网络无耗

$$S_{11}(s)S_{11*}(s)=1-G(-s^2)$$

分解反射函数

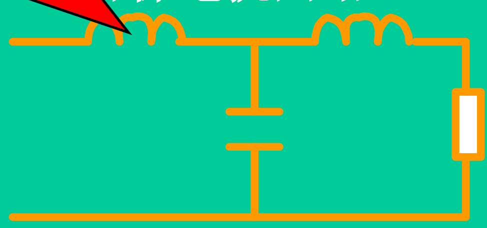
非唯一，可实现性

$$S_{11}(s)$$

阻抗函数

$$Z_{in}(s)=Z_0\frac{1\pm S_{11}(s)}{1\mp S_{11}(s)}$$

阶梯电抗网络



16. 低通原型和滤波器原理电路综合

一、低通原型及元件值

二、阻抗变换和频率变换

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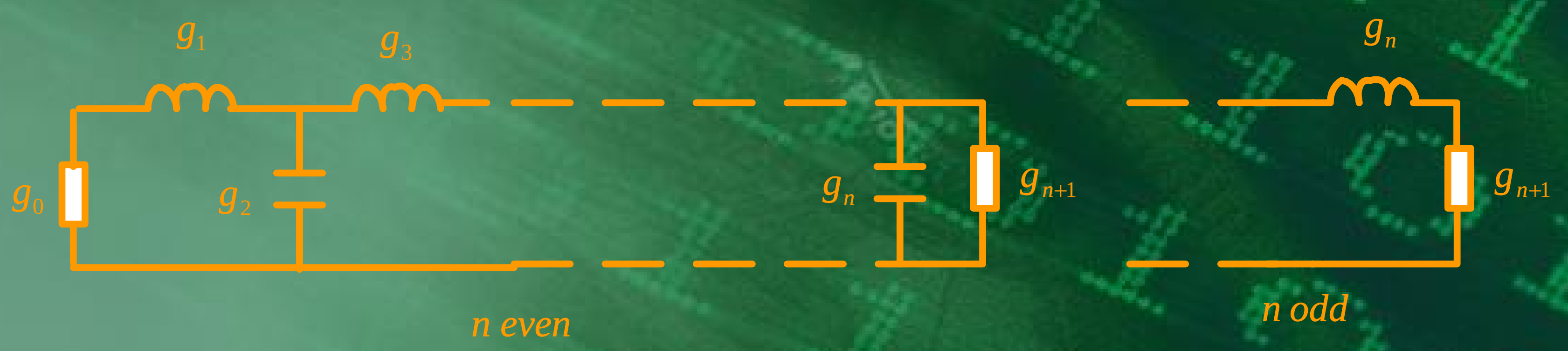
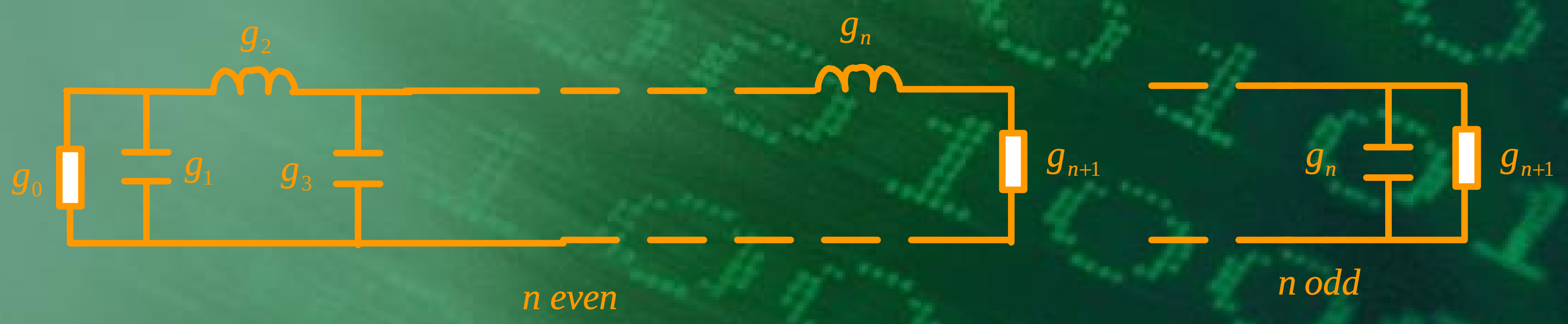
低通原型电路：特定低通电路

阻抗归一：特性阻抗为1；

频率归一：截止频率为1；





$$g_k\Big|_{k=1,2,\cdots n}=\begin{cases}\text{串联电感值}\\ \text{并联电容值}\end{cases}$$

$$g_0=\begin{cases}\text{源电阻}R_0',\text{若}g_1=C_1'\\ \text{源电导}G_0',\text{若}g_1=L_1'\end{cases}$$

$$g_{n+1}=\begin{cases}\text{负载电阻}R_{n+1}',\text{若}g_n=C_n'\\ \text{负载电导}G_{n+1}',\text{若}g_n=L_n'\end{cases}$$

阻抗归一 $g_0=1$ ；频率归一 $\omega_c'=1$

Butterworth低通原型的元件值

$$g_0 = 1.0$$

$$g_k = 2\sin\left[\frac{(2k-1)p}{2n}\right], k = 1,2, \mathbf{L}, n$$

$$g_{n+1} = 1.0$$

带外衰减：$\overline{w} = \overline{w}_s > 1$

$$L_B > AL$$

$$n = \mathrm{int}\left[\frac{\log\left(10^{\frac{AL}{10}} - 1\right)}{2\log \overline{w}_s}\right] + 1$$

Chebyshev低通原型的元件值

$$g_0 = 1.0$$

$$g_1 = 2\frac{a_1}{g}$$

$$g_k = \frac{4a_{k-1}a_k}{b_{k-1}g_{k-1}}, \quad k = 2,3,\ldots,n$$

$$g_{n+1} = \begin{cases} 1 \quad , & n\ odd \\ \coth^2\left(\frac{b}{4}\right) , & n\ even \end{cases}$$

其中，

$$b = \ln\left(\coth\frac{L_{Ar}}{17.37}\right)$$

$$g = \sinh\left(\frac{b}{2n}\right)$$

$$a_k = \sin\left[\frac{(2k-1)p}{2n}\right] \quad , \quad k = 1,2,\mathbf{L},n$$

$$b_k = g^2 + \sin^2\left(\frac{kp}{n}\right) \quad , \quad k = 1,2,\mathbf{L},n$$

当 $\overline{w}=\overline{w}_s>1$ 时，要求 $L_s>L_{As}$ ，

其中 L_{Ar} 分贝波纹

$$e=\sqrt{10^{\frac{L_{Ar}}{10}}-1}$$

$$n=\operatorname{int}\left[\frac{ch^{-1}\sqrt{\dfrac{10^{\frac{L_{As}}{10}}-1}{10^{\frac{L_{Ar}}{10}}-1}}}{ch^{-1}\overline{w}_s}\right]+1$$

讨论：

1、归一化思想

低通、带通、高通 → 低通

低通 → 低通原型

2、n较小时，(n+1)阶比n阶变化大；

n较大时，(n+1)阶比n阶变化小；

3、n>15的设计是罕见的。

如n=18，取n=14，中间断开，重复g6和g7两次即可。

【例】Cheyshev响应 $L_{Ar} = 0.1dB$

$$\overline{w}_s = 2\,,\, L_{As} \ge 40dB$$

解：求n

$$n = \mathrm{int}\left[\frac{ch^{-1}\sqrt{\dfrac{10^{0.1L_{As}}-1}{10^{0.1L_{Ar}}-1}}}{ch^{-1}\overline{w}_s}\right]+1$$

取n=6；

注意到偶数阶Chebyshev综合不对称，一般取n=7;

查表得到，带内波纹0.1dB，n=6时，元件值

g0	g1	g2	g3	g4	g5	g6	g7
1.0	1.1681	1.4039	2.0562	1.5170	1.9029	0.8618	1.3554

网络非对称，最后元件值不为1，意味着负载不等于特性阻抗，这与一般工程不符。

查表得到，带内波纹0.1dB，n=7时，元件值

g0	g1	g2	g3	g4	g5	g6	g7	g8
1.0	1.1811	1.4228	2.0966	1.5733	2.0966	1.4228	1.1811	1.0

网络对称，最后元件值为1，意味着负载等于特性阻抗。

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低通原型
（阻抗归一
频率归一）

阻抗变换
1 → Z_0
Impedance scaling
element transformation

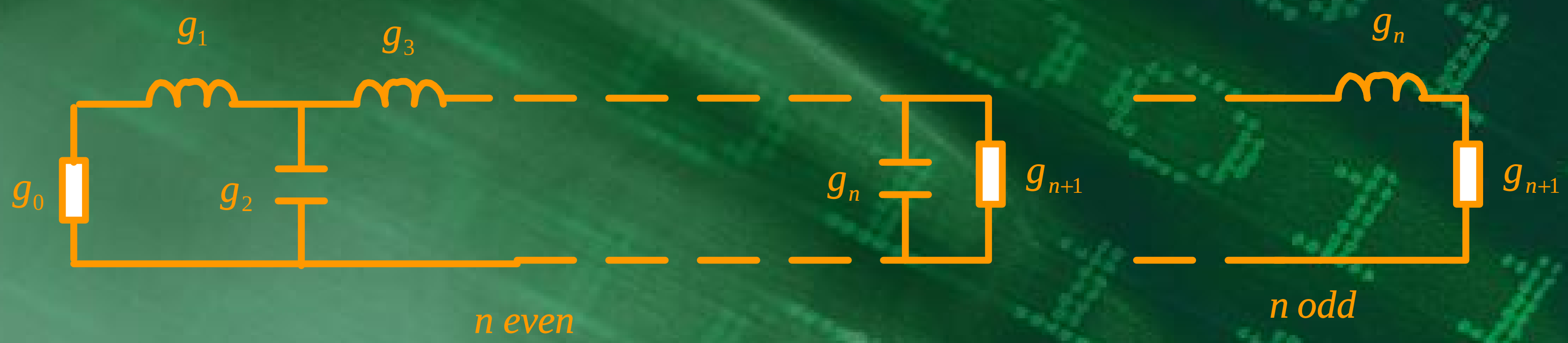
频率变换
低通原型 → 低通、带通等
Frequency mapping
Frequency transformation

1. 阻抗变换

原则上，同时做以下阻抗变换，滤波网络响应无变化

$$L \rightarrow g_0 L$$
$$R \rightarrow g_0 R$$
$$C \rightarrow C/g_0$$
$$G \rightarrow G/g_0$$

2. 低通变换



已知低通电抗阶梯网络阻抗函数为

$$Z_{in}(s)=L_1 s+\cfrac{1}{C_2 s+\cfrac{1}{L_3 s+\cfrac{\ddots}{+\cfrac{1}{M}}}}$$

同时进行阻抗归一和频率归一，则为

$$\bar{Z}_{in}(\bar{s}) = \frac{Z_{in}(s)}{Z_0} = \frac{L_1 w_c}{Z_0} \cdot \frac{s}{w_c} + \frac{1}{C_2 Z_0 w_c \dfrac{s}{w_c} + \mathbf{O}}$$

$$= \bar{L}_1 \bar{s} + \frac{1}{\bar{C}_s \bar{s} + }$$

$$\mathbf{O}$$

$$+ \frac{1}{\bar{M}}$$

即，归一化

$$
\begin{cases}
\overline{L}_i = \dfrac{L_i w_c}{Z_0} \\
\overline{C}_i = C_i w_c Z_0 \\
\overline{r}_l = r_l / Z_0
\end{cases}
$$

显然，反归一化

$$
\begin{cases}
L_i = \dfrac{\overline{L}_i Z_0}{w_c} \\
C_i = \dfrac{\overline{C}_i}{w_c Z_0} \\
r_l = \overline{r}_l Z_0
\end{cases}
$$

即，由低通原型元件值到任意低通的变换

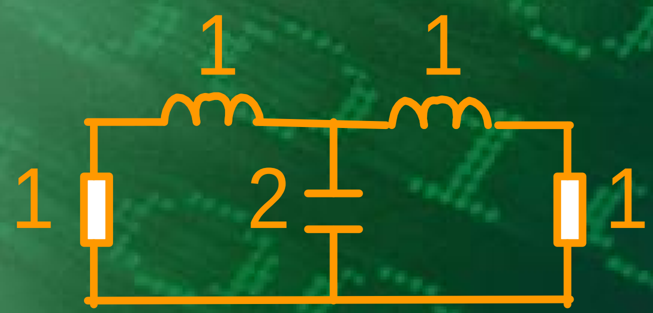
$$\begin{cases} L = \dfrac{Z_0 g}{w_c} \\ C_i = \dfrac{g}{w_c Z_0} \\ R = g_0 Z_0 \quad (G = g_0 / Z_0) \end{cases}$$

【例】3阶Butterworth低通滤波器设计，要求

$$f_c = 2GHz\ ,\ Z_0 = 50\Omega$$

解：查表得到3阶Butterworth低通原型的元件值

g0	g1	g2	g3	g4
1.0	1.0	2.0	1.0	2.0

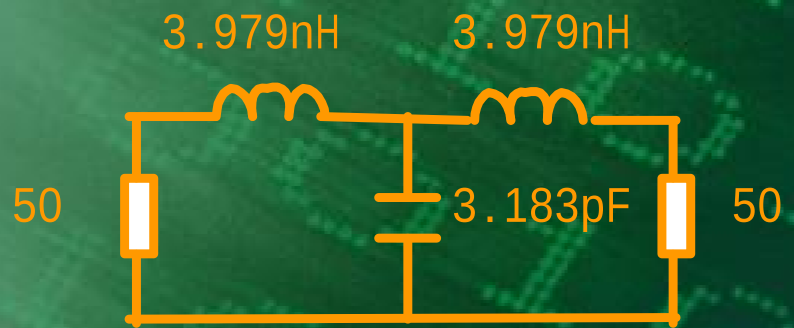


反归一

$$L_1 = \frac{Z_0 g_1}{\omega_c} = \frac{50*1}{2\pi*2*10^9} = 3.979nH$$

$$C_2 = \frac{g_2}{\omega_c Z_0} = \frac{2}{2\pi*2*10^9*50} = 3.183pF$$

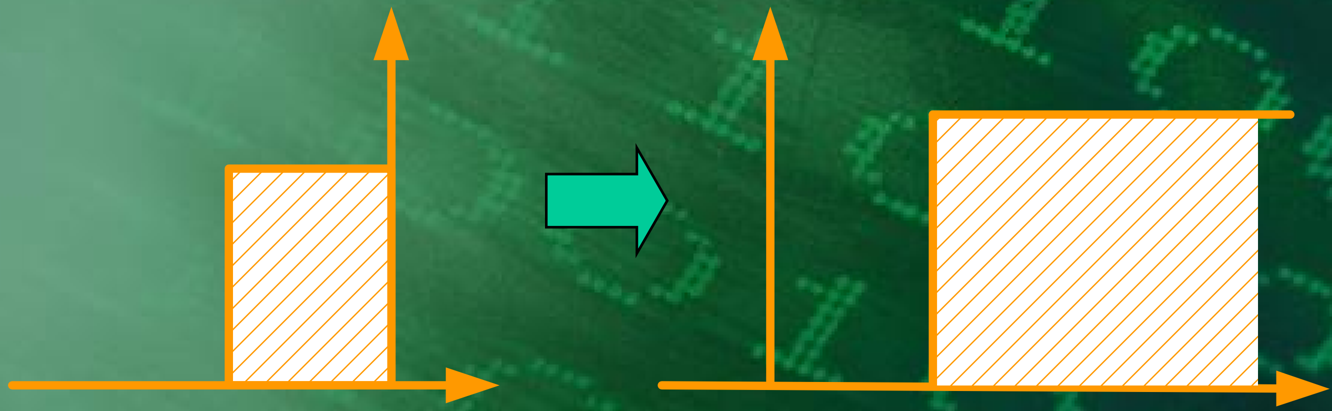
$$L_3 = L_1 = 3.979nH$$



3. 低通到高通

低通原型 w' ；滤波网络 w

$$\begin{cases} w' = \infty & w = 0 \\ w' = -1 & w = w_c \\ w' = 0 & w = \infty \end{cases}$$

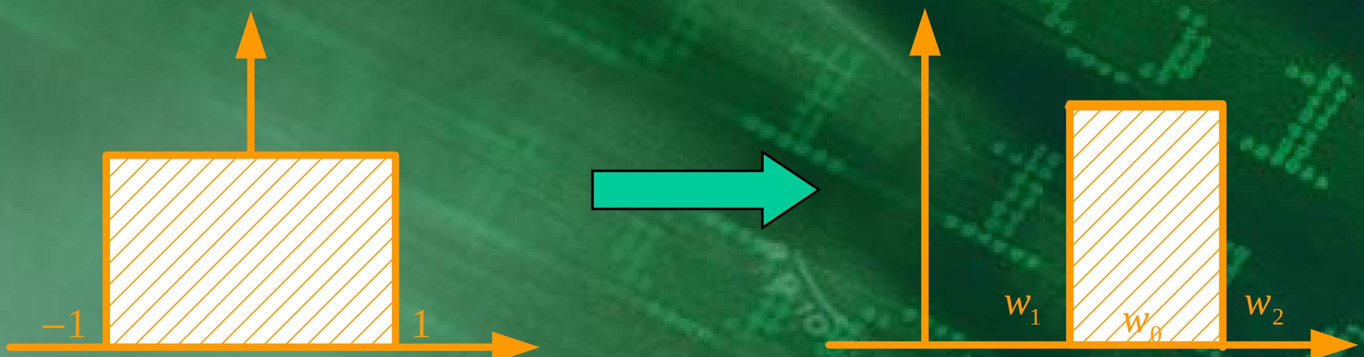


低通原型中的电感→高通中的电容

低通原型中的电容→高通中的电感

$$
\begin{cases}
C = \dfrac{1}{w_c}\dfrac{1}{Z_0 g} & g\text{表示电感} \\
L = \dfrac{1}{w_c}\dfrac{Z_0}{g} & g\text{表示电容}
\end{cases}
$$

4. 低通到带通



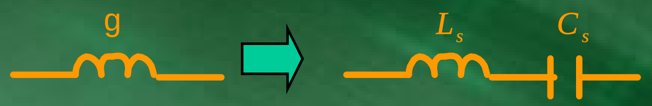
$$\begin{cases} w' = -\infty & w = 0 \\ w' = -1 & w = w_1 \\ w' = 0 & w = w_0 \\ w' = 1 & w = w_2 \\ w' = \infty & w = \infty \end{cases}$$

得到归一化公式

$$w' = \frac{1}{FBW}\left(\frac{w}{w_0} - \frac{w_0}{w}\right) \qquad FBW = \frac{w_2 - w_1}{w_0}$$

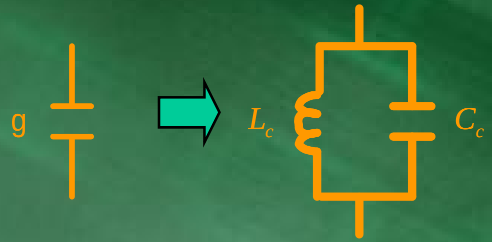
低通原型中的电感→带通中的串联谐振

低通原型中的电容→带通中的并联谐振



$$\begin{cases} L_s = \dfrac{Z_0 g}{FBW \cdot w_0} \\ C_s = \dfrac{FBW}{w_0 \cdot g \cdot Z_0} \end{cases}$$

$$L_s C_s = 1 / w_0^2$$



$$
\begin{cases}
L_c = \dfrac{FBW \cdot Z_0}{g \cdot w_0} \\[2ex]
C_c = \dfrac{g}{w_0 \cdot FBW \cdot Z_0}
\end{cases}
$$

$$L_c C_c = 1/w_0^2$$

【例】**f0=900MHz，BW=10MHz；**

△f=20MHz时，Las>30dB；Butterworth响应

解：

$$FBW = \frac{10}{900} = 0.0111$$

$$w' = \frac{1}{FBW}\left(\frac{w}{w_0} - \frac{w_0}{w}\right)$$

带外衰减要求**880MHz，920MHz**时，

$$f_{s1} = -4.045 \qquad f_{s2} = -3.9565$$

归一化带宽**10MHz**，带外衰减位置一般就取**4**；

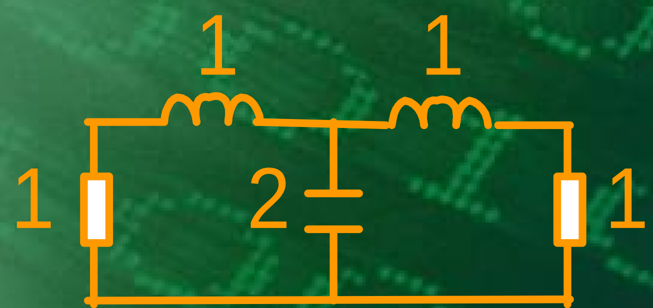
由带外衰减指标确定阶数n

$$n \geq \frac{\log\left(10^{0.1L_{As}}-1\right)}{2\log w_s} = 2.49$$

取n=3.

查表得到3阶Butterworth低通原型的元件值

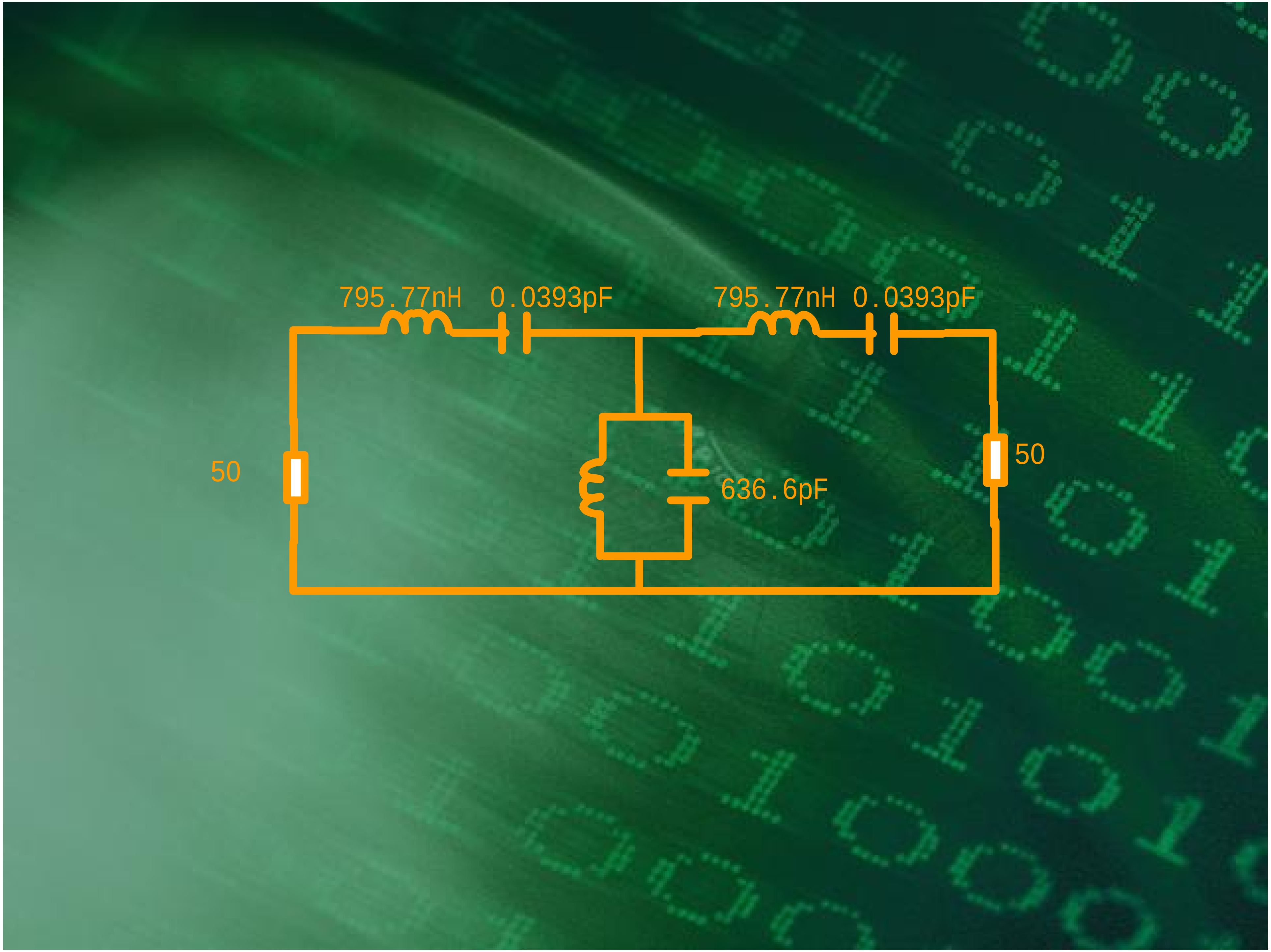
g0	g1	g2	g3	g4
1.0	1.0	2.0	1.0	2.0



$$\begin{cases} L_{s1} = \dfrac{Z_0 g}{FBW \cdot w_0} = \dfrac{50*1}{0.0111*2p*0.9*10^9} = 795.77nH \\ C_{s1} = \dfrac{FBW}{w_0 \cdot g \cdot Z_0} = \dfrac{0.0111}{2p*0.9*10^9*1*50} = 0.0393pF \end{cases}$$

$$\begin{cases} L_c = \dfrac{FBW \cdot Z_0}{g \cdot w_0} = \dfrac{0.0111*50}{2*2p*0.9*10^9} = 0.004912nH \\ C_c = \dfrac{g}{w_0 \cdot FBW \cdot Z_0} = \dfrac{2}{2p*0.9*10^9*0.0111*50} = 636.6pF \end{cases}$$

$$\begin{cases} L_{s1} = 795.77nH \\ C_{s1} = 0.0393pF \end{cases}$$


795.77nH
0.0393pF
795.77nH
0.0393pF
50
636.6pF
50