III. 散射参数最佳传输

西安电子科技大学，电子工程学院
苏 涛

Ⅲ. 散射参数最佳传输

一、散射参数（S参数）

二、散射参数最佳传输

三、散射参数研究的特殊性

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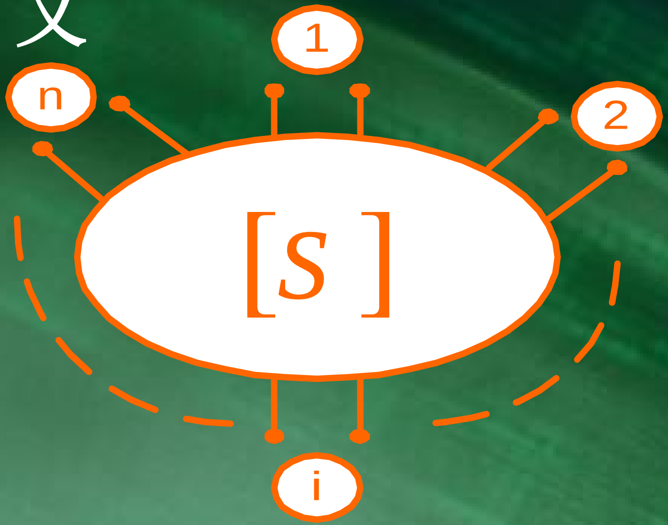
定　义



$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1n} \\ S_{21} & S_{22} & \cdots & S_{2n} \\ \vdots & \vdots & & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$

- **a,b 波参量；**
- **方向对称定义；**
- **按参量性质组合**

$$\begin{bmatrix} u_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} u_2 \\ i_2 \end{bmatrix}$$

- **u,i 参量；**
- **方向非对称定义；**
- **按端口组合**

参数含义

$$S_{11}=\left.\frac{b_1}{a_1}\right|_{a_2=0}$$

2 端口匹配时，1 端口自反射系数

$$S_{12}=\left.\frac{b_1}{a_2}\right|_{a_1=0}$$

1 端口匹配时，$2\rightarrow1$ 传输

$$S_{21}=\left.\frac{b_2}{a_1}\right|_{a_2=0}$$

2 端口匹配时，$1\rightarrow2$ 传输

$$S_{22}=\left.\frac{b_2}{a_2}\right|_{a_1=0}$$

1 端口匹配时，2 端口自反射系数

$$A_{11}=\left.\frac{u_1}{u_2}\right|_{i_2=0}$$

开路转移电压比

$$A_{12}=\left.\frac{i_1}{u_2}\right|_{i_2=0}$$

开路转移导纳

$$A_{21}=\left.\frac{u_1}{i_2}\right|_{u_2=0}$$

短路转移阻抗

$$A_{22}=\left.\frac{i_1}{i_2}\right|_{u_2=0}$$

短路转移电流比

性　质		
对称	$S_{11}=S_{22}$	$A_{11}=A_{22}$
互易	$S_{12}=S_{21}$	$\det[A]=1$
无耗	$[S]^{+}[S]=[I]$	A_{11},A_{22}　实数 A_{12},A_{21}　纯虚数

定　理

反射系数变换

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

阻抗变换

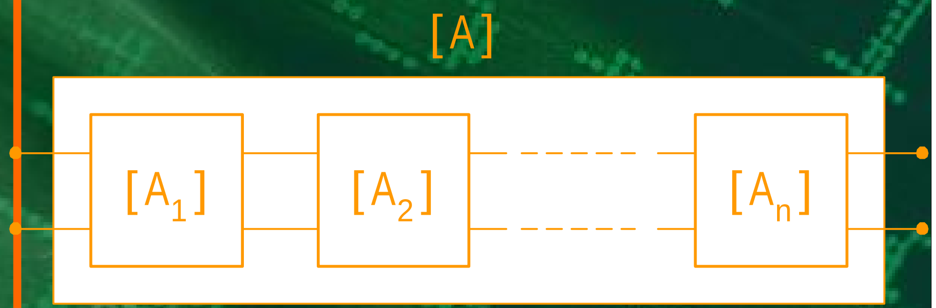
$$Z_{in} = \frac{A_{11}Z_L + A_{12}}{A_{21}Z_L + A_{22}}$$

定　理

无耗双端口网络

$$
\begin{cases}
|S_{11}| = |S_{22}| \\
|S_{12}| = |S_{21}| \\
(j_{12} + j_{21}) - (j_{11} + j_{22}) = \pm p
\end{cases}
$$

级联定理



$$[A] = \prod_{i=1}^{n} [A_i]$$

定　理

- 二端口无耗网络匹配定理　$S_{22} = \Gamma_L^*$
- 三端口无耗互易网络，三端口不能同时匹配；
- 四端口网络，耦合 / 隔离 / 匹配的关系；

注：S 参数是波参数，直指波的传输和反射。

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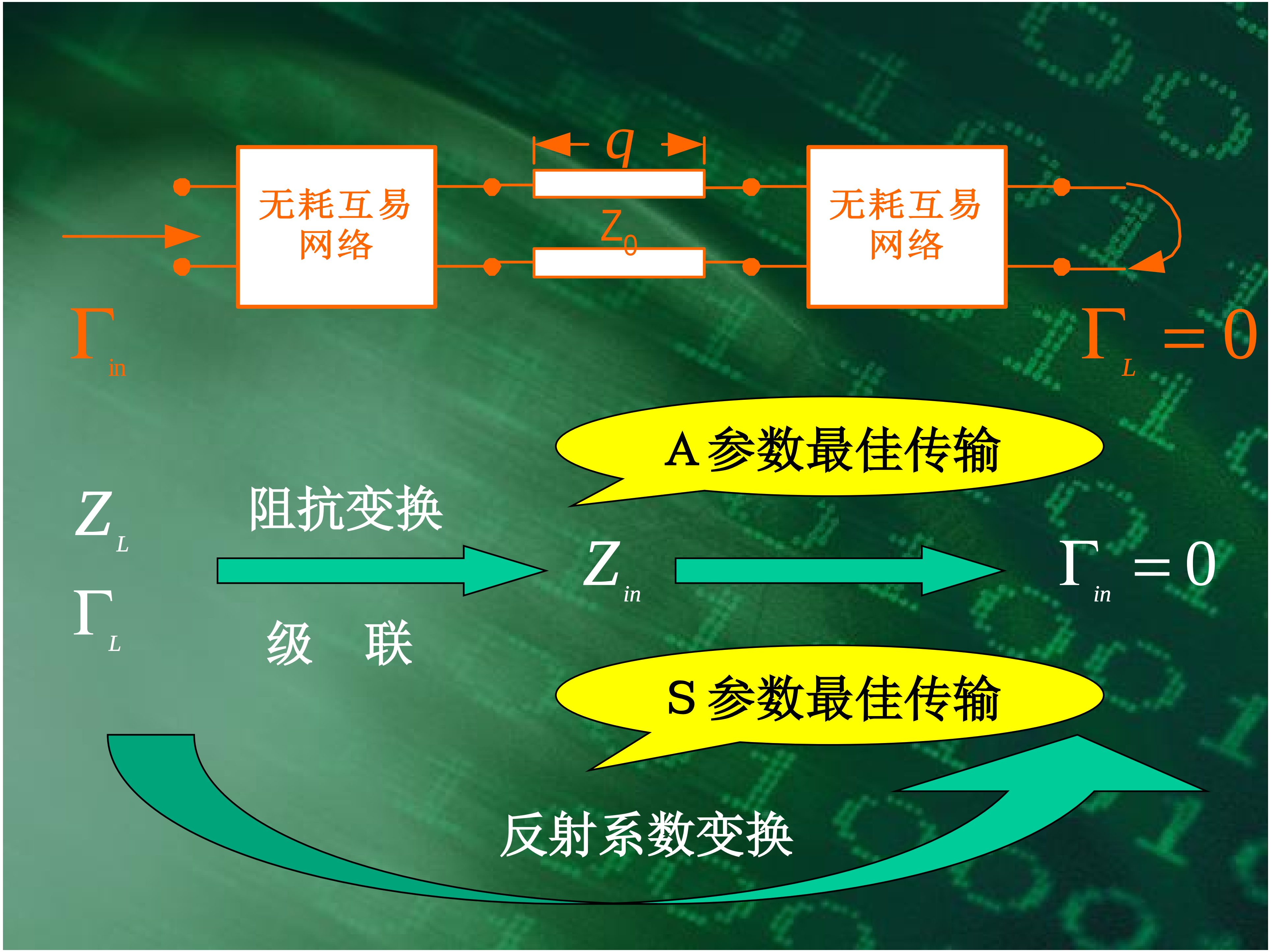
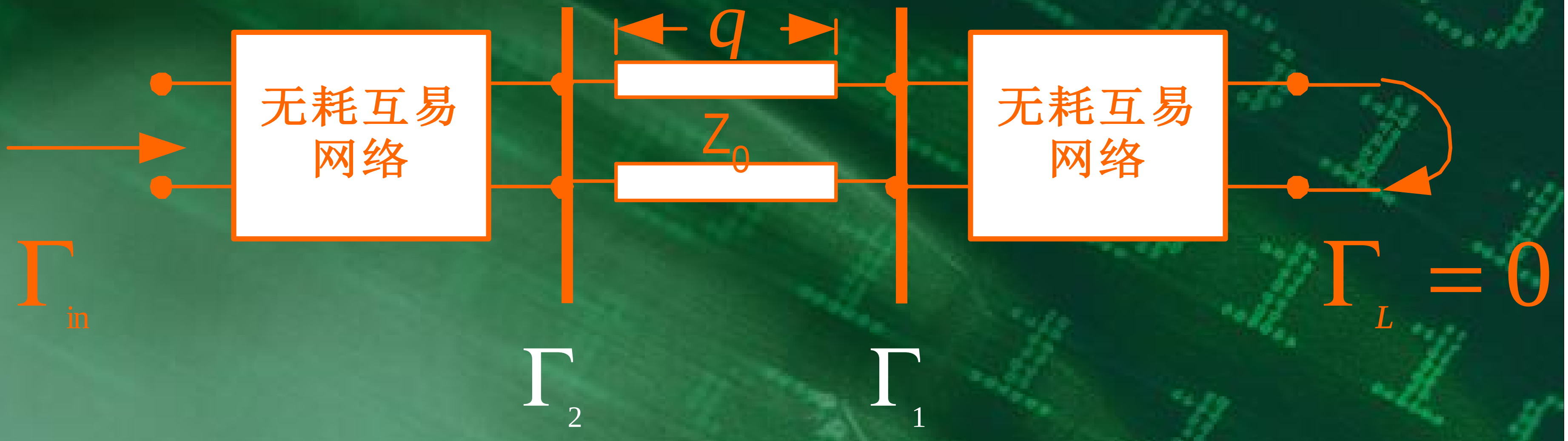


无耗互易网络
θ
Z_0
无耗互易网络
Γ_{in}
$\Gamma_L = 0$
A参数最佳传输
Z_L
Γ_L
阻抗变换
级 联
Z_{in}
$\Gamma_{in} = 0$
S参数最佳传输
反射系数变换



令无耗互易网络的 S 参数为：

$$[S]=\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

$$\Gamma_1 = S_{11}$$

$$\Gamma_2 = S_{11} e^{-j2q}$$

$$\Gamma_{in} = S_{11} + \frac{S_{12} S_{21} \Gamma_2}{1 - S_{22} \Gamma_2} = S_{11} + \frac{S_{12} S_{21} S_{11} e^{-j2q}}{1 - S_{22} S_{11} e^{-j2q}}$$

$$= \frac{S_{11} - S_{11}^2 S_{22} e^{-j2q} + S_{12} S_{21} S_{11} e^{-j2q}}{1 - S_{11} S_{22} e^{-j2q}}$$

考虑网络无耗互易条件，即

$$\begin{cases} |S_{11}| = |S_{22}| \\ |S_{12}| = |S_{21}| \\ (\varphi_{12} + \varphi_{21}) - (\varphi_{11} + \varphi_{22}) = \pm\pi \end{cases}$$

$$\Gamma_{in} = \frac{S_{11} - S_{11}^2 S_{22} e^{-j2\theta} + S_{12} S_{21} S_{11} e^{-j2\theta}}{1 - S_{11} S_{22} e^{-j2\theta}}$$

$$= \frac{S_{11} - S_{11}^2 S_{22} e^{-j2\theta} + \left(1 - |S_{11}|^2\right) S_{11} e^{j(\varphi_{12}+\varphi_{21}-2\theta)}}{1 - |S_{11}|^2 e^{j(\varphi_{11}+\varphi_{22}-2\theta)}}$$

令，$$j_{11} + j_{22} - 2q = y$$

$$\Gamma_{in} = \frac{S_{11} - S_{11}\left|S_{11}\right|^2 e^{jy} + \left(1 - \left|S_{11}\right|^2\right) S_{11} e^{j(\pm p + y)}}{1 - \left|S_{11}\right|^2 e^{jy}}$$

$$= \frac{S_{11} - S_{11}\left|S_{11}\right|^2 e^{jy} - S_{11} e^{jy} + S_{11}\left|S_{11}\right|^2 e^{jy}}{1 - \left|S_{11}\right|^2 e^{jy}}$$

$$= \frac{S_{11} - S_{11} e^{jy}}{1 - \left|S_{11}\right|^2 e^{jy}} = \frac{S_{11}\left(1 - e^{jy}\right)}{1 - \left|S_{11}\right|^2 e^{jy}}$$

分子模的平方：

$$\left|S_{11}\left(1-e^{jy}\right)\right|^2 = \left|S_{11}\right|^2 \left|1-\cos y - j\sin y\right|^2$$

$$= \left|S_{11}\right|^2 \left|2\sin\frac{y}{2}\left(\sin\frac{y}{2} - j\cos\frac{y}{2}\right)\right|^2$$

$$= 4\left|S_{11}\right|^2 \sin^2\frac{y}{2}$$

分母模的平方：

$$\left|1-\left|S_{11}\right|^{2} e^{jy}\right|^{2}$$

$$=\left|1-\left|S_{11}\right|^{2}(\cos y+j\sin y)\right|^{2}$$

$$=\left(1-\left|S_{11}\right|^{2}\cos y\right)^{2}+\left|S_{11}\right|^{4}\sin^{2} y$$

$$=1-2\left|S_{11}\right|^{2}\cos y+\left|S_{11}\right|^{4}\cos^{2} y+\left|S_{11}\right|^{4}\sin^{2} y$$

$$=1+\left|S_{11}\right|^{4}-2\left|S_{11}\right|^{2}+4\left|S_{11}\right|^{2}\sin^{2}\frac{y}{2}$$

$$|\Gamma_{in}|^2 = \frac{4|S_{11}|^2 \sin^2 \dfrac{y}{2}}{1+|S_{11}|^4 - 2|S_{11}|^2 + 4|S_{11}|^2 \sin^2 \dfrac{y}{2}}$$

$$= \frac{4|S_{11}|^2 \sin^2 \dfrac{y}{2}}{\left(1-|S_{11}|^2\right)^2 + 4|S_{11}|^2 \sin^2 \dfrac{y}{2}}$$

$$= \frac{1}{1+\left(\dfrac{1-|S_{11}|^2}{2|S_{11}|\sin\dfrac{y}{2}}\right)^2} = \frac{1}{1+\csc^2\dfrac{y}{2}\left(\dfrac{1-|S_{11}|^2}{2|S_{11}|}\right)^2}$$

当： $\sin\frac{y}{2}=0$ 时， $\Gamma_{in}=0$

得到：

$$q_p=\frac{1}{2}\left(j_{11}+j_{22}\right)$$

$$q_p=kp+\frac{j_{11}+j_{22}}{2},\quad k=0,\pm1,\pm2,\mathbf{L}$$

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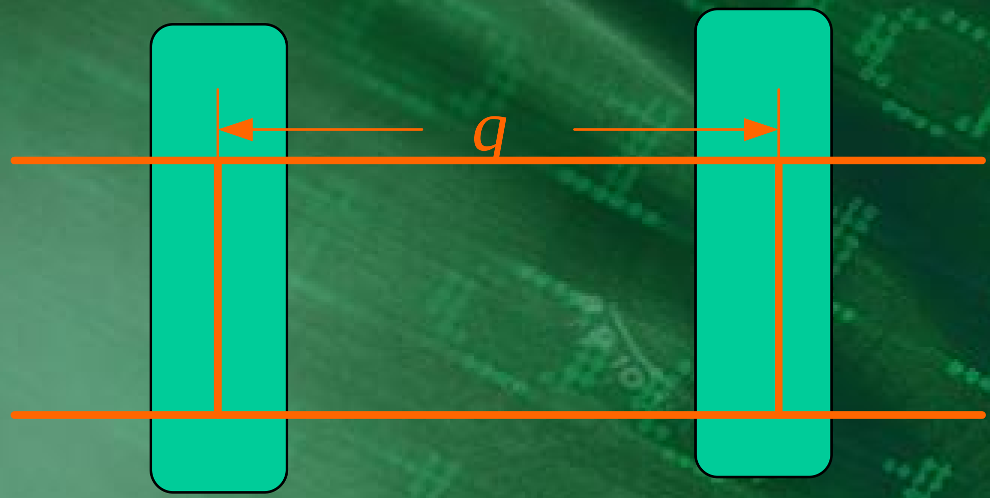
Case 1：对称网络情况——谐振腔



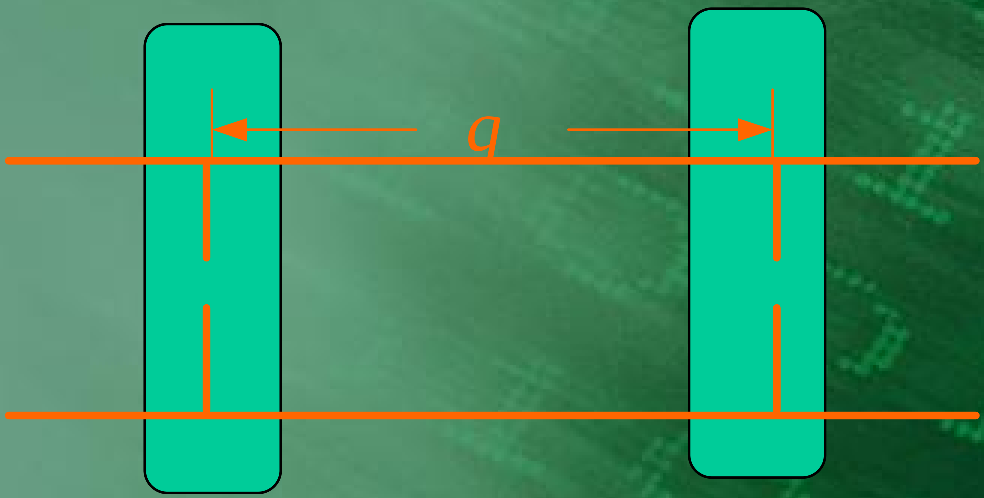
将两侧的短路壁看作网络，由于全反射

$$j_{11} = j_{22} = p$$

$$q_p = \frac{1}{2}\left(j_{11} + j_{22}\right) = p$$

即，

$$l = {l_g}/{2}$$



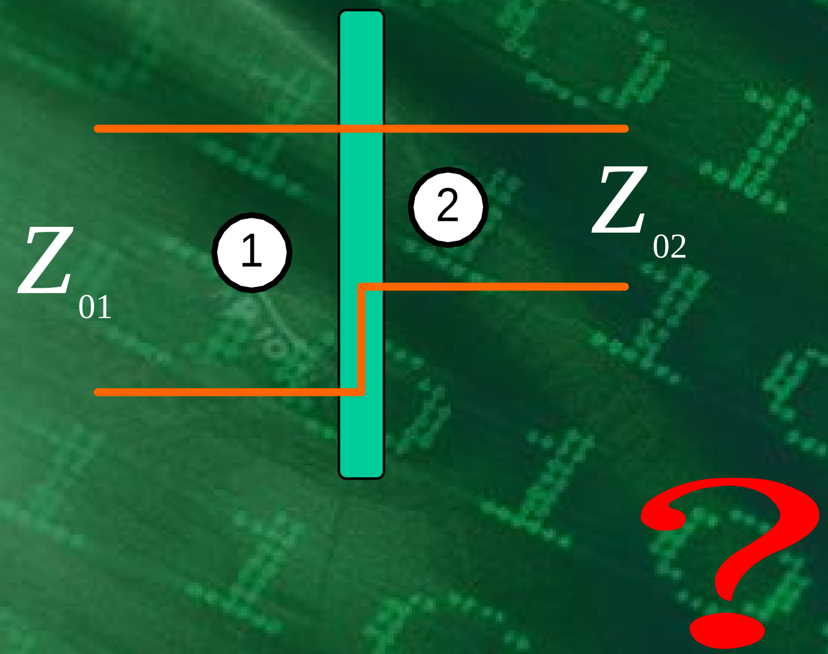
实际工作时，

通过式谐振腔

$$l \approx {l_g}/{2}$$

Case 2：反对称网络情况——1 / 4 波长阶梯阻抗变换器

波导阶跃阶梯



取波导阶跃面为网络，参考面均位于阶跃面

$$S_{11} = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}}$$

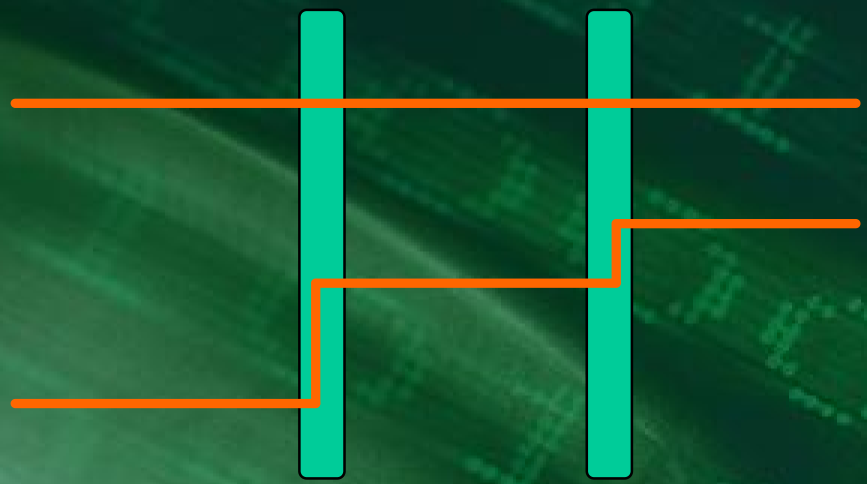
$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$

$$S_{22} = \frac{Z_{01} - Z_{02}}{Z_{01} + Z_{02}}$$

显然，反对称：$S_{11} = -S_{22}$

$$j_{11} = p + j_{22}$$

特性阻抗为实数，显然 $j_{11} = 0 .or. p$ ，



如上图的两个阶跃，如果S参数相同，最佳传输时

$$q_p = \frac{1}{2}\left(j_{11} + j_{22}\right) = \frac{p}{2} + j_{11}$$

$$q_p = \frac{p}{2}$$

即， 1/4波长阶梯阻抗变换器。

A 参数和 S 参数分析问题的异同：

- 等价的；
- S 参数直指反射。