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Error Vector Magnitude Analysis of Uplink Multiuser OFDMA and SC-FDMA Systems in the Presence of Nonlinear Distortion

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***Abstract*—We analyze the impact of nonlinear distortion on the performance of uplink multiuser orthogonal frequency division multiple access (OFDMA) and single carrier-FDMA (SC-FDMA) systems in terms of the error vector magnitude (EVM). Based on the power spectrum density (PSD) of the distortion noise, we derive a unified framework of theoretical EVM for both systems. The obtained results provide us approaches to evaluation of both the self-distortion and the inter-distortion interference from other users in multipath fading channels as well as the additive white Gaussian noise channel. In addition, simulation results agree well with our theoretical analyses.**

***Index Terms*—orthogonal frequency division multiple access (OFDMA), single carrier-FDMA (SC-FDMA), nonlinear distortion, error vector magnitude (EVM).**

I. INTRODUCTION

ONE major drawback of the orthogonal frequency division multiple access (OFDMA) signal is its high peak-to-average power ratio (PAPR). Though exhibiting lower PAPR compared to OFDMA, the single carrier-FDMA (SC-FDMA) signal also encounters a large PAPR for high-order modulations [1]. The high PAPR makes transmitted signals prone to the nonlinear distortion and leads to low efficiency of the power amplifier (PA), especially for the uplink with stringent low power consumption demand on user terminals.

The nonlinear distortion can either be intentional (from nonlinear processing for PAPR reduction, such as the clipping and filtering technique [2]) or not (e.g., from a nonlinear PA). Performance evaluation with traditional metrics such as the throughput and the bit error rate were analytically studied in [2]–[5] for nonlinearly distorted multicarrier signals. Due to the decision and decoding functions at the receiver, small variations of nonlinearity cannot necessarily be reflected on these bit-level metrics [6]. As an alternative, the error vector magnitude (EVM) is a symbol-level metric which defines the dispersion range of measured symbols from transmitted ones [6], [7]. Even a small variation of nonlinearity can be revealed by the fluctuation of theoretical EVM, providing benchmarks for the transceiver design and system parameter optimization [8]. Theoretical EVMs with nonlinear distortion were studied in [9], [10] for the single user scenario in the additive white Gaussian noise (AWGN) channel. However, OFDMA and SC-FDMA systems usually work over multipath fading channels in practice and the out of band radiation caused by nonlinearity degrades the performance of other users. Hence, the theoretical EVM analysis for uplink multiuser OFDMA and SC-FDMA systems in multipath fading channels is desired greatly to show the impact of nonlinear distortion.

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To this end, we derive a unified theoretical EVM for both systems based on the power spectrum density (PSD) of the distortion noise. The obtained results can help to evaluate the self-distortion interference (SDI) and inter-distortion interference (IDI) from other users and provide guidance to the transceiver design and parameter optimization for the multiuser system. Simulation results demonstrate the effectiveness of our theoretical analyses for both OFDMA and SC-FDMA signals.

II. SYSTEM MODEL

The modulations of OFDMA and SC-FDMA systems are the same except for the extra discrete Fourier transform (DFT) precoding for SC-FDMA signals. Hence, we consider them in a unified framework through a orthogonal precoding matrix $\mathbf{P}$ with $\mathbf{P}=\mathbf{I}_M$ for OFDMA and $\mathbf{P}=\mathbf{F}_M$ for SC-FDMA, where $\mathbf{I}_M$ is the identity matrix of size M, $\mathbf{F}_M$ is the M-point unitary DFT matrix and $(\cdot)^H$ denotes the conjugate transpose. Consider an uplink system containing U users. There are total N subcarriers in every OFDM-based symbol and each user occupies $M=N/L$ subcarriers, where $L(L\geq U)$ is the number of maximum users the system can support.

Specifically for the uth user, the data vector is denoted as $\mathbf{s}_u$=$[s_{u,0},...,s_{u,M-1}]^T$, where $(\cdot)^T$ is the transpose and $s_{u,k}(k=0,...,M-1)$ is selected from a constellation with power λ_u. Then $\mathbf{s}_u$ is fed to $\mathbf{P}$ and the output is mapped to M out of the N subcarriers by a an $N\times M$ matrix $\mathbf{T}_u$ whose elements are all zeros except a single "1" in each column. The indexes of rows with these '1's are denoted by the set $\mathcal{J}_u$ related to locations of the assigned subcarriers and the cardinality of $\mathcal{J}_u$ is M. There are two types of subcarrier assignment scheme: the localized scheme and the interleaved one. Here we consider the localized scheme since it is more robust to carrier mismatch than the later [1]. Then we have $\mathcal{J}_u=\{f_{u,k}|f_{u,k}=f_u+k, 0\leq f_{u,k}\leq N-1, k=0,...,M-1\}$, where f_u is the starting index regulated by the system. After an N-point unitary inverse DFT (IDFT), the time-domain signal is given by

$$\mathbf{x}_u=\mathbf{F}_N^H\mathbf{T}_u\mathbf{P}\mathbf{s}_u. \tag{1}$$

Then $\mathbf{x}_u=[x_{u,0},...,x_{u,N-1}]^T$ is submitted to the nonlinear module yielding the distorted signal $\mathbf{x}_u^C=[x_{u,0}^C,...,x_{u,N-1}^C]^T$ as $x_{u,n}^C=f(R)\exp[j\arg(x_{u,n})]$, where $R=|x_{u,n}|$ and $f(R)$ is the nonlinear function. As discussed in [11], the output can be decomposed as the sum of a useful component proportional to the input and a nonlinear distortion one, i.e., $\mathbf{x}_u^C=\alpha_u\mathbf{x}_u+\mathbf{d}_u$, where $\mathbf{d}_u=[d_{u,0},...,d_{u,N-1}]^T$ and

$$\alpha_u=\frac{\mathbb{E}\left[x_{u,n}^*x_{u,n}^C\right]}{\mathbb{E}[|x_{u,n}|^2]}=\frac{\int_0^\infty Rf(R)p(R)dR}{\lambda_u M/N}, \tag{2}$$

with $\mathbb{E}[\cdot]$ and $p(R)$ being the expectation and the probability density function, respectively. There is no general assumption

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on the correlation of $\mathbf{d}_u$ with $\mathbf{x}_u$ [11]. For OFDMA, if the subcarrier number M is high, $\mathbf{x}_u$ is complex Gaussian and $\mathbf{d}_u$ is thereby uncorrelated with $\mathbf{x}_u$ according to Bussgang's theorem [3]. Then R is Rayleigh distributed and α_u is obtained with (2). For SC-FDMA, $\mathbf{x}_u$ is no more Gaussian [12], making it difficult to theoretically quantify the correlation of $\mathbf{d}_u$ with $\mathbf{x}_u$ [11]. Fortunately, empirical approach can be useful when strict theoretical analysis is unavailable [3], [9]. Extensive numerical results verify that such correlation is weak enough to approximately assume that they are uncorrelated[1]. With the defined α_u in (2), the distortion power $P_{d,u}=\mathbb{E}\left[\left|x_{u,n}^C-\alpha_u x_{u,n}\right|^2\right]$ can be further written as $P_{d,u}=\mathbb{E}\left[\left|x_{u,n}^C\right|^2\right]-\alpha_u^2\mathbb{E}\left[\left|x_{u,n}\right|^2\right]$. The cumulative distribution function (CDF) of instantaneous power for SC-FDMA signals is given by [12]

$$F_p(z)=\frac{1}{L}\sum_{n=0}^{L-1}\sqrt{z}\int_0^\infty J_1(\sqrt{z}r)G(r;n)dr, \tag{3}$$

where $J_1(\cdot)$ is the first-kind Bessel function of the first order and $G(r;n)$ is a integral of characteristic function related to a specific modulation, which was derived in [12] and thus will not be repeated here. Since $p=R^2$, the CDF of the envelope R can be given by $F_R(z)=\Pr(R\leq z)=\Pr\left(R^2\leq z^2\right)=F_p\left(z^2\right)$. Then $P_{d,u}$ can be obtained with $\alpha_u=\frac{\int_0^\infty[f(z)+zf'(z)]\Gamma_R(z)dz}{\lambda_u M/N}$ and $\mathbb{E}\left[\left|x_{u,n}^C\right|^2\right]=2\int_0^\infty|f(z)||f(z)|'\Gamma_R(z)dz$, where $\Gamma_R(z)=1-F_R(z)$.

III. EVM Analysis

In this section, we derive the theoretical EVM to evaluate the impact of nonlinearity on OFDMA and SC-FDMA signals in uplink multiuser systems over multipath fading channels.

After removing the effects of CP and the N-point DFT, the received frequency domain (FD) signal is given by

$$\mathbf{Y}=\sum_{k=1}^{U}\mathbf{H}_k\mathbf{F}_N\mathbf{x}_k^C+\mathbf{V}, \tag{4}$$

where $\mathbf{H}_k$ is the $N\times N$ diagonal channel frequency response (CFR) matrix of the kth user and $\mathbf{V}$ is the $N{\times}1$ complex Gaussian noise vector with zero mean and variance matrix $\sigma^2\mathbf{I}_N$. In OFDMA and SC-FDMA systems, channel equalization is performed on per subcarrier basis with the zero-forcing (ZF) or the minimum mean square error (MMSE) principle [13]. Since the ZF equalizer suffers from noise enhancement especially in fading channels with severe frequency-selectivity, the MMSE one with better performance is mainly considered here.

For the uth user, the MMSE equalization is realized by multiplying the diagonal matrix $\mathbf{W}_u\triangleq\left(\mathbf{H}_u^H\mathbf{H}_u+\frac{1}{\gamma_u}\mathbf{I}_N\right)^{-1}\mathbf{H}_u^H$ to $\mathbf{Y}$ [13], where $\gamma_u\triangleq\lambda_u/\sigma^2$ is the signal-to-noise ratio (SNR). The M assigned subcarriers are selected with the matrix $\mathbf{T}_u^H$. Removing the precoding $\mathbf{P}$, the decision statistic is given by

$$\hat{\mathbf{s}}_u=\alpha_u\mathbf{G}_u\mathbf{s}_u+\underbrace{\mathbf{Q}_{uu}\mathbf{D}_u}_{\text{SDI}}+\underbrace{\sum_{k\neq u}\mathbf{Q}_{uk}\mathbf{D}_k}_{\text{IDI}}+\mathbf{P}^H\mathbf{T}_u^H\mathbf{W}_u\mathbf{V}, \tag{5}$$

where

$$\mathbf{G}_u=\mathbf{P}^H\mathbf{T}_u^H\mathbf{W}_u\mathbf{H}_u\mathbf{T}_u\mathbf{P},\mathbf{Q}_{uu}=\mathbf{P}^H\mathbf{T}_u^H\mathbf{W}_u\mathbf{H}_u,\mathbf{Q}_{uk}=\mathbf{P}^H\mathbf{T}_u^H\mathbf{W}_u\mathbf{H}_k \tag{6}$$

and $\mathbf{D}_u\triangleq\mathbf{F}_N\mathbf{d}_u$, $\mathbf{D}_k\triangleq\mathbf{F}_N\mathbf{d}_k$ are the FD nonlinear distortions.

Based on (5), the EVM with nonlinear distortion is given by [6], [7]

$$\mathrm{EVM}_u=\sqrt{\frac{\mathbb{E}[\boldsymbol{\Delta}_u^H\boldsymbol{\Delta}_u]}{\mathbb{E}[\mathbf{s}_u^H\mathbf{s}_u]}}=\sqrt{\frac{\mathbb{E}[\mathrm{Tr}(\boldsymbol{\Delta}_u\boldsymbol{\Delta}_u^H)]}{M\lambda_u}}, \tag{7}$$

where $\boldsymbol{\Delta}_u\triangleq\hat{\mathbf{s}}_u-\mathbf{s}_u$ is the symbol error vector. Assuming that the data for different users is independent of each other, the resulting nonlinear distortion is thus mutually independent [5]. Recalling that the distortion is uncorrelated with the input data, one has that distortion terms $\mathbf{D}_k(k=1,...,U)$ are independent of each other as well as the data and noise, then we write

$$\mathbb{E}[\mathrm{Tr}(\boldsymbol{\Delta}_u\boldsymbol{\Delta}_u^H)]=\lambda_u\alpha_u^2\underbrace{\mathbb{E}\left[\mathrm{Tr}\left(\mathbf{G}_u'\mathbf{G}_u'^H\right)\right]}_{t_1}+\underbrace{\mathbb{E}[\mathrm{Tr}(\mathbf{Q}_{uu}\mathbf{D}_u\mathbf{D}_u^H\mathbf{Q}_{uu}^H)]}_{t_2}$$
$$+\sum_{k\neq u}\underbrace{\mathbb{E}\left[\mathrm{Tr}(\mathbf{Q}_{uk}\mathbf{D}_k\mathbf{D}_k^H\mathbf{Q}_{uk}^H)\right]}_{t_3}+\sigma^2\underbrace{\mathbb{E}\left[\mathrm{Tr}(\mathbf{P}^H\mathbf{T}_u^H\mathbf{W}_u\mathbf{W}_u^H\mathbf{T}_u\mathbf{P})\right]}_{t_4}, \tag{8}$$

where $\mathrm{Tr}(\cdot)$ denotes the trace and $\mathbf{G}_u'\triangleq\mathbf{G}_u-\mathbf{I}_M$. The first term denoted as t_1 in (8) can be expressed as

$$t_1=M-2\Re\underbrace{\{\mathbb{E}\left[\mathrm{Tr}(\mathbf{G}_u)\right]\}}_{t_{1a}}+\underbrace{\mathbb{E}\left[\mathrm{Tr}\left(\mathbf{G}_u\mathbf{G}_u^H\right)\right]}_{t_{1b}}, \tag{9}$$

where $\Re(\cdot)$ denotes the real part of a complex number. In the next, we start with t_{1a} and obtain

$$\begin{aligned}t_{1a}&=\mathbb{E}\left[\mathrm{Tr}\left(\mathbf{P}^H\mathbf{T}_u^H\mathbf{W}_u\mathbf{H}_u\mathbf{T}_u\mathbf{P}\right)\right]\\&=\mathbb{E}\left[\mathrm{Tr}\left(\mathbf{W}_u\mathbf{H}_u\mathbf{T}_u\mathbf{P}\mathbf{P}^H\mathbf{T}_u^H\right)\right]=\mathbb{E}\left[\mathrm{Tr}\left(\mathbf{W}_u\mathbf{H}_u\tilde{\mathbf{T}}_u\right)\right],\end{aligned} \tag{10}$$

where $\tilde{\mathbf{T}}_u\triangleq\mathbf{T}_u\mathbf{T}_u^H$ is a diagonal matrix with nonzero elements "1"s only at indexes in $\mathcal{J}_u$. Denote h_{un} as the nth diagonal element of $\mathbf{H}_u$, then (10) can be rewritten as

$$t_{1a}=\sum_{n\in\mathcal{J}_u}\mathbb{E}\left[\frac{|h_{un}|^2}{|h_{un}|^2+\gamma_u^{-1}}\right]. \tag{11}$$

Consider a multipath Rayleigh fading channel whose CFR h_{un} can be viewed as zero-mean complex Gaussian with variance σ_h^2. Then $|h_{un}|^2$ is exponentially distributed with parameter $\theta=\mathbb{E}\left[|h_{un}|^2\right]=\sigma_h^2$ and (11) is given by

$$t_{1a}=\sum_{n\in\mathcal{J}_u}\int_0^\infty\frac{xe^{\frac{-x}{\theta}}/\theta}{x+\gamma_u^{-1}}dx=M\left[1+\rho_u^{-1}\exp\left(\rho_u^{-1}\right)\mathrm{Ei}\left(-\rho_u^{-1}\right)\right], \tag{12}$$

where $\rho_u\triangleq\theta\gamma_u$ stands for the effective SNR and $\mathrm{Ei}(x)=\int_{-\infty}^{x}\exp(\tau)/\tau d\tau$ denotes the exponential integral function [14]. Similarly, t_{1b} in (9) is written as

$$\begin{aligned}t_{1b}&=\mathbb{E}\left[\mathrm{Tr}\left(\mathbf{W}_u\mathbf{H}_u\mathbf{H}_u^H\mathbf{W}_u^H\tilde{\mathbf{T}}_u\right)\right]\\&=\sum_{n\in\mathcal{J}_u}\mathbb{E}\left[\frac{|h_{un}|^4}{\left(|h_{un}|^2+\gamma_u^{-1}\right)^2}\right]=\sum_{n\in\mathcal{J}_u}\int_0^\infty\frac{x^2e^{\frac{-x}{\theta}}/\theta}{\left(x+\gamma_u^{-1}\right)^2}dx\\&=M\left[1+\rho_u^{-1}+\rho_u^{-1}\left(\rho_u^{-1}+2\right)\exp\left(\rho_u^{-1}\right)\mathrm{Ei}\left(-\rho_u^{-1}\right)\right].\end{aligned} \tag{13}$$

[1]Numerical experiments with various settings of M, N and the input backoff (IBO) obtain similar observations (weak correlation in order of 10^{-3} or lower) and they are omitted here due to the lack of space.

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With (5), (6), t_2 in (8) related to the SDI is given by

$$t_2 = \mathbb{E}\left[\mathrm{Tr}\left(\mathbf{D}_u\mathbf{D}_u^H\mathbf{W}_u\mathbf{H}_u\mathbf{H}_u^H\mathbf{W}_u^H\tilde{\mathbf{T}}_u\right)\right] = \sum_{n\in\mathcal{J}_u}\mathbb{E}\left[\frac{|h_{un}|^4}{\left(|h_{un}|^2+\gamma_u^{-1}\right)^2}\right]\mathbb{E}\left[|D_{un}|^2\right] = \left[1+\rho_u^{-1}+\rho_u^{-1}\left(\rho_u^{-1}+2\right)\exp\left(\rho_u^{-1}\right)\mathrm{Ei}\left(-\rho_u^{-1}\right)\right]\sum_{n\in\mathcal{J}_u}S_{D,u}(n), \quad (14)$$

where $S_{D,u}(n)=\mathbb{E}\left[|D_{un}|^2\right]$ is the power spectral density (PS-D) of the distortion noise $d_{u,n}$. Similar to (14), we write the IDI-related term t_3 in (8) as

$$t_3 = \mathbb{E}\left[\mathrm{Tr}\left(\mathbf{D}_k\mathbf{D}_k^H\mathbf{W}_u\mathbf{H}_k\mathbf{H}_k^H\mathbf{W}_u^H\tilde{\mathbf{T}}_u\right)\right] = \sum_{n\in\mathcal{J}_u}\mathbb{E}\left[\frac{|h_{un}|^2}{\left(|h_{un}|^2+\gamma_u^{-1}\right)^2}\right]\mathbb{E}\left[|h_{kn}|^2\right]\mathbb{E}\left[|D_{kn}|^2\right] = -\left[1+\left(\rho_u^{-1}+1\right)\exp\left(\rho_u^{-1}\right)\mathrm{Ei}\left(-\rho_u^{-1}\right)\right]\sum_{n\in\mathcal{J}_u}S_{D,k}(n), \quad (15)$$

where $S_{D,k}(n)$ is the distortion PSD of the kth ($k \neq u$) user. Finally, the noise-related term t_4 in (8) is evaluated as

$$t_4 = \mathbb{E}\left[\mathrm{Tr}\left(\mathbf{W}_u\mathbf{W}_u^H\tilde{\mathbf{T}}_u\right)\right] = \sum_{n\in\mathcal{J}_u}\mathbb{E}\left[\frac{|h_{un}|^2}{\left(|h_{un}|^2+\gamma_u^{-1}\right)^2}\right] = -\frac{M}{\theta}\left[1+\left(\rho_u^{-1}+1\right)\exp\left(\rho_u^{-1}\right)\mathrm{Ei}\left(-\rho_u^{-1}\right)\right]. \quad (16)$$

Substituting (12), (13), (14), (15), (16), and (9) into (7), (8), the final expression of EVM for the nonlinearly distorted signal can be obtained as $\mathrm{EVM}_u=\sqrt{\beta_0}$, where

$$\beta_0 = \alpha_u^2\left[1-2\Re(\beta_1)+\beta_2\right]+\beta_3\rho_u^{-1} + \underbrace{\beta_2\sum_{n\in\mathcal{J}_u}S_{D,u}(n)/(M\lambda_u)}_{\text{SDI}} + \underbrace{\beta_3\sum_{k\neq u}\sum_{n\in\mathcal{J}_u}S_{D,k}(n)/(M\lambda_u)}_{\text{IDI}},$$
$$\beta_1 = 1+\rho_u^{-1}\exp\left(\rho_u^{-1}\right)\mathrm{Ei}\left(-\rho_u^{-1}\right),$$
$$\beta_2 = 1+\rho_u^{-1}+\rho_u^{-1}\left(\rho_u^{-1}+2\right)\exp\left(\rho_u^{-1}\right)\mathrm{Ei}\left(-\rho_u^{-1}\right),$$
$$\beta_3 = -\left[1+\left(\rho_u^{-1}+1\right)\exp\left(\rho_u^{-1}\right)\mathrm{Ei}\left(-\rho_u^{-1}\right)\right]. \quad (17)$$

The EVM in (17) involves no precoding $\mathbf{P}$ and can be used as a unified framework for both OFDMA and SC-FDMA signals to evaluate the SDI and IDI. It is also applicable to scenarios without nonlinear distortion by simply setting the distortion PSD to zero.[2] For OFDMA signals, the required PSD $S_{D,u}(n)$ can be obtained from the DFT of its autocorrelation given using the inter-modulation product (IMP) analysis by [4]

$$R_{d_u}(m) = \mathbb{E}\left[d_{u,n}^* d_{u,n+m}\right] = \sum_{\gamma=1}^{+\infty}2P_{u,2\gamma+1}\frac{\left[R_{x_u}(m)\right]^{\gamma+1}\left[R_{x_u}^*(m)\right]^{\gamma}}{\left[R_{x_u}(0)\right]^{2\gamma+1}}, \quad (18)$$

where the coefficient $P_{u,2\gamma+1}=\frac{\left|\int_0^\infty Rf(R)c_{2\gamma+1}\left(\frac{R}{\sqrt{\lambda_u}}\right)dR\right|^2}{2\gamma!(\gamma+1)!}$ denotes the power associated to the IMP of order $2\gamma+1$ with $c_{2\gamma+1}(x)=\frac{\gamma!}{2}e^{-x^2}xL_\gamma^{(1)}(x^2)$ and $L_\gamma^{(1)}(\cdot)$ denotes a generalized Laguerre polynomial of order γ [4]. With (1), the autocorrelation of $x_{u,n}$ is obtained as

$$R_{x_u}(m) = \mathbb{E}\left[x_{u,n}^* x_{u,n+m}\right] = \frac{\lambda_u}{N}e^{\frac{j\pi m(M-1+2f_u)}{N}}\frac{\sin\left(\frac{\pi Mm}{N}\right)}{\sin\left(\frac{\pi m}{N}\right)}. \quad (19)$$

From (2), (18), (19), we learn that α_u and the distortion PSD of OFDMA signals is independent of the modulation and results in a modulation-independent EVM in (17) for OFDMA.

For SC-FDMA systems, the Gaussian assumption is violated and the IMP analysis in (18) is not applicable, making it difficult to analytically quantify the distortion PSD. Thus for the kth ($k=1,...,U$) user, we propose the effective distortion ratio (EDR) defined as the average distortion PSD in the band of interest $\mathcal{J}_u$ to that in the whole band, i.e.,

$$\mu_k^{(u)} = \left(\sum_{n\in\mathcal{J}_u}S_{D,k}(n)/|\mathcal{J}_u|\right) / \left(\sum_{n=0}^{N-1}S_{D,k}(n)/N\right), \quad (20)$$

where $|\mathcal{J}_u|$ is the cardinality of $\mathcal{J}_u$. From Parseval's theorem, we write $P_{d,k}=\mathbb{E}\left[|d_{k,n}|^2\right]=\sum_{n=0}^{N-1}S_{D,k}(n)/N$. Then one has

$$\frac{\sum_{n\in\mathcal{J}_u}S_{D,u}(n)}{M} = \mu_u^{(u)}P_{d,u}, \quad \frac{\sum_{n\in\mathcal{J}_u}S_{D,k}(n)}{M} = \mu_k^{(u)}P_{d,k}. \quad (21)$$

The EVM of SC-FDMA signals in (17) is re-expressed as

$$\beta_0 = \alpha_u^2\left[1-2\Re(\beta_1)+\beta_2\right]+\beta_3\rho_u^{-1} + \underbrace{\beta_2\frac{\mu_u^{(u)}}{\lambda_u}P_{d,u}}_{\text{SDI}} + \underbrace{\beta_3\sum_{k\neq u}\frac{\mu_k^{(u)}}{\lambda_u}P_{d,k}}_{\text{IDI}}. \quad (22)$$

The EDR $\mu_k^{(u)}(k=1,...,U)$ is reasonable only when nonlinear distortion exists and can be obtained by simulations. Unlike analytical results for OFDMA, the EDR and the uncorrelation assumption of distortion with data are empirically obtained for SC-FDMA, thus the derived EVM is termed semi-analytical as in [9]. Nevertheless, good agreements confirmed by later simulations provide confidence that both the uncorrelation assumption and empirical estimates of $\mu_k^{(u)}$ are valid. Different from OFDMA, the EVM in (22) for SC-FDMA is influenced by the modulation since α_u is modulation-dependent.

Note that by replacing $\mathbf{W}_u$ with $\mathbf{H}_u^{-1}$, the EVM with ZF equalization can be similarly obtained. In addition, the derived EVM in (17) can be applied to the AWGN channel by simply assigning $\mathbf{H}_k = \mathbf{I}_N, k = 1,...,U$, and (17) is simplified with $\beta_1=\left(1+\gamma_u^{-1}\right)^{-1}, \beta_2=\beta_3=\beta_1^2$.

IV. Simulation Results

Simulations are conducted to verify the proposed theoretical EVM for OFDMA and SC-FDMA systems under 16-quadrature amplitude modulation (QAM) and 64-QAM. There are $U=3$ users with equal power and each occupies $M=128$ subcarriers from the total $N = 1024$ ones. The solid state

[2]Ideal channel state information (CSI) is assumed in the above derivation. For imperfect CSI, the CFR estimate can be written as $(\hat{h}_{un}=h_{un}+e) \sim \mathcal{CN}(0,\sigma_h^2+\sigma_e^2)$, where e is the estimation error which can be considered as zero-mean Gaussian with variance σ_e^2 [13]. Then $|\hat{h}_{un}|^2$ is still exponentially distributed with parameter $\theta=\mathbb{E}\left[|\hat{h}_{un}|^2\right]=\sigma_h^2+\sigma_e^2$. By assigning $\rho_u=\theta\gamma_u=(\sigma_h^2+\sigma_e^2)\gamma_u$ in (17), the EVM with imperfect CSI can thus be obtained.

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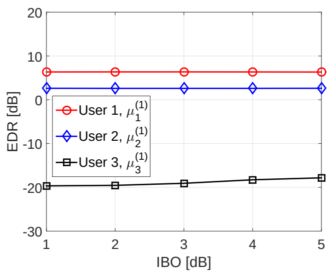

Fig. 1. The EDR $\mu_k^{(1)}(k = 1, 2, 3)$ in $\mathcal{J}_1$ for SC-FDMA systems with16-QAM.

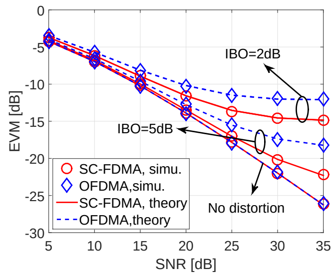

Fig. 2. EVM results under various IBOs for 16-QAM in the VA channel.

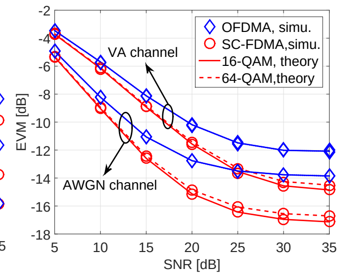

Fig. 3. EVM under various modulations in the AWGN and VA channel, IBO = 2dB.

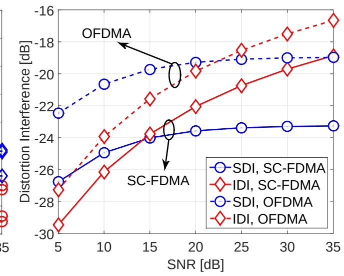

Fig. 4. Theoretical SDI and IDI for 16-QAM in the VA channel, IBO = 2dB.

power amplifier (SSPA) [15] with a smoothness factor $p = 3$ is used at the transmitter. The Vehicular-A (VA) channel [13] is adopted[3]. Without loss of generality, we take the first user whose band of interest $\mathcal{J}_1$ locates left most for example.

Fig. 1 shows the EDR in $\mathcal{J}_1$ for SC-FDMA systems with 16-QAM. We see that the EDR for the first user is much larger than others. This is because that the EDR is determined by the distortion power in $\mathcal{J}_1$ as defined in (20).The band $\mathcal{J}_1$ contains the mainlobe of the distortion PSD for the first user. While for others, only sidelobes lie in $\mathcal{J}_1$ and attenuate rapidly with the increasing distance to $\mathcal{J}_1$. It is also observed that the EDR for the third user is very small due to its weak distortion PSD in $\mathcal{J}_1$, indicating that the current user is mainly interfered by distortion from its adjacent users. For a specific IBO, the EDR only needs to be calculated once and then stored offline, leading to no big complexity increase.

Fig. 2 depicts the EVM results for both OFDMA and SC-FDMA signals with IBO = 2dB, 5dB and no distortion under 16-QAM and the VA channel. It is observed that simulation results perfectly match theoretical ones for all cases, validating our analytical results for OFDMA and the effectiveness of semi-analytical approaches used for SC-FDMA. A larger IBO results in a smaller EVM but leads to lower power efficiency of the PA, indicating that a proper IBO is required to balance the system performance and the PA efficiency. From Fig. 2, one also sees that for a certain IBO, SC-FDMA suffers from less nonlinearity than OFDMA due to its lower PAPR.

Fig. 3 justifies our theoretical analyses for different modulations in the AWGN and the VA channel with IBO = 2dB. Theoretical curves show good agreement with the simulation-based curves for both 16-QAM and 64-QAM either in the AWGN or the VA channel. The modulation-independency of EVM for OFDMA is observed while the EVM of SC-FDMA increases with the modulation order as previously analyzed. In addition, we can see that both OFDMA and SC-FDMA signals exhibit a larger EVM in the VA channel than the AWGN one due to the existence of channel frequency-selectivity.

Fig. 4 illustrates the theoretical SDI and IDI obtained from (17), (22) for 16-QAM in the VA channel with IBO = 2dB. It is seen that for both OFDMA and SC-FDMA signals, the SDI is larger than the IDI at low-to-moderate SNRs. While at high SNRs, the nonlinear interference from other users has a more significant impact. Therefore, mitigation of IDI from other users should be considered in the design of multiuser detection algorithm, especially at high SNRs.

[3]Ideal CSI with unitary power is used as in many literatures, i.e., $\sigma_h^2 = 1$.

V. Conclusion

Based on the distortion PSD, we obtained a unified expression of theoretical EVM for uplink multiuser OFDMA and SC-FDMA systems with nonlinear distortion. The derived results help to evaluate EVMs with nonlinearity in both AWGN and multipath fading channels and is confirmed by simulations. In addition, it reveals that distortion from other users also has a significant impact on the system performance and should be considered in multiuser detection, especially at high SNRs.

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