



# Theoretical study on tetragonal transition metal dinitrides from first principles calculations



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## ABSTRACT

Three new transition metal dinitrides TMN<sub>2</sub> (TM = Ta, W, and Re) with the *P4/mbm* structure are investigated by the first principles calculations method based on the density functional theory. The elastic constants and phonons calculations have confirmed that these three compounds are all mechanical and dynamically stable at ambient pressure. The distributions of elastic moduli of these dinitrides have been systematically studied and the obtained results indicate that the (001) plane may be viewed as the cleavage plane for TaN<sub>2</sub> and WN<sub>2</sub> as well as (100) plane for ReN<sub>2</sub>. Moreover, TMN<sub>2</sub> within this tetragonal structure are found to be ultra-incompressible and hard, among which WN<sub>2</sub> exhibits the largest bulk modulus (389 GPa) and Vickers hardness (38.5 GPa). Density of states calculation revealed that the strong TM–N covalent bonding is the driving force for the high bulk and shear modulus as well as small Poisson's ratio of the studied dinitrides.

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## 1. Introduction

Transition metal (TM) nitrides have attracted considerable attentions from both theoretical and experimental studies due to their extreme hardness and durability as well as the outstanding mechanical, electronic, magnetic and optoelectronic properties [1–3]. Most of the early transition metal mononitrides are well known hard materials. For example, TiN and CrN hard coatings are widely used in cutting tools industry [4]. Recently, great interest for transition metal nitrides has re-emerged based on the design concept for intrinsically superhard compounds that the interaction of lights elements (e.g., B, C, N, and O) into the transition metal lattices to form strong covalent bonds yet keeping a high valence-electron density and bulk moduli [5,6]. Taking advantage of high-pressure techniques, bulk TM<sub>3</sub>N<sub>4</sub> (TM = Zr, Hf) [3] with high elastic moduli and hardness has been obtained, opening new avenues for the synthesis of other TM nitrides. Subsequently, the platinum-metal (such as Pt, Ir, Os, and Pd) dinitrides were successfully synthesized under high pressure and high temperature (HPHT) [7–10]. The anomalously ultra-high incompressibility of these nitrides (428 GPa for IrN<sub>2</sub>), comparable to that of *c*-BN, suggests that they are potential (super)hard materials. These pioneering studies have stimulated considerable research enthusiasm for

5d TM nitrides. More recently, a hexagonal MoS<sub>2</sub>-type ReN<sub>2</sub> was synthesized by metathesis reaction using X-ray diffraction under high pressure by Kawamura et al. [11]. However, a following theoretical work [12] has revealed that this MoS<sub>2</sub>-ReN<sub>2</sub> is mechanical unstable and it is actually nitrogen-vacancies in MoS<sub>2</sub>-ReN<sub>2</sub>. Strikingly, theoretical calculations for this nitrogen-vacancies phase correctly reproduce the experimental X-ray diffraction pattern. Their further structural searching identifies a possible ground state monoclinic structure and a high-pressure tetragonal phase (space group: *P4/mbm*, *Z* = 2) for ReN<sub>2</sub>. Especially, this *P4/mbm*-ReN<sub>2</sub> exhibits an unusual incompressibility along the *c* axis, close to that of diamond.

Up to now, however, TMN<sub>2</sub> (TM = Hf, Ta, and W) have not been synthesized in crystalline form when compared to other 5d TMN<sub>2</sub> (TM = Os, Ir, and Pt) compounds. Therefore, many theoretical studies [13–23] have proposed a series of hypotheses on the structures and properties of TMN<sub>2</sub> (TM = Hf, Ta, and W). Recently, HfN<sub>2</sub> and TaN<sub>2</sub> within four different structures fluorite-type, pyrite-type, *P6<sub>3</sub>/mmc*, and *P6<sub>3</sub>m2* structures have been systematically studied [13–15], and these two compounds within pyrite-type structure are potential ultra-incompressible. For WN<sub>2</sub>, Wang et al. predicted [16] two types of hexagonal structures which are ultra-incompressible and are energetically superior to the previously proposed cotunnite phase [17]. Meanwhile, numerous theoretical studies have proposed different structures for ReN<sub>2</sub> [18–23], among which a tetragonal phase for ReN<sub>2</sub> suggested by Du et al. is potential superhard. Therefore, as a possible metastable phase, the

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$P4/mbm$ - $\text{ReN}_2$ -type structure mentioned above can be extended by those of Hf, Ta, and W which can provide further details are highly desirable. However, the calculated phonon dispersion curves in the present work indicate that only  $\text{TaN}_2$  and  $\text{WN}_2$  with the  $P4/mbm$  structure are dynamically stable. Accordingly, to explore such possibilities and provide guidance for future experimental efforts, here we perform first principles density functional theory (DFT) calculations to investigate the structural stability, mechanical properties, electronic properties, and chemical bonding of  $\text{TaN}_2$  and  $\text{WN}_2$  in comparison with  $\text{ReN}_2$ . The calculated results indicate that these dinitrides are mechanically stable and exhibit excellent mechanical properties. We hope our results can stimulate further experimental research to synthesize these ultra-incompressible and potential (super)hard compounds.

## 2. Computational methods

The DFT calculations have been performed within generalized gradient approximation (GGA) [24], as implemented in the Vienna *ab initio* simulation package (VASP) [25]. The electron and core interactions were included by using the frozen-core all-electron projector augmented wave (PAW) potential [26] of the metal atoms including  $d$  electrons as valence states. The integration in the Brillouin zone was employed using the Monkhorst–Pack scheme [27] with a grid of  $0.03 \text{ \AA}^{-1}$ , an energy cutoff of 800 eV, and a tetrahedron method with Blöchl corrections for the energy calculation and Gaussian smearing for the stress calculations. During the structural optimizations, all forces on atoms were converged to the order of  $0.001 \text{ eV/\AA}$ , and the external stresses were reduced to be less than 0.01 GPa. The phonon calculations were carried out by using a supercell approach as implemented in the PHONOPY code [28]. Single crystal elastic constants were calculated from evaluation of stress tensor generated small strain, and the bulk modulus, shear modulus, Young's modulus, and Poisson's ratio were thus derived from the Voigt–Reuss–Hill approximation [29].

## 3. Results and discussion

The considered  $P4/mbm$  crystal structure of  $\text{TMN}_2$  is shown in Fig. 1. It can be seen that this  $P4/mbm$  phase consists of a fundamental building block connected by edge along the  $c$ -axis: a tetragonal sublattice (solid line cell in Fig. 1) which can be viewed as a distorted CsCl-type structure. Table 1 lists the calculated lattice constants, equilibrium volumes, bond lengths, bulk moduli, and their pressure derivatives of  $\text{TMN}_2$  (TM = Ta, W, and Re) along with previous theoretical values of  $\text{ReN}_2$  [12]. The bulk moduli and their pressure derivatives are obtained by fitting pressures and cell volumes with the third-order Birch–Murnaghan equation of state (EOS) [30]. From this table, the calculated lattice parameters, equilibrium volumes and bond lengths of  $\text{ReN}_2$  are in good agreement with previous theoretical results. However, for other compounds of  $\text{TaN}_2$  and  $\text{WN}_2$ , there are no available experimental data and theoretical values for comparison. Therefore, the present results could provide useful information for further experimental or theoretical investigations. Moreover, as in the Birch–Murnaghan EOS treatment, we also obtained the values of equilibrium bulk moduli and their pressure derivatives for these three dinitrides presented in Table 1. It can be seen that these compounds possess a large value of bulk moduli which are comparable to those of synthesized platinum-metal dinitrides ( $\text{PtN}_2$ : 372 GPa [7],  $\text{OsN}_2$ : 358 GPa [9],  $\text{IrN}_2$ : 428 GPa [9]), but larger than previous proposed structures of  $\text{TaN}_2$  [13,14],  $\text{WN}_2$  [16], and  $\text{ReN}_2$  [18–20]. Thus, one might expect their excellent ultra-incompressibility. At zero-temperature a stable crystalline structure requires all phonon frequencies to be

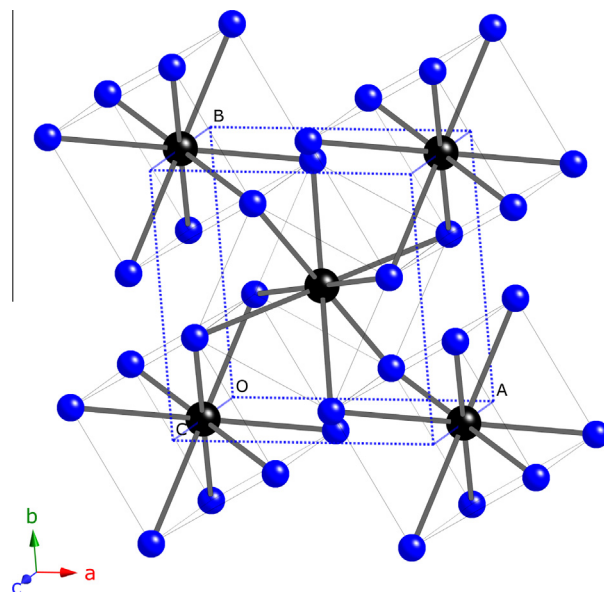


Fig. 1. Crystal structure of the tetragonal  $P4/mbm$  phase. The black and blue spheres represent TM and N atoms, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

positive. Therefore, we have performed the full phonon dispersion calculations for  $\text{TMN}_2$  (TM = Ta, W, and Re) at 0 GPa. As shown in Fig. 2, no imaginary phonon frequency was detected in the whole Brillouin zone, indicating the dynamical stability of the  $\text{TaN}_2$ ,  $\text{WN}_2$ , and  $\text{ReN}_2$ , respectively. It is important to explore the thermodynamic stability of  $\text{TMN}_2$  with  $P4/mbm$  structure for further experimental synthesis. The thermodynamic stability at ambient pressure with respect to decomposition is quantified in terms of the formation enthalpy, using  $\Delta H_f = H_{(\text{TMN}_2)} - H_{(\text{TM})} - H_{(\text{N}_2)}$ . The  $\Delta H_f$  is the formation enthalpy, the body-centered-cubic Ta and W (space group:  $Im-3m$ ), hexagonal Re (space group:  $P6_3/mmc$ ), and  $\alpha$ -phase  $\text{N}_2$  are chosen as the reference phases. The calculated  $\Delta H_f$  per formula unit (f.u.) of these compounds are listed in Table 1, together with other theoretical values for  $\text{WN}_2$ . Compared to the negative formation enthalpy values of previous candidates  $P6_3/mmc$  [16],  $P6m2$  [16], and  $P21/c$  [22], the  $P4/mbm$ - $\text{WN}_2$  is metastable with positive formation enthalpy comparable to that of  $P4/mmm$  phase proposed by Du et al. [20]. Therefore, high temperatures are needed to synthesize this  $P4/mbm$  structure for  $\text{WN}_2$  in actual experiment. Results obtained by this reaction route for  $\text{TaN}_2$ , on the contrary, have demonstrated its stability against the decomposition into the mixture of Ta and  $\text{N}_2$ , suggesting that it is most likely to be synthesized at ambient pressure.

In order to provide some insights into the pressure behavior of  $\text{TMN}_2$ , the pressure acting on the system as functions of lattice parameters ( $a/a_0$  and  $c/c_0$ ) and unit cell volume ( $V/V_0$ ) are plotted in Fig. 3, where  $a_0$ ,  $c_0$ , and  $V_0$  are the equilibrium structural parameters at zero pressure. The structural behaviors of synthesized hexagonal  $\text{Re}_2\text{N}$  and  $\text{Re}_3\text{N}$  as well as  $c$ -BN under pressure were also presented in Fig. 3 for comparison. Firstly, it can be seen that the incompressibility along the  $c$ -axis is larger than that along the  $a$ -axis for each  $\text{TMN}_2$  compound, suggesting their clear elastic anisotropy. The incompressibility of  $\text{WN}_2$  is almost identical to that of  $\text{ReN}_2$ , but larger than that of  $\text{TaN}_2$ . Secondly, both  $\text{WN}_2$  and  $\text{ReN}_2$  exhibit a larger  $a$ -axis incompressibility and similar volume incompressibility when compared to recently synthesized ultra-incompressible  $\text{Re}_2\text{N}$  and  $\text{Re}_3\text{N}$ . Thirdly, we notice that the volume incompressibility of  $\text{WN}_2$  and  $\text{ReN}_2$  exceeds that of  $c$ -BN at high pressure, although  $c$ -BN has the higher bulk at zero pressure. Take  $\text{WN}_2$  for example, by fitting obtained data with least squares meth-

**Table 1**  
Calculated equilibrium lattice constants  $a_0$  (Å),  $c_0$  (Å), equilibrium volume  $V_0$  (Å<sup>3</sup>), bond length (Å), EOS fitted bulk modulus  $B_0$  (GPa), its pressure derivative  $B'_0$ , and formation enthalpy  $\Delta H_f$  (eV/f.u.) for the  $P4/mbm$ -TMN<sub>2</sub>, respectively.

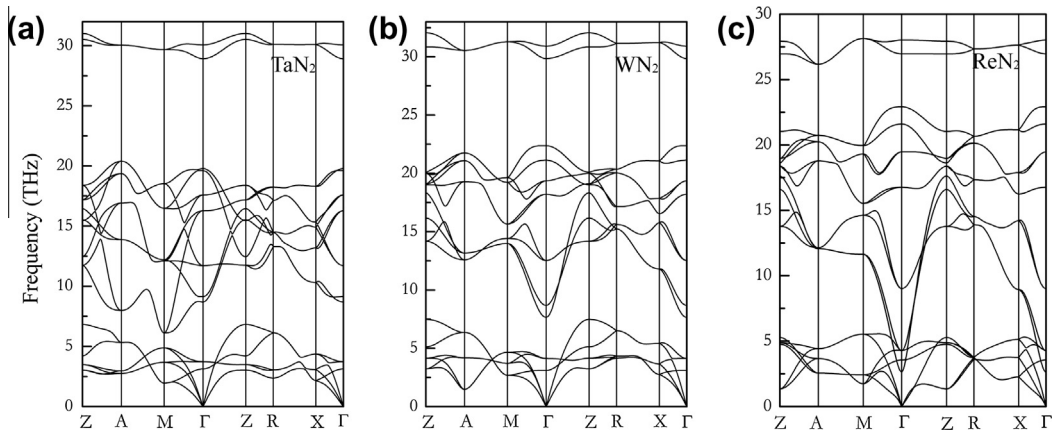
| TMN <sub>2</sub> | Space group  | $a_0$ | $b_0$ | $c_0$ | $V_0$  | $d_{TM-N}$ | $d_{N-N}$ | $B_0$ | $B'_0$ | $\Delta H_f$ |
|------------------|--------------|-------|-------|-------|--------|------------|-----------|-------|--------|--------------|
| TaN <sub>2</sub> | $P4/mbm$     | 4.403 |       | 2.839 | 55.041 | 2.272      | 1.410     | 343   | 4.477  | −0.951       |
| WN <sub>2</sub>  | $P4/mbm$     | 4.331 |       | 2.761 | 51.784 | 2.221      | 1.408     | 390   | 4.590  | 0.537        |
|                  | $P6_3/mmc^a$ | 2.934 |       | 7.790 | 58.07  | 2.102      | 1.405     |       |        | −0.942       |
|                  | $P6m2^a$     | 2.928 |       | 3.916 | 29.069 | 2.101      | 1.420     |       |        | −0.941       |
|                  | $P4/mmm^b$   | 2.71  |       | 7.38  | 54.20  | 2.23       | 1.41      | 378   | 4.58   | 0.74         |
|                  | $P21/c^c$    | 4.83  | 4.88  | 5.10  |        |            |           |       |        | −2.67        |
| ReN <sub>2</sub> | $P4/mbm$     | 4.390 |       | 2.663 | 51.330 | 2.211      | 1.424     | 387   | 4.766  | 1.938        |
|                  | $P4/mbm^d$   | 4.390 |       | 2.644 | 50.955 | 2.20       | 1.453     |       |        |              |

<sup>a</sup> Ref. [16].

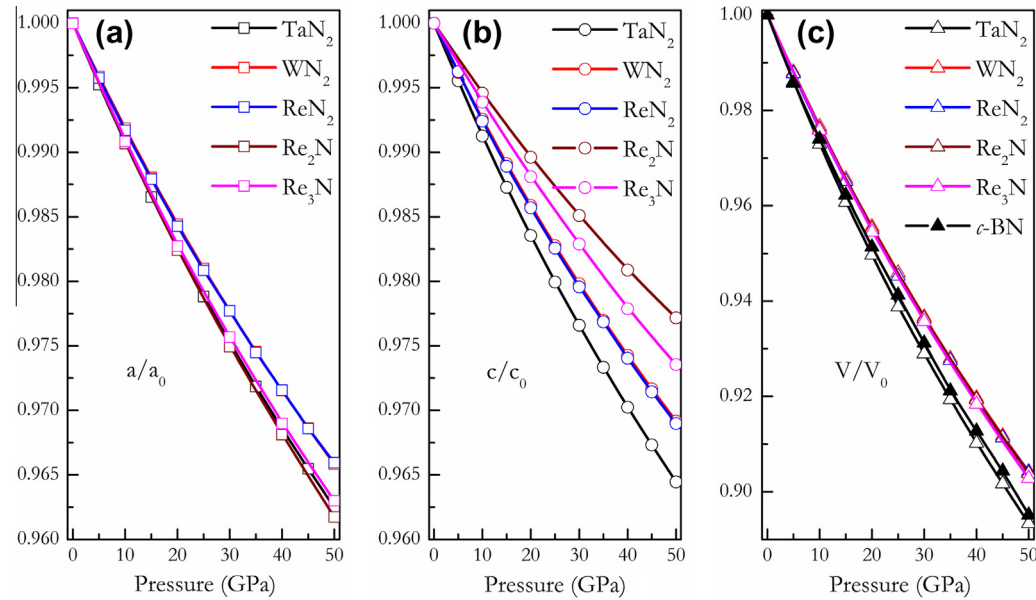
<sup>b</sup> Ref. [20].

<sup>c</sup> Ref. [22].

<sup>d</sup> Ref. [12].



**Fig. 2.** Phonon dispersion curves of TaN<sub>2</sub> (a), WN<sub>2</sub> (b), and ReN<sub>2</sub> (c) within  $P4/mbm$  structure at 0 GPa.



**Fig. 3.** Calculated  $a$ -axis compressions ( $a/a_0$ ) (a),  $c$ -axis compressions ( $c/c_0$ ) (b), and volume compressions ( $V/V_0$ ) (c) as a function of pressure TMN<sub>2</sub>, respectively.

od, we obtained their relationships at the temperature of 0 K as the following relations:

$$a/a_0 = 0.9995 - 8.36028 \times 10^{-4}P + 3.10152 \times 10^{-6}P^2 \quad (1)$$

$$c/c_0 = 0.99992 - 7.58873 \times 10^{-4}P + 2.90318 \times 10^{-6}P^2 \quad (2)$$

$$V/V_0 = 0.99981 - 2.41 \times 10^{-3}P + 1.00029 \times 10^{-5}P^2 \quad (3)$$

The mechanical properties (elastic constants and elastic moduli) are essential for understanding the macroscopic mechanical properties of solids and for the design of (super)hard materials and their potential technological applications. We calculated the zero-pressure elastic constants  $C_{ij}$  of these TMN<sub>2</sub> compounds by the strain–stress method. A small finite strain was applied on the optimized structure and the atomic positions were fully optimized. Then, the elastic constants were obtained from the stress of the strained structure. The calculated elastic constants  $C_{ij}$  are listed in Table 2, along with the previous theoretical values of the TMN<sub>2</sub> (TM = Ta, W, and Re) [12–13,16,18–20,22]. It is well known that the elastic stability is a necessary condition for a stable crystal. A tetragonal crystal has to obey the following restrictions of its elastic constants [31]:  $C_{11} > 0$ ,  $C_{33} > 0$ ,  $C_{44} > 0$ ,  $C_{66} > 0$ ,  $(C_{11} - C_{12}) > 0$ ,  $(C_{11} + C_{33} - 2C_{13}) > 0$ , and  $2(C_{11} + C_{12}) + C_{33} + 4C_{13} > 0$ . As shown in Table I, these conditions are clearly satisfied for tetragonal stability, confirming that these compounds are mechanically stable. It also can be seen that these compounds possess a remarkable value of  $C_{33}$ , especially for WN<sub>2</sub> (894 GPa), which is comparable to those of P6<sub>3</sub>/mmc and P6m2 phases and smaller than that of diamond (1079 GPa) [32]. These results indicate that their *c*-axis directions are extremely stiff. Elastic constant  $C_{44}$  is an important parameter of the material's indentation hardness. Among the three compounds within P4/mbm structure and previous proposed candidates, the  $C_{44}$  of WN<sub>2</sub> (275 GPa) is the largest one which is close to that of superhard material ReB<sub>2</sub> (276 GPa) [33], indicating that WN<sub>2</sub> has the strong resistance to shear stress. The shear anisotropic factors of the tetragonal TaN<sub>2</sub>/WN<sub>2</sub>/ReN<sub>2</sub> deduced from the present results are  $A_1 = 4C_{44}/(C_{11} + C_{33} - 2C_{13}) = 0.70/0.82/0.75$  within (100) planes between  $\langle 011 \rangle$  and  $\langle 0\bar{1}0 \rangle$  directions,  $A_2 = 2C_{66}/(C_{11} - C_{12}) = 0.97/1.03/0.60$  within (001) shear planes between  $\langle 110 \rangle$  and  $\langle 010 \rangle$  directions, indicating a certain degree of elastic anisotropy.

Based on the calculated  $C_{ij}$ , the corresponding six independent elastic compliance constants  $s_{ij}$  of the tetragonal crystal given by Sirdeshmukh were determined [34]. For engineering applications that make use of single crystals, it is necessary to know the values of bulk modulus, Young's modulus, and shear modulus as a function of crystal orientation. As outlined by He et al. [35], executing the appropriate coordinate system transformations for the compliances allows the determination of the variation of Young's moduli  $E$  and shear modulus  $G$  with crystallographic direction,  $[uvw]$ , for a

given crystallographic plane,  $(hkl)$ , containing these directions, i.e.,  $E_{[uvw]}$  and  $G_{(hkl)[uvw]}$  are obtained. For P4/mbm crystal structure, the Young's modulus  $E$  can be expressed as:

$$E^{-1} = s_{11}(\alpha^4 + \beta^4) + s_{33}\gamma^4 + 2s_{12}\alpha^2\beta^2 + 2s_{13}(\beta^2\gamma^2 + \alpha^2\gamma^2) + s_{44}(\beta^2\gamma^2 + \alpha^2\gamma^2) + s_{66}\alpha^2\beta^2 \quad (4)$$

where  $\alpha$ ,  $\beta$ , and  $\gamma$  are the direction cosines of  $[uvw]$  direction. The three-dimensional surface representations showing the variation of Young's modulus are plotted in Fig. 4. For an isotropic system, one would see a spherical shape. The degree of elastic anisotropy in a system can be directly reflected from the degree of deviation in shape from a sphere. As shown in Fig. 4, all the TMN<sub>2</sub> compounds exhibit the elastic anisotropy since the shape of the Young's modulus representation deviates from a spherical shape, especially for ReN<sub>2</sub> (Fig. 4c). For more detail in Fig. 5, the variation of Young's modulus in the (001) plane for the quadrant of directions  $[uvw]$  between  $[100]$  ( $\theta = 0^\circ$ ) and  $[010]$  ( $\theta = 90^\circ$ ). It is clearly seen that Young's moduli of TaN<sub>2</sub> and WN<sub>2</sub> exhibit little change on the whole, on the contrary, ReN<sub>2</sub> possesses a maximum of  $E_{[100]} = E_{[010]} = 738$  GPa and a minimum of  $E_{[110]} = 533$  GPa. For the (100) plane in Fig. 5 displays the variation of Young's modulus for directions  $[0vw]$  between  $[001]$  ( $\theta = 0^\circ$ ) and  $[010]$  ( $\theta = 90^\circ$ ), the variation tendencies of Young's moduli for TaN<sub>2</sub> and WN<sub>2</sub> are similar among three compounds, with  $E_{[001]} = 735/838/764$  GPa and  $E_{[011]} = 523/689/630$  GPa for TaN<sub>2</sub>/WN<sub>2</sub>/ReN<sub>2</sub>. For  $(1\bar{1}0)$  plane, Young's moduli  $E$  of TaN<sub>2</sub> and WN<sub>2</sub> behave again very similar for the quadrant of directions  $[uuw]$  between  $[001]$  and  $[110]$ , and the values of  $E_{[111]}$  are 520 GPa and 691 GPa for TaN<sub>2</sub> and WN<sub>2</sub>, respectively. The Young's moduli  $E$  of WN<sub>2</sub> decrease quickly from  $[001]$  to  $[010]$  directions within  $(1\bar{1}0)$  plane.

To understand plastic deformations in TMN<sub>2</sub>, the dependence of the shear modulus on stress direction is also plotted in Fig. 6. The shear modulus  $G$  on the  $(hkl)$  shear plane with shear stress applied along  $[uvw]$  direction is given by:

$$G^{-1} = 4s_{11}(\alpha_1^2\alpha_2^2 + \beta_1^2\beta_2^2) + 4s_{33}\gamma_1^2\gamma_2^2 + 8s_{12}\alpha_1\alpha_2\beta_1\beta_2 + s_{66}(\alpha_1\beta_2 + \alpha_2\beta_1)^2 + 8s_{13}(\beta_1\beta_2\gamma_1\gamma_2 + \alpha_1\alpha_2\gamma_1\gamma_2) + s_{44}[(\beta_1\gamma_2 + \beta_2\gamma_1)^2 + (\alpha_1\gamma_2 + \alpha_2\gamma_1)^2] \quad (5)$$

where  $\alpha_1$ ,  $\beta_1$ ,  $\gamma_1$ ,  $\alpha_2$ ,  $\beta_2$ ,  $\gamma_2$  are the direction cosines of the  $[uvw]$  and  $[HKL]$  directions in the coordinate systems, where the  $[HKL]$  denotes

**Table 2**

Calculated elastic constants  $C_{ij}$  (in GPa), isotropic bulk modulus  $B_H$ , shear modulus  $G_H$ , Young's modulus  $E_H$ , and the hardness  $H_v$  in unit of GPa for the P4/mbm-TMN<sub>2</sub>. Also shown are Poisson's ratio  $\nu_H$  and  $B_H/G_H$  ratio.

| Crystal          | Phases                            | $C_{11}$ | $C_{22}$ | $C_{33}$ | $C_{44}$ | $C_{55}$ | $C_{66}$ | $C_{12}$ | $C_{13}$ | $C_{23}$ | $B_H$ | $G_H$ | $E_H$ | $\nu_H$ | $B_H/G_H$ | $H_v$ |
|------------------|-----------------------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|-------|-------|-------|---------|-----------|-------|
| TaN <sub>2</sub> | P4/mbm                            | 651      |          | 771      | 204      |          | 205      | 228      | 125      |          | 340   | 225   | 552   | 0.2296  | 1.51      | 26.3  |
|                  | Pyrite <sup>a</sup>               | 255      |          |          | 82       |          |          | 322      |          |          | 299   | 37    |       |         |           |       |
| WN <sub>2</sub>  | P4/mbm                            | 785      |          | 894      | 275      |          | 308      | 188      | 165      |          | 389   | 300   | 716   | 0.1953  | 1.33      | 38.5  |
|                  | P6 <sub>3</sub> /mmc <sup>b</sup> | 579      |          | 973      | 233      |          |          | 195      | 211      |          | 364   | 228   |       |         |           |       |
|                  | P6m2 <sup>b</sup>                 | 588      |          | 973      | 232      |          |          | 191      | 206      |          | 364   | 231   |       |         |           |       |
|                  | P4/mmm <sup>c</sup>               | 853      |          | 861      | 203      |          | 276      | 122      | 147      |          | 378   | 295   |       | 0.19    |           |       |
|                  | P21/c <sup>d</sup>                | 537      | 609      | 562      | 137      | 131      | 239      | 214      | 201      | 223      | 332   | 173   |       |         |           |       |
| ReN <sub>2</sub> | P4/mbm                            | 795      |          | 829      | 240      |          | 191      | 160      | 176      |          | 375   | 259   | 632   | 0.2192  | 1.45      | 30.5  |
|                  | P4/mbm <sup>e</sup>               | 820      |          | 967      | 258      |          | 252      | 165      | 111      |          | 374   | 297   |       |         |           |       |
|                  | P4/mmm <sup>c</sup>               | 881      |          | 936      | 212      |          | 162      | 134      | 116      |          | 378   | 295   |       | 0.19    |           |       |
|                  | Pbcn <sup>f</sup>                 | 365      | 553      | 610      | 138      | 230      | 83       | 307      | 232      | 242      | 334   | 127   | 337   | 0.3319  |           |       |
|                  | Pmmn <sup>g</sup>                 | 788      | 889      | 573      | 131      | 339      | 235      | 125      | 308      | 76       | 358   | 238   | 584   | 0.23    |           |       |

<sup>a</sup> Ref. [13].

<sup>b</sup> Ref. [16].

<sup>c</sup> Ref. [20].

<sup>d</sup> Ref. [22].

<sup>e</sup> Ref. [12].

<sup>f</sup> Ref. [19].

<sup>g</sup> Ref. [18].

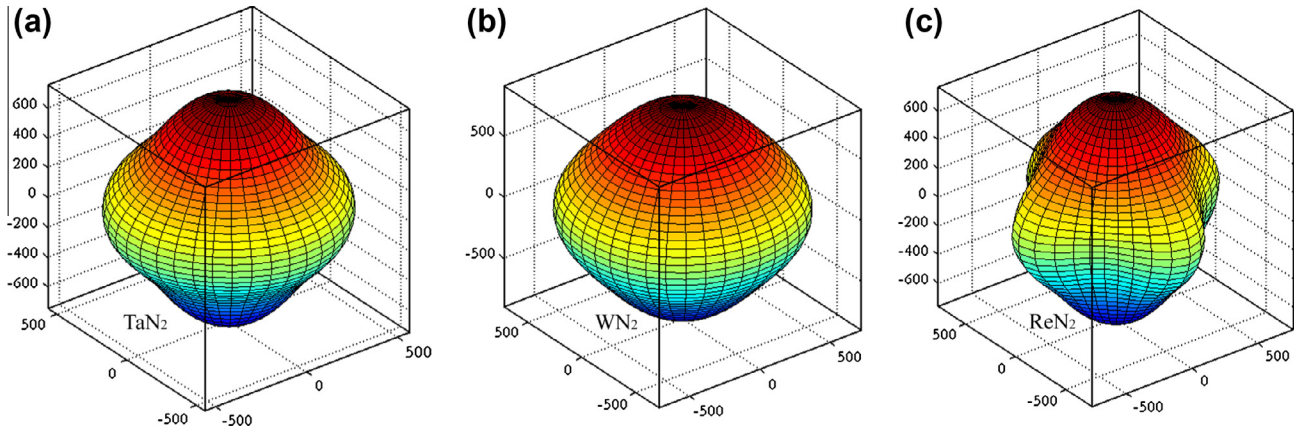


Fig. 4. Three-dimensional surface representations of the Young's modulus for TaN<sub>2</sub> (a), WN<sub>2</sub> (b), and ReN<sub>2</sub> (c).

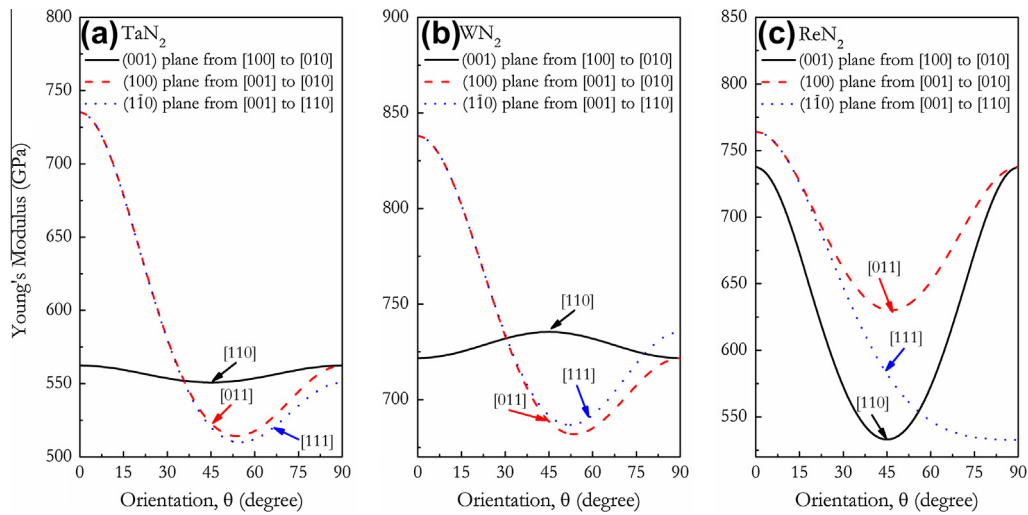


Fig. 5. Orientation dependence of the Young's modulus for TaN<sub>2</sub> (a), WN<sub>2</sub> (b), and ReN<sub>2</sub> (c).

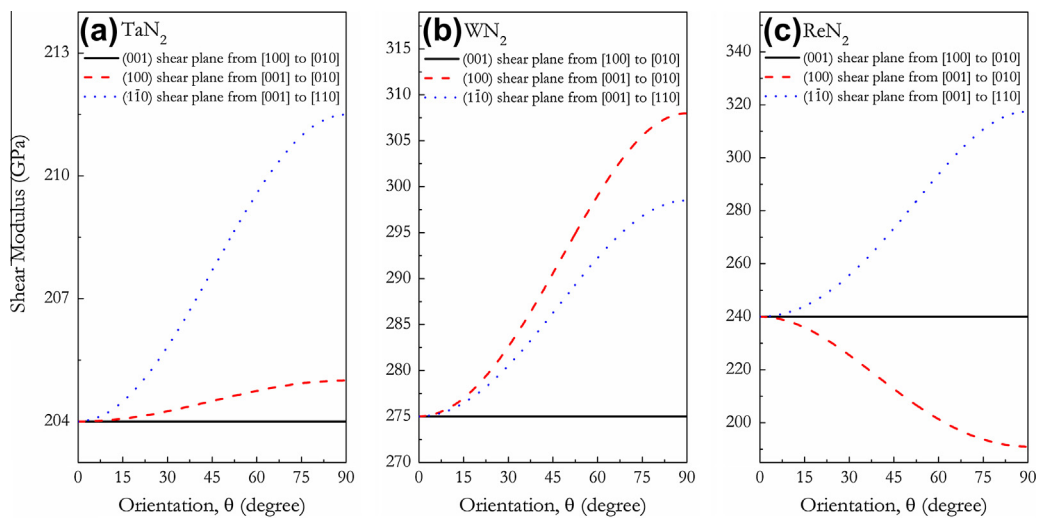


Fig. 6. Orientation dependence of the Shear modulus for TaN<sub>2</sub> (a), WN<sub>2</sub> (b), and ReN<sub>2</sub> (c).

the vector normal to the (*hkl*) shear plane. For shear plane (001) with the shear stress direction rotated from [100] to [010], the direction cosines are  $\alpha_1 = \cos\theta$ ,  $\beta_1 = \sin\theta$ ,  $\gamma_1 = 0$ ,  $\alpha_2 = \beta_2 = 0$ , and

$\gamma_2 = 1$ , where  $\theta$  is the angle between the [100] and the shear stress direction. By solving Eq. (5), the  $G = 1/s_{44} = C_{44}$ , which means that within (001) plane, the shear modulus of the TMN<sub>2</sub> is independent

of the shear stress direction. For shear plane (100) with the shear stress directions  $[0vw]$  varying from  $[100]$  to  $[010]$ , the direction cosines are  $\alpha_1 = 0$ ,  $\beta_1 = \sin\theta$ ,  $\gamma_1 = \cos\theta$ ,  $\alpha_2 = 1$ ,  $\beta_2 = \gamma_2 = 0$ , and  $G^{-1} = s_{66} + (s_{44} - s_{66}) \cos^2 \theta$ . Because  $s_{44} > s_{66}$  in  $\text{TaN}_2$  and  $\text{WN}_2$ , their shear moduli are the largest along  $[010]$  and the smallest along  $[001]$ . Contrarily,  $\text{ReN}_2$  possesses a maximum of  $E_{[001]}$  and a minimum of  $E_{[010]}$  due to  $s_{44} < s_{66}$ . For shear plane  $(1\bar{1}0)$  with the shear stress directions  $[uvw]$  between  $[001]$  and  $[110]$  in Fig. 6, the variation tendencies of shear moduli are similar. From Fig. 6, it is clear that the shear moduli are the smallest on the (100) plane for  $\text{TaN}_2$ , (100) plane for  $\text{WN}_2$ , and (100) plane for  $\text{ReN}_2$ , respectively. This means that the (001) plane may be viewed as the cleavage plane for  $\text{TaN}_2$  and  $\text{WN}_2$  as well as (100) plane for  $\text{ReN}_2$ .

Based on the calculated elastic constants, the isotropic bulk modulus ( $B_H$ ) and shear modulus ( $G_H$ ) for the  $\text{TMN}_2$  polycrystalline are calculated by the Voigt–Reuss–Hill approximation [29] in Table 2. The Young's modulus  $E_H$  and Poisson's ratio  $\nu_H$  are obtained from the equations of  $E_H = 9B_H G_H / (3B_H + G_H)$  and  $\nu_H = (3B_H - 2G_H) / (6B_H + 2G_H)$ . The calculated bulk moduli of the  $\text{TMN}_2$  are above 340 GPa, indicating ultra-incompressible structural nature. Strikingly, it should be noted that the calculated bulk moduli agree well with those directly obtained from the fitting of the third-order Birch–Murnaghan equation of state (EOS) (see Table 1), which further demonstrates the reliability of our elastic calculations. A material with high bulk modulus is not enough to ensure its high hardness. For the partially covalent transition metal-based materials, the shear modulus of a material quantifies its resistance to the shear deformation and is a better indicator of potential hardness. The value of Poisson's ratio is indicative of the degree of directionality of the covalent bonding, and small Poisson's ratio indicates a strong degree of covalent bonding, which contributes to the materials' hardness. In Table 2, the  $\text{WN}_2$  is found to have the highest shear modulus (300 GPa) and lowest Poisson's ratio (0.195), which are two important elastic properties thought to be strongly correlated to hardness. Indeed, the estimated Vickers hardness ( $H_v$ ) for  $\text{WN}_2$  is 38.5 GPa, the largest one among the studied  $\text{TMN}_2$  compounds based on the empirical formula for hardness prediction proposed by Chen et al. [36]. Future synthesis of this nitride is thus of great interest and important for utility of this excellent mechanical property. In addition, the ratio between the bulk and the shear modulus  $B/G$  are used to predict the brittle or ductile behavior of materials. According to the Pugh criterion [37], the ductile behavior is predicted when  $B/G > 1.75$ , otherwise the material would fail in a brittle manner. It can be clearly seen in Table 2 that all these  $\text{TMN}_2$  compounds are strongly prone to brittle manner.

We have also obtained other fundamental properties using above calculated mechanical quantities. The Debye temperature

closely relates to many physical properties of solids, such as specific, dynamic properties, and melting temperature. At low temperature, it can be calculated from the elastic constants using the average sound velocity  $v_m$ , by the following equation [38],

$$\Theta_D = \frac{h}{k} \left[ \frac{3n}{4\pi} \left( \frac{\rho N_A}{M} \right) \right]^{\frac{1}{3}} v_m \quad (6)$$

where  $h$  is Planck's constant,  $k$  is Boltzmann's constant,  $N_A$  is Avogadro's number,  $n$  is the number of atoms per formula unit,  $M$  is the molecular mass per formula unit, and  $\rho$  is the density. The average sound velocity  $v_m$  is given by

$$v_m = \left[ \frac{1}{3} \left( \frac{2}{v_t^3} + \frac{1}{v_l^3} \right) \right]^{-\frac{1}{3}} \quad (7)$$

here  $v_t$  and  $v_l$  are the transverse and longitudinal elastic wave velocities of the polycrystalline materials, which can be obtained using the polycrystalline bulk modulus and shear modulus from Navier's equation [38]. The calculated Debye temperatures of these  $\text{TMN}_2$  compounds increase in the following sequence:  $\text{TaN}_2$  (666 K) <  $\text{ReN}_2$  (697 K) <  $\text{WN}_2$  (753 K), which is the same as the trend in estimated hardness of  $\text{TMN}_2$  listed Table 2. This can be naturally explained by the empirical linear relationship between hardness and Debye temperature proposed by Abrahams and Hsu [39].

To illustrate the mechanical properties on a fundamental level, the total and site projected electronic densities of states (DOS) of  $\text{TMN}_2$  (TM = Ta, W, and Re) at 0 GPa were plotted in Fig. 7, where the vertical dot line is the Fermi level. Clearly, all these structures exhibit metallic behavior by evidence of the finite electronic DOS at the Fermi level ( $E_F$ ). From the partial DOS, it reveals that the TM-5d orbital has a significant hybridization with N-2p orbital localized in the energy range from  $-10$  eV to 0 eV, indicating the strong TM–N covalent bonding nature. Therefore, the strong covalent Ta–N bond is the main driving force for their high bulk and shear moduli as well as small Poisson's ratio. In addition, the contribution to Fermi level is mainly due to TM-5d orbital which is the principal cause for their metallicity. The typical feature of the total DOS is the presence of so-called pseudogap [40], which is considered as the borderline between the bonding and antibonding states. It can be seen that the  $E_F$  of  $\text{WN}_2$  and  $\text{ReN}_2$  nearly lying on the pseudogap, respectively, indicating the  $p$ - $d$  bonding states started to be saturated. This nearly full occupation of the bonding states enhances the stability of  $\text{WN}_2$  and  $\text{ReN}_2$ . We also performed the Mulliken population analysis of these nitrides and found a charge transfer from TM to N, implying an ionic contribution to the TM–N bonding. We thus conclude that the chemical bonding in these molybdenum borides is a complex mixture of covalent, ionic, and metallic

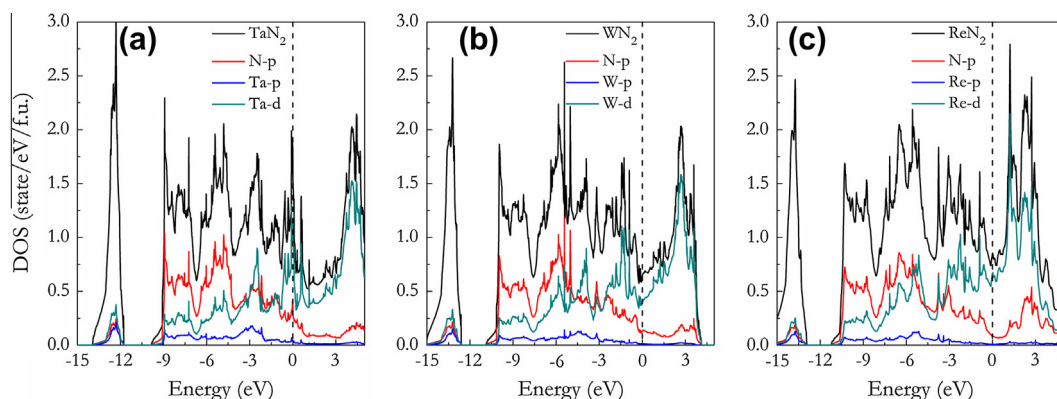


Fig. 7. Total and partial densities of states of  $\text{TaN}_2$  (a),  $\text{WN}_2$  (b), and  $\text{ReN}_2$  (c) at 0 GPa. The vertical dashed line is the Fermi energy.

characters. Such a conclusion was also found in other transition metal nitrides.

#### 4. Conclusions

In summary, by means of density functional theory calculations, we have systematically study the structural stability, mechanical properties, electronic structure and chemical bonding of TMN<sub>2</sub> (TM = Ta, W, and Re) within *P4/mbm*-ReN<sub>2</sub> structure. The elastic constants and phonons calculations show that these three compounds are all mechanical and dynamically stable. The Young's modulus and shear modulus as a function of crystal orientation for the TMN<sub>2</sub> were systematically investigated. The results indicate that the (001) plane may be viewed as the cleavage plane for TaN<sub>2</sub> and WN<sub>2</sub> as well as (100) plane for ReN<sub>2</sub>. The calculated bulk moduli and hardness suggested that these nitrides are ultra-incompressible and hard materials. The electronic densities of states analysis have demonstrated that the strong covalent TM–N bonding plays a key role in the ultra-incompressibility and hardness of the TMN<sub>2</sub>. These findings will inevitably stimulate extensive experimental works on synthesizing these technologically important materials.

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