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k-Connectivity Analysis of One-Dimensional Linear VANETs

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***Abstract*—In a 1-D linear vehicular ad hoc network (1-DL-VANET), some vehicles may leave the network (e.g., at highway exits), which may make the 1-DL-VANET disconnected. Thus, it is important to analyze the connectivity of the 1-DL-VANET. When removal of any $(k-1)$ arbitrary nodes from a network does not disconnect the network, the network is said to be k-connected. In this paper, we investigate the k-connectivity of the 1-DL-VANET. Sufficient and necessary conditions are derived for the 1-DL-VANET to be k-connected, and based on this, a method is provided, with the help of matrix decomposition, to obtain expression of the probability of the 1-DL-VANET being k-connected. The expectation of the maximum number of tolerable vehicle departures is also derived. Simulation results confirm the accuracy of our analysis and indicate that the expectation of the maximum number of tolerable vehicle departures almost linearly increases with the total number of vehicles.**

***Index Terms*—k-connectivity, 1-D linear networks, vehicular ad hoc networks (VANETs).**

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I. Introduction

A vehicular ad hoc network (VANET) consists of a group of moving vehicles and probably a fixed infrastructure (such as roadside units), supporting intervehicle communications, and vehicle-to-infrastructure communications. Typical information transmitted in a VANET includes safety messages (such as accident notifications, road condition warnings, and emergency braking alarms) and interactive communications (such as instant message and online games). In this work, we consider intervehicle communications among a group of vehicles on a highway. Considering that the road width of highways is usually much smaller than the wireless transmission range and that the curves on highways are usually not sharp, we can approximately model the group of vehicles as a 1-D linear VANET (1-DL-VANET). A similar model is adopted in [1] and [2]. Here, we consider only intervehicle communications, and thus, roadside units are not involved. In a 1-DL-VANET, some vehicles may leave or quit the current network, e.g., due to arriving at their exits on the highway or due to mechanical faults. Upon departures of those vehicles, it is desired that any two remaining vehicles can still communicate with each other. In other words, the high connectivity level of the 1-DL-VANET is desired. In specific, if there exists a one- or multiple-hop communication path between any two nodes in a network, the network is said to be *connected*; otherwise, the network is said to be *unconnected* or *disconnected*. A network is called k-*connected* if removal of any $(k-1)$ arbitrary nodes does not disconnect the network [3]. In particular, biconnectivity means that $k=2$, which is a popular connectivity measure [4]–[8].

In the literature, connectivity has been well investigated for ad hoc networks. Existing research efforts are focused on how to achieve connectivity or biconnectivity by the following: 1) setting the wireless transmission range or node density or 2) changing the network topology by adjusting transmission power or by node movement inside the network. On the other hand, research on the expression of the probability of a network being connected, biconnected, or k-connected is still in its infancy. Current limited research efforts are focused on the probability of a 1-D network being connected or biconnected.

1) The probability of a 1-D network being connected is investigated in [9]–[12]. In [9], by considering all realizable networks as in a polytope, the probability of a network being connected is derived in closed form. In [10], the probability of a network consisting of at most $C(\geq 1)$ clusters is first calculated, which is equal to the probability that the Cth largest spacing (the distance between consecutive vehicles) is smaller than the wireless communication range denoted R. In particular, when $C=1$, the probability of all spacings smaller than R is actually the probability that the network is connected. A queuing model is utilized in [11] to analyze the connectivity of 1-D networks. The exact results of the coverage probability, the node isolation probability, and the connectivity distance for several node placements are obtained. In [12], asymptotic analysis for 1-D network connectivity is obtained. It is concluded that, as the number of nodes goes to infinity, the probability of the network being connected is approximately 1 when the wireless transmission range (i.e., R) is larger than a threshold and 0 when R is smaller than that threshold.
2) In [13], the probability of a 1-D network being biconnected is approximated as the product of the probabilities of two events: the network is connected, and there are no cut nodes. (If the removal of a node makes the remaining network disconnected, the node is called a *cut node.*) The independence of the two events is validated through simulation.

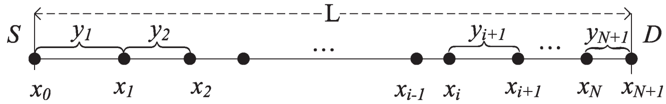

Fig. 1. System model for a 1-DL-VANET.

To the best knowledge of the authors, there is no research in the literature to investigate the probability of a network being k-connected.

In this paper, we investigate the k-connectivity of a 1-DL-VANET. In particular, we find an insightful sufficient and necessary condition for the 1-DL-VANET to be k-connected, i.e., the sum of any k consecutive spacings is less than the communication range. Based on the sufficient and necessary condition, a method is provided to derive the probability of the 1-DL-VANET being k-connected. The expectation of the maximum number of tolerable vehicle departures is also derived. Simulation results validate our analysis. It is also shown that the expectation of the maximum number of tolerable vehicle departures in the 1-DL-VANET almost linearly increases with the total number of vehicles.

The rest of this paper is organized as follows: Section II describes the system model. Section III gives sufficient and necessary conditions for a 1-DL-VANET to be k-connected, and Section IV derives the probability of a 1-DL-VANET to be k-connected and the expectation of the maximal number of tolerable vehicle departures. Section V is devoted to performance evaluation, followed by concluding remarks in Section VI.

II. System Model

We consider a snapshot of the 1-DL-VANET, as shown in Fig. 1, where one source node S and one destination node D are located at the ends of a finite length of road, e.g., $[0, L]$. Here, S and D can be the first and last vehicles of a 1-DL-VANET on a highway. In the sequel, the terms "vehicle" and "node" are interchangeably used. Empirical measurement has shown that intervehicle spacings can be modeled as exponentially distributed i.i.d. random variables [14]. Then, according to [15], the positions of the N vehicles between S and D have the same distribution as the order statistics[1] corresponding to N i.i.d. uniform random variables over $[0, L]$. Therefore, in the following, we assume that there are N nodes in our 1-DL-VANET over $[0, L]$ between S and D, and these N nodes are uniformly distributed over $[0, L]$. The indexes of the N nodes are denoted according to the positions of the nodes in $[0, L]$, which are denoted as $\mathcal{S} = \{1, 2, \ldots, N\}$. The communication range of S, D, and all nodes in $\mathcal{S}$ is R. Let x_i $(i = 1, \ldots, N)$ denote the position of the ith node. For consistency, we define S's location to be $x_0 = 0$ and D's location to be $x_{N+1} = L$. Let $y_j = x_j - x_{j-1}$ $(j = 1, \ldots, N+1)$ be the distance between two consecutive nodes, which is referred to as *spacing*.

We have four definitions.

1) *Neighbor:* Two nodes u and v are said to be neighbors of each other if their distance is less than R.
2) *Path:* A path is defined as a one-hop or multihop communication link from a source node to its destination node. We use the vector of nodes along a path to denote the path, where any two consecutive nodes in the vector are neighbors. For example, vector $(a_1, a_2, \ldots, a_l)$ denotes the path $a_1 \rightarrow a_2 \rightarrow \cdots \rightarrow a_l$, in which nodes a_i and a_{i+1} $(i = 1, 2, \ldots, l-1)$ are neighbors. Without loss of generality, for a path, assume the node at the left end as the source node and the node at the right end as the destination node. In this paper, we consider only paths that *always* travel toward the right-hand side. This is because, if a path turns back toward the left-hand side at some point, we can easily find a new path that always travels toward the right-hand side and is formed by a subset of the original node set of the original path.
3) *Node-disjoint paths:* If $(s, a_1, a_2, \ldots, a_l, t)$ and $(s, b_1, b_2, \ldots, b_m, t)$ are two paths from node s to node t and $\{a_1, a_2, \ldots, a_l\} \cap \{b_1, b_2, \ldots, b_m\} = \emptyset$ (empty set), then these two paths are said to be node-disjoint paths.
4) *k-connectivity of two nonneighboring nodes:* Two nonneighboring nodes are said to be k-connected if they can still be connected upon removal of any $(k-1)$ arbitrary nodes in the network.[2] An equivalent definition for two nonneighboring nodes being k-connected is as follows: If there are k node-disjoint paths between two nonneighboring nodes, then the two nodes are k-connected.

In this paper, we are interested in the expression of the probability of the 1-DL-VANET being k-connected, which is denoted as $P_{\geq}(k)$. Note that, if the 1-DL-VANET is $(k+n)$-connected $(n > 0)$, then it is also k-connected. Let $P(k)$ denote the probability that the 1-DL-VANET is k-connected but not $(k+1)$-connected. Then, we have

$$\begin{aligned} P(k) &= P_{\geq}(k) - P_{\geq}(k+1) \\ P_{\geq}(k) &= \sum_{i=k}^{k_{\max}} P(i) \end{aligned} \tag{1}$$

where $k_{\max}$ is the maximal possible connectivity level of the network, i.e., the network can be at most $k_{\max}$-connected. Recall that there are N nodes between S and D. Thus, we have $k_{\max} \leq N+1$. When $k_{\max} = N+1$, it means that S and D are neighbors. A tighter bound of $k_{\max}$ will be given in Section IV-C.

III. Sufficient and Necessary Conditions for k-Connectivity of the 1-D Linear Vehicular Ad Hoc Network

Without loss of generality, we assume that nodes S and D are nonneighboring, i.e., $L > R$.

Based on the definitions of the k-connectivity of a network and two nonneighboring nodes, it can be concluded that the 1-DL-VANET is k-connected if and only if any two nonneighboring nodes are k-connected. Furthermore, because of the linear topology of the 1-DL-VANET, if nodes S and D can be connected upon removal of any $(k-1)$ arbitrary nodes in the 1-DL-VANET (i.e., S and D are k-connected), apparently any two nonneighboring nodes in $\mathcal{S} \cup \{S, D\}$ can also be connected upon removal of any $(k-1)$ arbitrary nodes (i.e., the two nodes are also k-connected). Thus, the 1-DL-VANET is k-connected if and only if nodes S and D are k-connected.

Based on the equivalent definition of k-connectivity of two nonneighboring nodes in Section II, we have the following lemma:

Lemma 1: Assume that two nonneighboring nodes, e.g., nodes a and b, are k-connected, and that node a is on the left-hand side of node b. For node a, denote its k nearest nodes on the right-hand side of itself as $a_1, a_2, \ldots, a_k$. For node b, denote its k nearest nodes on the left-hand side of itself as $b_1, b_2, \ldots, b_k$. Then, there exist k node-disjoint paths from node a to node b, and in each path, the node immediately after node a is from $\{a_1, a_2, \ldots, a_k\}$, and the node immediately before node b is from $\{b_1, b_2, \ldots, b_k\}$.

[1] For a number of statistical samples, the kth smallest value is the kth-order statistic [16].

[2] Any two neighboring nodes are, of course, still connected upon removal of any $(k-1)$ arbitrary nodes. Thus, we focus on nonneighboring nodes when discussing k connectivity of two nodes.

Proof: See Appendix A. ■

Lemma 1 means that node a_i $(i = 1, 2, \ldots, k)$ exists in one and only one of the k paths, as the node immediately after node a in the path, whereas node b_i $(i = 1, 2, \ldots, k)$ exists in one and only one of the k paths, as the node immediately before node b in the path.

Then, we have the following two theorems for the k-connectivity of the 1-DL-VANET:

Theorem 1: The 1-DL-VANET is k-connected if and only if two conditions hold.

1) Node S has at least k neighbors, and node D has at least k neighbors.
2) Any two nonneighboring nodes in $\mathcal{S}$ are k-connected.

Proof: See Appendix B. ■

Theorem 2: A sufficient and necessary condition for the 1-DL-VANET being k-connected is that the sum of any k consecutive spacings in the 1-DL-VANET is less than R.

Proof: See Appendix C. ■

IV. Probability of the 1-D Linear Vehicular Ad Hoc Network Being k-Connected

In the sequel, we use upper case boldface letters (e.g., $\mathbf{X}$) to denote matrices and lower case boldface letters (e.g., $\mathbf{x}$) to denote vectors.

We first use an example to illustrate how to calculate the probability of the 1-DL-VANET being k-connected, i.e., $P_{\geq}(k)$. Suppose that $N = 5$ and $k = 4$. Recall that y_i $(i = 1, 2, \ldots, N+1)$ means the spacing between two consecutive nodes. Then, according to Theorem 2, we have

$$\begin{aligned} P_{\geq}(4) &= \text{Prob}\begin{pmatrix} y_1+y_2+y_3+y_4 < R \\ y_2+y_3+y_4+y_5 < R \\ y_3+y_4+y_5+y_6 < R \end{pmatrix} \\ &= \text{Prob}(\mathbf{A}_{3,6}\mathbf{y} \prec \mathbf{r}) \end{aligned} \tag{2}$$

where

$$\mathbf{A}_{3,6} = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$\mathbf{y} = (y_1, y_2, y_3, y_4, y_5, y_6)^T$, and $\mathbf{r} = (R, R, R)^T$. Here, $\text{Prob}(\cdot)$ means the probability of an event, and superscript T means transpose operation. $\mathbf{v}_1 \prec \mathbf{v}_2$ (or $\mathbf{v}_1 \succeq \mathbf{v}_2$) means that any element in vector $\mathbf{v_1}$ is smaller than (or no less than) its counterpart in vector $\mathbf{v_2}$.

For the general case of $P_{\geq}(k)$, we have

$$\begin{aligned} P_{\geq}(k) &= \text{Prob}(y_1+y_2+\cdots+y_k < R,\ y_2+y_3+\cdots+y_{k+1} \\ &\qquad < R, \ldots,\ y_{N-k+2}+y_{N-k+3}+\cdots+y_{N+1} < R) \\ &= \text{Prob}(\mathbf{A}_{N-k+2,N+1}\mathbf{y} \prec \mathbf{r}) \end{aligned} \tag{3}$$

where $\mathbf{A}_{N-k+2,N+1}$ is an $(N-k+2) \times (N+1)$ matrix whose ith $(1 \leq i \leq N-k+2)$ row has k consecutive elements being 1s, starting from the ith column, and has other elements being 0s; $\mathbf{y} = (y_1, y_2, \ldots, y_{N+1})^T$; and $\mathbf{r} = (R, R, \ldots, R)^T$, with $|\mathbf{r}| = N-k+2$.

To calculate (3), we consider a more general case of $\mathbf{A}_{N-k+2,N+1}$. Define $\mathbf{A}$ as a $p \times q$ $(p \leq q \leq N+1)$ binary matrix with the following structure:

1) The ith $(1 \leq i \leq p)$ row has a block of consecutive 1s, starting from the a_ith column to the b_ith column; other elements in the ith row are all 0s.
2) $1 = a_1 < a_2 < \cdots < a_p; b_1 < b_2 < \cdots < b_p = q$, and $a_{i+1} \leq b_i + 1, i \in \{1, 2, \ldots, p-1\}$.

If, for any $1 \leq i \leq p-1$, we have $a_{i+1} = b_i + 1$, then we say that $\mathbf{A}$ is *disjoint*[3]; otherwise, we say that $\mathbf{A}$ is *overlapped*. For overlapped matrix $\mathbf{A}$, if, for any $1 \leq i \leq p-1$, we have $a_{i+1} < b_i + 1$, we say that $\mathbf{A}$ is *nondiagonal overlapped*; otherwise (i.e., there exists $i \in \{1, 2, \ldots, p-1\}$ such that $a_{i+1} = b_i + 1$), we say that $\mathbf{A}$ is *diagonal overlapped*.

As an example, matrix

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

is disjoint (since $a_2 = b_1 + 1$ and $a_3 = b_2 + 1$), matrix

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

is nondiagonal overlapped (since $a_2 < b_1 + 1$ and $a_3 < b_2 + 1$), and matrix

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

is diagonal overlapped (since $a_2 < b_1 + 1$ but $a_3 = b_2 + 1$).

In the following, $\text{Prob}(\mathbf{A}\mathbf{y} \prec \mathbf{r})$ is derived for disjoint $\mathbf{A}$ and overlapped $\mathbf{A}$, respectively.

A. Case With Disjoint $\mathbf{A}$

Define $S_i = \sum_{a_i \leq j \leq b_i} y_j$ and $l_i = b_i - a_i$ for $1 \leq i \leq p$. Then, from [17, eq. (9)], we have

$$\begin{aligned} \text{Prob}(\mathbf{A}\mathbf{y} \succeq \mathbf{r}) &= \text{Prob}(S_1 \geq R, S_2 \geq R, \ldots, S_p \geq R) \\ &= \sum_{k_1=0}^{l_1}\sum_{k_2=0}^{l_2}\cdots\sum_{k_p=0}^{l_p}\binom{N}{k_1, \ldots, k_p} \\ &\quad \cdot \left(\frac{R}{L}\right)^{\sum_{i=1}^{p} k_i}\left(1 - p\frac{R}{L}\right)_+^{N-\sum_{i=1}^{p} k_i} \end{aligned} \tag{4}$$

where $(x)_+ = \max(0, x)$ and $k_1, k_2, \ldots, k_p$ are integers, and

$$\binom{N}{k_1, \ldots, k_p} = \frac{N!}{k_1!k_2!\cdots k_p!\left(N - \sum_{i=1}^{p} k_i\right)!}.$$

For simplicity of representation, define

$$Q_N(\alpha, \beta) = \begin{cases} \binom{N}{\alpha}\left(\frac{R}{L}\right)^{\alpha}\left(1 - \beta\frac{R}{L}\right)^{(N-\alpha)}, & \text{if } \beta\,\frac{R}{L} < 1 \\ 0, & \text{if } \beta\,\frac{R}{L} \geq 1 \end{cases}$$

with nonnegative integers α and β. In terms of Q_N, (4) can be rewritten as

$$\begin{aligned} &\text{Prob}(\mathbf{A}\mathbf{y} \succeq \mathbf{r}) \\ &\quad = \sum_{k_1=0}^{l_1}\sum_{k_2=0}^{l_2}\cdots\sum_{k_p=0}^{l_p}\binom{\sum_{i=1}^{p} k_i}{k_1, \ldots, k_p} Q_N\left(\sum_{i=1}^{p} k_i, p\right). \end{aligned} \tag{5}$$

Recall that $\mathbf{A}_{N-k+2,N+1}$ in (3) is a special case of $\mathbf{A}$. Since $\mathbf{A}_{N-k+2,N+1}$ is a disjoint matrix when $k = 1$, we can use (5) to solve (3) with $k = 1$, as (6), shown at the bottom of the next page, where $\lfloor \cdot \rfloor$ denotes the floor function, (a) comes from the i.i.d. feature of y_i's $(i \in \{1, 2, \ldots, N+1\})$, and (b) comes from (5). Note that (6) denotes the probability of the 1-DL-VANET to be connected $(k = 1)$.

[3]If $\mathbf{A}$ has only one row, it can be viewed as a special case of disjoint matrix.

A similar result is also obtained in [10] and [11] in different ways. In [10], a connected component of a 1-D network is called a *cluster*, and the probability of the network containing at most C clusters is obtained by calculating $\text{Prob}(y_{(C)} < R)$, where $y_{(C)}$ is the Cth largest spacing. When $C = 1$, the probability of only one cluster is just the probability of network connected, where

$$y_{(1)} = \max_{1\le i\le N+1} y_i.$$

Therefore, the probability of the network being connected is

$$\text{Prob}\left(\max_{1\le i\le N+1} y_i < R\right).$$

A queueing model is introduced in [11], where the constant servant time corresponds to the wireless communication radius R, and the interarrival times between customers correspond to the intervehicle spacings y_i's. A time duration is called a *busy period* if all the $N+1$ interarrival times of customers are less than R. The probability of the network connected is equal to the probability of a busy period larger than L, which is the distance between S and D.

B. Case With Overlapped $\mathbf{A}$

In the following, we discuss how to calculate $\text{Prob}(\mathbf{Ay} \prec \mathbf{r})$ for nondiagonal-overlapped matrix $\mathbf{A}$. When $\mathbf{A}$ is diagonal overlapped, $\mathbf{A}$ can be expressed in the form of a number of nondiagonal-overlapped matrices and $\mathbf{0}$s. Here, $\mathbf{0}$ means a matrix with all elements being 0s. As an example

$$\mathbf{A} = \begin{pmatrix} \mathbf{B} & \mathbf{0} \\ \mathbf{0} & \mathbf{C} \end{pmatrix}$$

where $\mathbf{B}$ and $\mathbf{C}$ are nondiagonal-overlapped matrices. Then, methods similar to that for the nondiagonal-overlapped matrix (as follows) can be individually applied to $\mathbf{B}$ and $\mathbf{C}$. See [17] for details.

According to [18, Th. 1], for the nondiagonal-overlapped $p \times q$ matrix $\mathbf{A}$, we have

$$\text{Prob}(\mathbf{Ay} \prec \mathbf{r}) = \sum_{i=1}^{q} c_i \text{Prob}(\mathbf{A}_i(\mathbf{w})\mathbf{y} \prec \mathbf{r}) \tag{7}$$

where $\mathbf{c} = (c_1, \ldots, c_q)^T$ satisfies $\sum_{i=1}^{q} c_i = 1$, and $\mathbf{w} = \mathbf{Ac}$. Submatrix $\mathbf{A}_i(\mathbf{w})$ is obtained by replacing the ith column of $\mathbf{A}$ by $\mathbf{w}$. The process in (7) is called *decomposition*, i.e., it decomposes matrix $\mathbf{A}$ into multiple submatrices $\mathbf{A}_i(\mathbf{w})$. For simplicity of presentation, we can write (7) in a *short form* as

$$\mathbf{A} \Rightarrow \sum_{1\le i\le q,\ c_i\ne 0} c_i \mathbf{A}_i(\mathbf{w}).$$

For decomposition in (7), we need to determine vector $\mathbf{c}$. We adopt the marking algorithm for a binary matrix proposed in [17] to determine $\mathbf{c}$. Recall that each row of $\mathbf{A}$ has a consecutive block of element 1s, and a_i and b_i $(1 \le i \le p)$ are the indexes of the ith row's starting and ending points of its block of element 1s, respectively. If $a_j = b_i + 1$ $(1 \le i, j \le p, i \ne j)$, we call the jth row as the "adjacent" row of the ith row (but not *vice versa*). Then, the marking process is given as follows: Initialize $\mathbf{c} = (c_1, \ldots, c_q)^T = (0, 0, \ldots, 0)^T$; let the first row be the marking row, i.e., let $m = 1$; and mark $c_{a_m} = 1$ and $c_{b_m} = -1$. If the mth row (the marking row) has adjacent row j and $j > m$ (i.e., $a_j = b_m + 1$), then let $m = j$ (which means that the jth row becomes the marking row) and mark $c_{a_m} = 1$, $c_{b_m} = -1$. The aforementioned marking process is repeated until the marking row does not have an adjacent row. Finally, if $b_m = q$ (i.e., the last marking row is the last row of $\mathbf{A}$), mark $c_{b_m} = 0$; otherwise, mark $c_{b_m+1} = 1$.

It can be seen that we have $\sum_{i=1}^{q} c_i = 1$ when the marking process is completed. If the last marking row is the last row of $\mathbf{A}$, we have $\mathbf{w} = (0, 0, \ldots, 0, 1)^T$; otherwise, we have $\mathbf{w} = (0, 0, \ldots, 0, 0)^T$.

After the marking process, operations defined in the following property [17] can be applied to submatrices $\mathbf{A}_i(\mathbf{w})$.

Property 1: For any binary matrix $\mathbf{B}$, $\text{Prob}(\mathbf{By} \prec \mathbf{r})$ remains the same when matrix $\mathbf{B}$ has one or more of four operations.

1) Delete a row, e.g., the ith row, if the following condition is satisfied: there exists a row, e.g., the jth row, such that, for any element equal to 1 in the ith row, the counterpart element in the jth row is also equal to 1.
2) Delete a column if all its elements are equal to 0. (The cardinality of $\mathbf{y}$ and $\mathbf{r}$ is also reduced by one.)
3) Permute the columns (due to the i.i.d. feature of y_i's, $i \in \{1, 2, \ldots, N+1\}$).
4) Permute the rows.

$$\begin{aligned}
P_{\ge}(1) &= \text{Prob}(\mathbf{A}_{N+1,N+1}\mathbf{y} \prec \mathbf{r}) = \text{Prob}(y_1 < R, y_2 < R, \ldots, y_{N+1} < R) \\
&= 1 - \text{Prob}\left(\{y_1 \ge R\} \cup \{y_2 \ge R\} \cup \cdots \cup \{y_{N+1} \ge R\}\right) \\
&= 1 - \left(\sum_{1\le i\le N+1} \text{Prob}(y_i \ge R) - \sum_{1\le i_1<i_2\le N+1} \text{Prob}(y_{i_1} \ge R, y_{i_2} \ge R)\right. \\
&\qquad \left. + \sum_{1\le i_1<i_2<i_3\le N+1} \text{Prob}(y_{i_1} \ge R, y_{i_2} \ge R, y_{i_3} \ge R) - \cdots + (-1)^N \text{Prob}(y_1 \ge R, \ldots, y_{N+1} \ge R)\right) \\
&\overset{(a)}{=} 1 - \binom{N+1}{1}\text{Prob}(y_1 \ge R) + \binom{N+1}{2}\text{Prob}(y_1 \ge R, y_2 \ge R) \\
&\quad - \binom{N+1}{3}\text{Prob}(y_1 \ge R, y_2 \ge R, y_3 \ge R) + \cdots + (-1)^{N+1}\text{Prob}(y_1 \ge R, \ldots, y_{N+1} \ge R) \\
&\overset{(b)}{=} \sum_{i=0}^{\min\{\lfloor \frac{L}{R} \rfloor, N+1\}} (-1)^i \binom{N+1}{i} Q_N(0, i) = \sum_{i=0}^{\min\{\lfloor \frac{L}{R} \rfloor, N+1\}} (-1)^i \binom{N+1}{i} \left(1 - i\frac{R}{L}\right)^N
\end{aligned} \tag{6}$$

In specific, if $c_i \neq 0$, we have the following operation to $\mathbf{A}_i(\mathbf{w})$.

1) When $\mathbf{w} = (0, 0, \ldots, 0, 0)^T$, we can remove the ith column in $\mathbf{A}_i(\mathbf{w})$ (which is $(0, 0, \ldots, 0, 0)^T$).
2) When $\mathbf{w} = (0, 0, \ldots, 0, 1)^T$, we can move the ith column in $\mathbf{A}_i(\mathbf{w})$ (which is $(0, 0, \ldots, 0, 1)^T$) to be the last column of $\mathbf{A}_i(\mathbf{w})$.

By these two operations and possibly the first two operations defined in Property 1, $\mathbf{A}_i(\mathbf{w})$ has the same structure as $\mathbf{A}$. (Recall that the structure of $\mathbf{A}$ is described at the beginning of Section IV.) If $\mathbf{A}_i(\mathbf{w})$ is disjoint, we can use the method in Section IV-A to calculate $\text{Prob}(\mathbf{A}_i(\mathbf{w})\mathbf{y} \prec \mathbf{r})$; otherwise, we need to use (7) again to decompose $\mathbf{A}_i(\mathbf{w})$, i.e., a new iteration of decomposition is needed to decompose $\mathbf{A}_i(\mathbf{w})$'s that are not disjoint. This procedure is repeated until all submatrices are disjoint.

Then, a question is raised: Can the procedure be completed within finite iterations? The answer is yes, with the reasons given as follows.

It can be seen that the decomposition of $\mathbf{A}$ to $\mathbf{A}_i(\mathbf{w})$ is actually to delete a column in $\mathbf{A}$ or to replace a column that is not in the form of $(0, 0, \ldots, 0, 1)^T$ by $(0, 0, \ldots, 0, 1)^T$ and move the column to be the last column of the matrix. Therefore, after at most $(q-1)$ iterations of deleting columns and $(q-1)$ iterations of replacing columns, the obtained submatrices, after operations specified in Property 1, are either disjoint or in the form of (1, 1, …, 1) (which is actually also considered disjoint). In other words, the number of iterations is bounded by $(2q-2)$.

Now, we return to our original research problem in (3). For $\mathbf{A}_{N-k+2,N+1}$ in (3), the number of iterations of decomposition is bounded by $2(N+1) - 2 = 2N$. After all the iterations are completed, matrix $\mathbf{A}_{N-k+2,N+1}$ is decomposed to disjoint submatrices. Then, similar to (5) and (6), $P_{\geq}(k) = \text{Prob}(\mathbf{A}_{N-k+2,N+1}\mathbf{y} \prec \mathbf{r})$ can be expressed in terms of $Q_N(\cdot,\cdot)$. Note that the first integer parameter in $Q_N(\cdot,\cdot)$ is bounded by N, whereas the second integer parameter is bounded by $\lfloor L/R \rfloor$ (because when the second parameter is larger than the bound, $Q_N(\cdot,\cdot)$ equals 0). Therefore, in the expression of $P_{\geq}(k)$, the number of terms $Q_N(\cdot,\cdot)$ is bounded by $\lfloor L/R \rfloor (N+1)$, and the computational complexity for $P_{\geq}(k)$ is $O(N)$.

We use

$$\mathbf{A}_{3,6} = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

as an example, i.e., $N = 5$ and $k = 4$, to demonstrate the iterations of decomposition and show how to calculate (2). First, let the first row be the marking row. Since the first row does not have an adjacent row, we have $\mathbf{c} = (1\ 0\ 0\ -1\ 1\ 0)^T$, and therefore, $\mathbf{w} = \mathbf{A}_{3,6}\mathbf{c} = (0\ 0\ 0)^T$. Since there are three nonzero items in $\mathbf{c}$, $\mathbf{A}_{3,6}$ can be decomposed [based on (7)] into the following three submatrices:

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}.$$

After operations in Property 1, we have the following short form for the decomposition:

$$\mathbf{A}_{3,6} \Rightarrow \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}.$$

Since none of the three submatrices is disjoint, we continue to decompose each of them, and eventually, we have

$$\mathbf{A}_{3,6} \Rightarrow 3\,(1\quad 1\quad 1\quad 1) - 3\,(1\quad 1\quad 1) + \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{8}$$

In (8), each submatrix is disjoint. Therefore, we have

$$\begin{aligned} P_{\geq}(4) &= \text{Prob}(\mathbf{A}_{3,6}\mathbf{y} \prec \mathbf{r}) \\ &= 3\text{Prob}(y_1 + y_2 + y_3 + y_4 < R) \\ &\quad - 3\text{Prob}(y_1 + y_2 + y_3 < R) \\ &\quad + \text{Prob}(y_1 + y_2 < R, y_3 + y_4 < R) \\ &\quad - \text{Prob}(y_1 < R, y_2 + y_3 + y_4 < R) \\ &\quad + \text{Prob}(y_1 + y_2 < R, y_3 < R) \\ &\overset{(c)}{=} 1 - 3\text{Prob}(y_1 + y_2 + y_3 + y_4 \geq R) \\ &\quad + 4\text{Prob}(y_1 + y_2 + y_3 \geq R) \\ &\quad - 3\text{Prob}(y_1 + y_2 \geq R) \\ &\quad + \text{Prob}(y_1 + y_2 \geq R, y_3 + y_4 \geq R) \\ &\quad - \text{Prob}(y_1 \geq R, y_2 + y_3 + y_4 \geq R) \\ &\quad + \text{Prob}(y_1 + y_2 \geq R, y_3 \geq R) \\ &= 1 - 2Q_5(0,1) - 2Q_5(1,1) + Q_5(2,1) \\ &\quad - 3Q_5(3,1) + Q_5(0,2) + 2Q_5(1,2) + Q_5(2,2) \end{aligned}$$

where (c) follows a similar method in (6).

As another example, when $k = N$, according to the marking process, we have

$$\begin{aligned} P_{\geq}(N) &= \text{Prob}(\mathbf{A}_{2,N+1}\mathbf{y} \prec \mathbf{r}) \\ &= 2\text{Prob}(y_1 + y_2 + \cdots + y_N < R) - \text{Prob}(\mathbf{A}_{2,N}\mathbf{y} \prec \mathbf{r}) \\ &= \cdots = 2\text{Prob}(y_1 + y_2 + \cdots + y_N < R) \\ &\quad - 2\text{Prob}(y_1 + y_2 + \cdots + y_{N-1} < R) \\ &\quad + 2\text{Prob}(y_1 + y_2 + \cdots + y_{N-2} < R) \\ &\quad - \cdots + (-1)^N \cdot 2\text{Prob}(y_1 + y_2 < R) \\ &\quad + (-1)^{(N-1)}\text{Prob}(y_1 < R, y_2 < R) \\ &= \begin{cases} 1 - Q_N(0,2) \\ \quad -2\sum_{i=1}^{N/2} Q_N(2i-1,1) & \text{(for even } N\text{)} \\ 1 + Q_N(0,2) \\ \quad -2\sum_{i=0}^{(N-1)/2} Q_N(2i,1) & \text{(for odd } N\text{).} \end{cases} \end{aligned}$$

C. Tolerable Vehicle Departures

Let N_f denote the maximum number of arbitrary vehicle departures such that the remaining network is still connected. The expectation of N_f is given as

$$E[N_f] = \sum_{k=2}^{k_{\max}} (k-1)P(k). \tag{9}$$

To compute $E[N_f]$, $P(k)$ and $k_{\max}$ should be known. Note that $P(k)$ can be computed using (1), where $P_{\geq}(k)$ can be calculated as in Sections IV-A and B. Next, we determine $k_{\max}$, which is the maximal level of connectivity. From Theorem 2, the sum of any $k_{\max}$ consecutive spacings is less than R. Then, for the whole distance from node S to node D, the first distance of R includes at least $k_{\max}$ nodes (excluding node S), and the jth ($j = 2, 3, \ldots, \lfloor L/R \rfloor$) distance of R also includes at least $k_{\max}$ nodes. As the total number of nodes (excluding node S) is $N + 1$, we have $k_{\max}\lfloor L/R \rfloor \leq N + 1$, which leads to

$$k_{\max} \leq \frac{(N+1)}{\lfloor L/R \rfloor}.$$

Therefore, we use

$$k_{\max} = \left\lfloor \frac{(N+1)}{\lfloor L/R \rfloor} \right\rfloor$$

when we calculate $E[N_f]$ using (9).

V. Performance Evaluation

Simulation is carried out to validate our analysis. To simulate the maximum number of node-disjoint paths between S and D of a specific 1-DL-VANET topology, we use flow network simulation [19]. In specific, a flow in a flow network can be viewed as a water flow in a water network, where the amount of water flow should be no greater than the capacity of the water pipes. We view each node in the 1-DL-VANET to be a water pipe with unit capacity and each link between two neighboring nodes to be a water pipe with infinite capacity. Then, the 1-DL-VANET is mapped to a water network (or flow network). A path from S to D in the 1-DL-VANET corresponds to a water flow from the source to the sink in the flow network, and the maximum number of node-disjoint paths between S and D in the 1-DL-VANET corresponds to the maximum amount of flow in the flow network. The push-relabel Algorithm in [19] can be used to compute the maximum amount of flow of a flow network.

In our analysis and simulation, communication range R varies from 100 to 700 m with a step size of 60 m. For each R value, 10 000 random scenarios are generated, where $N = 14$ nodes are uniformly distributed over a line segment of length $L = 1000$ m. Fig. 2 shows the analyzed and simulated average probability that there are at least k node-disjoint paths between S and D, i.e., $P_{\geq}(k)$. It can be seen that the simulation and the analysis match well, which confirms the accuracy of our analysis.

The expected maximum number of tolerable vehicle departures $(E[N_f])$ is another interesting metric for a 1-DL-VANET. Let $L = 1000$ m, N vary from 10 to 17 and R vary from 300 to 800 m. Fig. 3 shows the values of $E[N_f]$. As the node number N and/or communication range R increases, $E[N_f]$ also increases. With fixed R, $E[N_f]$ almost linearly increases with N, which can be explained as follows.

Consider two 1-DL-VANETs: Case-1 network with $N = N_1$ and Case-2 network with $N = N_2$. Assume that the expected maximum number of tolerable vehicle departures in the two networks is A_1 and A_2, respectively. If we arbitrarily remove A_1 nodes and A_2 nodes from the Case-1 and Case-2 networks, respectively, the remaining networks in the two cases are expected to be connected but not biconnected. Considering the random positions of nodes in the two original networks, if we randomly add one node into the two remaining networks, the two remaining networks should have the same probability, which is denoted as ρ, to become biconnected. In other words, one additional node is with probability ρ to make either the remaining network tolerate one more arbitrary node departure (without network disconnection). This means that, when N changes from N_1 to $N_1 + 1$ or from N_2 to $N_2 + 1$, the value of $E[N_f]$ is increased by the same value ρ. This explains the linearity of the curves in Fig. 3.

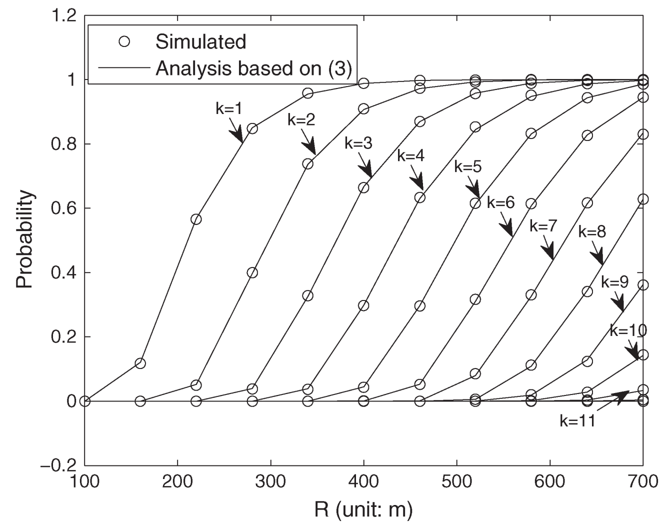

Fig. 2. $P_{\geq}(k)$ versus R.

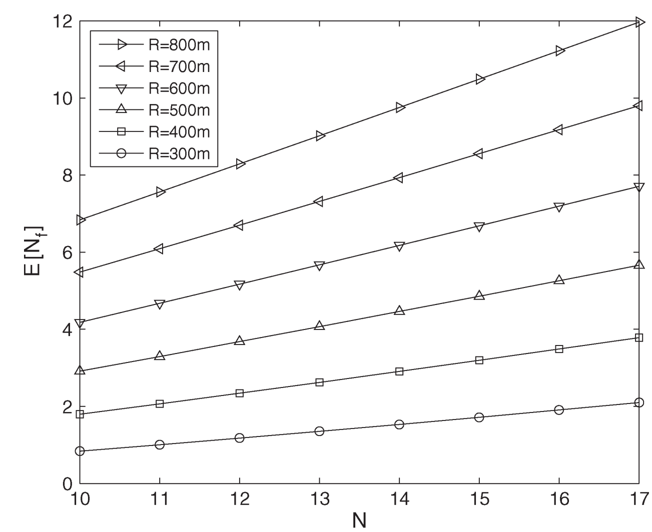

Fig. 3. Expected maximum number of tolerable vehicle departures.

An upper bound of the value of ρ can be given as follows. For a 1-DL-VANET, we arbitrarily remove $E[N_f]$ nodes one by one. Then, before the last removal, the network is biconnected; and after the last removal, the network is connected but not biconnected. Assume that node v is the last removed node. Before the removal of node v, the two nearest nodes on the left-hand side of node v are denoted nodes a and b, whereas the two nearest nodes on the right-hand side of node v are denoted as nodes c and e, as shown in Fig. 4. The distance between node a and node b, between node b and node c, and between node c and node e is d_1, x, and d_2, respectively.

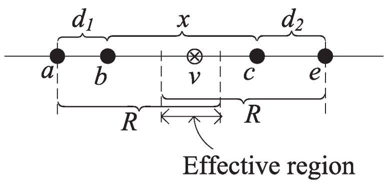

Fig. 4. Network topology when node v is removed.

Since the network is biconnected before the removal of node v but becomes not biconnected after that, according to Theorem 2, we have $x < R$, $d_1 + x + d_2 < 2R$, and $\max(d_1, d_2) + x > R$. After the removal of node v, we randomly add one node into the remaining network. If the additional node is within the *effective region* shown in Fig. 4, then the remaining network becomes biconnected, i.e., can tolerate one more arbitrary node departure. The length of the effective region is $\Delta = (R - d_1) + (R - d_2) - x = 2R - (d_1 + d_2 + x) < 2R - (\max(d_1, d_2) + x) < 2R - R = R$. Then, the probability that the additional node makes the remaining network (after the removal of node v) become biconnected is the probability that the additional node is within the effective region and is denoted as $\rho = \Delta/L$, which is bounded by R/L. Since the slope of curve $E[N_f]$ versus N is ρ, the slope is also bounded by R/L. In particular, Fig. 3 shows that the slope of the $E[N_f]$ versus N curve is $0.50R/L$, $0.70R/L$, $0.79R/L$, $0.84R/L$, $0.88R/L$, and $0.92R/L$, when R is 300, 400, 500, 600, 700, and 800 m, respectively.

VI. Conclusion

In this work, we have analyzed the k-connectivity of a 1-DL-VANET. We have found a simple but insightful sufficient and necessary condition for the 1-DL-VANET to be k-connected, i.e., the sum of any k consecutive spacings is less than the communication range. The probability of the 1-DL-VANET being k-connected has been then derived with the help of matrix decomposition. The results are helpful in evaluating the tolerance level of the 1-DL-VANET to simultaneous vehicle departures (such as at highway exits) and other faults. Future research topics include the evaluation of the sojourning time when the 1-DL-VANET keeps k-connected and the sojourning time when the 1-DL-VANET does not remain k-connected, considering a specific mobility model of the vehicles.

Appendix A
Proof of Lemma 1

Since node a and node b are k-connected, we can have k node-disjoint paths from node a to node b, which are denoted as $\mathbf{P}_1, \mathbf{P}_2, \ldots,$ and $\mathbf{P}_k$. We have the following modification to path $\mathbf{P}_i$ ($i \in \{1, 2, \ldots, k\}$): if two or more nodes in path $\mathbf{P}_i$ are neighbors of node a (which means that those nodes are consecutive nodes after node a in path $\mathbf{P}_i$), then we remove all those nodes, except the last one from path $\mathbf{P}_i$. By this means, in path $\mathbf{P}_i$, we have one and only one neighbor of node a, which is the node immediately after node a in the modified path.

Among the k modified paths ($\mathbf{P}_1, \mathbf{P}_2, \ldots,$ and $\mathbf{P}_k$), denote l as the number of paths in which the node immediately after node a is from $\{a_1, a_2, \ldots, a_k\}$. The nodes immediate after node a in the l paths form a set $\mathcal{A}$ ($|\mathcal{A}| = l$). Then, we can insert the $(k - l)$ nodes in $\{a_1, a_2, \ldots, a_k\} \setminus \{\mathcal{A}\}$ immediately after node a in the other $(k - l)$ paths, respectively (with one node inserted to one path). By this means, the k paths are still node disjoint, and in each path, the node immediately after node a is from $\{a_1, a_2, \ldots, a_k\}$.

Fig. 5. $k + 1$ nearest nodes from S and $k + 1$ nearest nodes from D.

Similarly, we can prove that, after some modifications, the k paths are still node disjoint, and in each path, the node immediately before node b is from $\{b_1, b_2, \ldots, b_k\}$.

Appendix B
Proof of Theorem 1

The necessity is obvious. Next, we focus on sufficiency.

As shown in Fig. 5, denote the $(k + 1)$ nearest nodes from node S as $s_1, s_2, \ldots, s_{k+1}$ and the $(k + 1)$ nearest nodes from node D as $d_{k+1}, d_k, \ldots, d_2, d_1$. Then, $s_1, s_2, \ldots, s_k$ are neighbors of node S, and $d_2, d_3, \ldots, d_{k+1}$ are neighbors of node D.

First, consider the case when nodes s_1 and d_{k+1} are neighbors. Then, any two nodes between s_1 and d_{k+1} are neighbors.

1) If there are common nodes between sets $\{s_1, s_2, \ldots, s_k\}$ and $\{d_2, d_3, \ldots, d_{k+1}\}$: Assume that the number of common nodes is m. Then, the common nodes are s_{k-m+1} (which is also d_2), s_{k-m+2} (which is also d_3), $\ldots$, and s_k (which is also d_{m+1}). We can find k node-disjoint paths from S to D: the first $(k - m)$ paths are (S, s_1, d_{m+2}, D), $(S, s_2, d_{m+3}, D), \ldots,$ and (S, s_{k-m}, d_{k+1}, D); and the remaining m paths are $(S, s_{k-m+1}, D), (S, s_{k-m+2}, D), \ldots,$ and (S, s_k, D). Thus, nodes S and D are k-connected.
2) If there is no common node between sets $\{s_1, s_2, \ldots, s_k\}$ and $\{d_2, d_3, \ldots, d_{k+1}\}$: Then, we can find k node-disjoint paths from S to D, with the ith ($1 \le i \le k$) path as (S, s_i, d_{i+1}, D). Thus, nodes S and D are k-connected.

Next, we consider the case when nodes s_1 and d_{k+1} are not neighbors. Then, nodes s_1 and d_{k+1} are k-connected since any two nonneighboring nodes in $\mathcal{S}$ are k-connected. According to Lemma 1, there exist k node-disjoint paths from s_1 to d_{k+1}, and each path is in the form of $(s_1, s_i, \ldots, d_j, d_{k+1})$, where $i \in \{2, 3, \ldots, k + 1\}$ and $j \in \{1, 2, \ldots, k\}$. We have the following transformation for path $(s_1, s_i, \ldots, d_j, d_{k+1})$.

1) If $i \le k$, then replace node s_1 with node S; if $i = k + 1$, then add node S in front of node s_1.
2) If $j = 1$, then add node D after node d_{k+1}; if $j \ge 2$, then replace node d_{k+1} with node D.

It can be seen that the newly formed k paths are k node-disjoint paths from node S to node D. Therefore, node S and node D are k-connected, which means that the 1-DL-VANET is k-connected.

Appendix C
Proof of Theorem 2

First, we prove the necessity. Consider any node $u \in \mathcal{S} \cup \{S\}$, and consider the k consecutive spacings starting from node u (to the right-hand side). Note that we need to consider only node u that has at least k nodes on its right-hand side. If nodes u and D are neighbors, then apparently the sum of the k spacings starting from node u is less than R. If nodes u and D are not neighbors, then according to Theorem 1, nodes u and D are k-connected, and then, node u has at least k neighbors on its right-hand side. Thus, the sum of the k consecutive spacings starting from node u is less than R.

Next, we prove the sufficiency. Recall that the number of nodes in $\mathcal{S} = \{1, 2, \ldots, N\}$ is N. We express N in the format $N = mk + n$, where m is a nonnegative integer, and $n \in \{0, 1, 2, \ldots, k - 1\}$. Then, we have the following k node-disjoint paths from node S to node D, for cases when $n = 0$ and $n \neq 0$, respectively.

1) When $n = 0$: The ith $(1 \leq i \leq k)$ path is $(S, i, i + k, i + 2k, \ldots, i + (m-1)k, D)$. Between any two consecutive nodes in the ith path there are no more than k spacings. Therefore, any two consecutive nodes in the ith path are neighbors. The k paths are node disjoint.
2) When $n \neq 0$: For $i \in \{1, 2, \ldots, n\}$, the ith path is $(S, i, i + k, i + 2k, \ldots, i + (m-1)k, i + mk, D)$; for $i \in \{n+1, n+2, \ldots, k\}$, the ith path is $(S, i, i + k, i + 2k, \ldots, i + (m-1)k, D)$. Between any two consecutive nodes in the ith path, there are no more than k spacings. Therefore, any two consecutive nodes in the ith path are neighbors. The k paths are node disjoint.

Therefore, nodes S and D are k-connected, which means that the 1-DL-VANET is k-connected.

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A Multihop Transmission Scheme With Detect-and-Forward Protocol and Network Coding in Two-Way Relay Fading Channels

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***Abstract*—In this paper, we propose and analyze a multihop transmission scheme based on detect-and-forward (DF) relay protocol and network coding (NC) for two-way relay channels. In this scheme, the odd relays perform hard detection and then forward the detected signals to the next hop, whereas the even relays perform NC on the detected signals from the two adjacent nodes and broadcast them to the next hop. By separating the network into multiple two-hop subsystems, we develop closed-form expressions for bit error rate (BER) in flat Rayleigh fading channels. Our results are given as both lower bound and asymptotic expression based on an accurate upper bound of the end-to-end signal-to-noise ratio (SNR). It is shown that the proposed scheme has the same asymptotic BER performance and a much higher throughput compared with the conventional bidirectional relay scheme based on four transmission phases. Therefore, the proposed scheme is efficient for practical wireless applications. Simulation results are provided to validate the analysis.**

***Index Terms*—Bidirectional relaying, detect-and-forward (DF), fading channels, multihop transmission, network coding (NC).**

I. Introduction

The application of network coding (NC) [1] in a two-way relay channel (TWRC) brings the benefits of overall network throughput increase and spectral efficiency improvement [2]–[4]. A popular three-node dual-hop TWRC model in additive white Gaussian noise (AWGN) channel has been widely studied in the recent literature, in particular from information-theoretic perspective. Achievable rate

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