

Achieving Compliant Spherical Linkage Designs from Compliant Planar Linkages Based on PRBM: A Spherical Young Mechanism Case Study

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Abstract— Many studies about the compliant mechanisms has been done and most of them are planar configurations. Spherical linkages, as a transition between planar and spatial linkages, play an important role in the modern precision instruments and robots. In this paper, we explore spherical linkage designs from available compliant planar linkages with a method based on the pseudo-rigid-body model and the rigid-body replacement synthesis approach. A typical Young mechanism is utilized as an illustration, which is called the spherical Young mechanism. The kinetostatic results of the spherical Young mechanism, which have been calculated based on the pseudo-rigid-body method, show that it has bistability behaviors.

I. INTRODUCTION

Compliant mechanisms have been playing increasingly important roles in high precision manufacturing, minimally invasive surgeries, scientific instruments, and microelectromechanical systems (MEMS), etc. Unlike the rigid-body mechanisms, compliant mechanisms achieve at least some of their mobility through the deflections of flexible members, thus can provide precise and frictionless motion in their workspace [1]. Since flexible members may undergo large deflections, the nonlinearity associated with large deflections often complicates the design and analysis of compliant mechanisms.

Most of the compliant mechanisms we have studied are planar[2] or parallel [3], [4] configurations, while there are very few compliant spatial linkages. Chiang [5], [6] and Murray [7] classified the spherical linkages with a modified Grashof's law. Smith and Lusk [8], [9] studied a bistable spherical compliant mechanism which is capable of providing an out-of-plane stable position. Tanik and Parlaktas [10], [11] introduced a new type of compliant spatial four-bar mechanism and an enumeration and novel approach for the analysis and design of such mechanisms. Hoover and Fearing[12] studied the compliant Sarrus mechanisms and used them to produce straight-line motion in a millimeter scale robot. And, Chen et al. [13] discussed the multistable behaviors of compliant Sarrus mechanisms.

As we know, spherical linkages are a special class of spatial linkages, and provide spatial motions and accomplish complex tasks beyond what planar linkages are capable of performing [14], [15]. And there have been many applications in medical[16], biological and mechanical[17] fields for

This work is supported by the National Natural Science Foundation of China under Grant No. 51175396, and the program for new century excellent talents in university under Grant No. NCET-11-0689

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its special flexible locus. Therefore, the study of compliant spherical linkages is of practical importance. A spherical mechanism with compliant joints or links can be defined as “spherical compliant mechanism”. With the pseudo-rigid-body model (PRBM), which has been discussed and applied to planar compliant mechanisms [1], [18] and spherical compliant linkages [20], [21], [22], the positions of the end point of the compliant segments in a spherical compliant mechanism are determined approximately. In this paper, we present a PRBM-based approach [23], [24] to obtain compliant spherical linkage designs from available compliant planar linkages. The Young mechanism [25] is utilized as a case study.

The rest of this paper is organized as follows. Section II introduces the PRBM of the compliant curved flexible beam. With the help of a PRBM-based approach, a typical Young mechanism is transformed to its corresponding spherical compliant mechanism, which is described in Section III. Besides, the kinetostatics of the spherical Young mechanism are formulated based on the PRB method. The conclusions and future work are included in Section IV.

II. PRBM OF FLEXIBLE BEAMS

The large deflection of planar flexible cantilever beam can be approximated by the PRBM, which greatly simplifies the nonlinear deflection analysis of compliant mechanisms [1], [26], [27], [28]. Similarly, Jagirdar[21] developed the PRBM for curved flexible beam and gave the corresponding PRBM parameters through nonlinear Finite Element Analysis. As shown in Fig. 1, similar to the planar compliant cantilever beam, the PRBM for flexible curved beams is mainly constituted of two components: a torsional spring and a rigid cantilever beam. The beam is obtained from the planar mechanism by making straight beam curved (bending along a certain spherical surface). The main flexible direction of the curved beam is along the spherical surface, while the main stiffness points to the center of the sphere.

By applying an approximative load to make a compliant curved beam move in the manner consistent with a spherical kinetics, we can get the coordinates of the tip of the compliant curved beam in terms of angle Θ by the PRBM[21]

$$\vartheta = (1 - \gamma_\Theta) \rho + \arctan(\tan \gamma_\Theta \rho \cos \Theta) \quad (1)$$

$$\mu = \arcsin(\sin \gamma_\Theta \rho \sin \Theta) \quad (2)$$

$$\theta = C_\theta \Theta \quad (3)$$

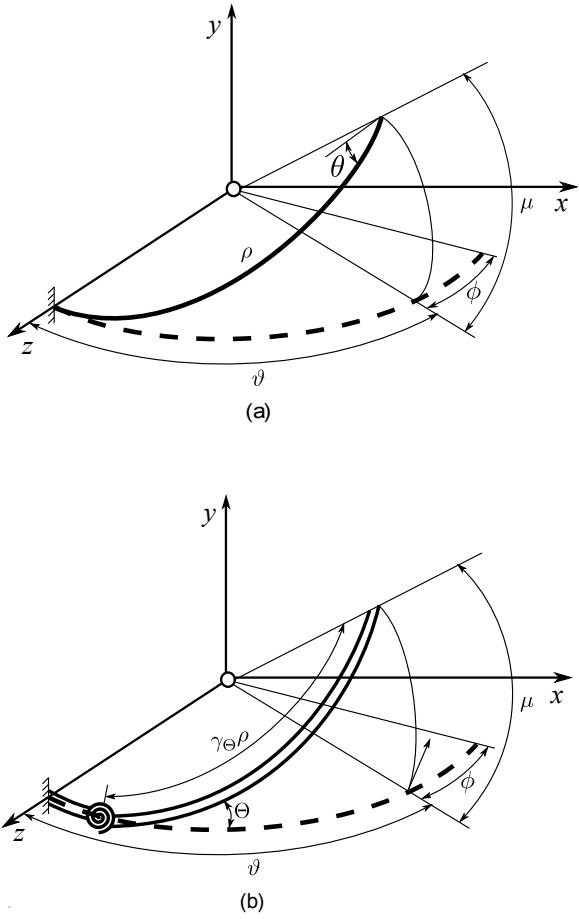


Fig. 1. A compliant circular cantilever: (a) a compliant curved beam, and (b) the pseudo-rigid-body model (PRBM) of the compliant curved beam

where Θ is the pseudo-rigid-body angle and γ_θ is the characteristic radius factor, shown in Fig. 1.

This model can be used to calculate the position of the end point of a compliant curved beam, thus can be applied to modeling and analysis of spherical compliant mechanisms. Therefore, by utilizing the available knowledge and experience from available compliant planar mechanisms, we can design, model and analyze spherical compliant linkages.

Although the method of modeling and analysis of compliant curved beams have been proved available, there are still numerous troubles in designing the compliant curved beam. To ensure a compliant curved beam move along the spherical surface, we must reduce the vertical motion. Thus, the beam must be modeled such that highly compliant at the most flexible direction, while highly rigid at the stiffest direction. In this paper, a beam of a rectangular cross-section is employed to synthesize the spherical compliant linkages.

The stiffness of a curved beam is related to the material characteristics, the length L , and the moment of inertia, which is related to the bending directions of the beam and depended on the cross-section including the height h and the width b . And the relations are listed as following [22]

$$K_{stiff} \approx \pi \gamma_\theta^2 \frac{EI_{stiff}}{L} = \pi \gamma_\theta^2 \frac{Eb h^3}{12L} \quad (4)$$

and

$$K_{slender} \approx \pi \gamma_\theta^2 \frac{EI_{slender}}{L} = \pi \gamma_\theta^2 \frac{Eb h^3}{12L} \quad (5)$$

where K_{stiff} is the stiffness of the stifferst direction, $K_{slender}$ is the stiffness of the most flexible direction, and γ_θ is a constant value ($\gamma_\theta = 0.85$). The ratio μ of K_{stiff} to $K_{slender}$ is

$$\mu = \frac{K_{stiff}}{K_{slender}} = \left(\frac{h}{b}\right)^2 \quad (6)$$

Equation (6) shows that, the stiffness ratio μ depends on the height h and the width b . Consequently, To ensure the compliant curved beam move along the spherical surface accurately, we should make the compliant beams have a larger stiffness ratio (μ). And then, a curved beam with a rectangular cross-section and larger ratio of height h to width w , must be the best choice in spherical linkages.

III. SPHERICAL YOUNG MECHANISMS

The principles of exact constraint [29] state that two parallel constraints can be equivalently treated as two intersecting constraints that intersect at infinity. That is to say, by moving the intersecting point of the revolute joints' axes infinity to a finite distance, a planar linkage is changed into a spherical linkage. Similarly, with the axes of the revolute joints intersecting at infinity, a planar linkage can be considered as a special spherical linkage (with an infinite spherical radius and the center of the sphere locate at infinity), in which case we assume that, the radius of the planar linkage is infinite and the center of the sphere located at infinity.

With two revolute pairs, the Young mechanisms [25] have double links which is a continuous body between two hinges, and there are two compliant segments in every link. Figure 2 shows a compliant Young mechanism which can be modeled as a planar four-bar mechanism using the PRBM [18], [19], as shown in Fig. 3. The revolute joints of this four-bar mechanism intersect at infinity. According to the aforementioned rule, a spherical four-bar PRBM is obtained by moving the intersecting point from infinity to a finite distance, as shown in Fig. 4. Thereof, the rigid-body replacement synthesis approach [23], which is combined with the rigid analytical synthesis is used to synthesize the flexible members, can be employed to achieve compliant designs of the PRBM [24]. Figure 5 shows one possible design for the spherical Young mechanism, whose design parameters are listed in Tab. I.

The spherical Young bistable mechanism shown in Fig. 5 has two compliant links, each of which has one end rigidly connected to a common coupler and the other articulated to the ground. The coupler moves approximately on the spherical surface, centered at point O . And its pseudo-rigid-body model is shown in Fig. 4, which is a rigid spherical four-bar mechanism, with torsional springs attached at the

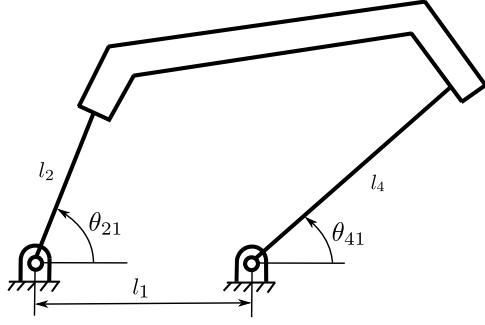


Fig. 2. Young mechanism

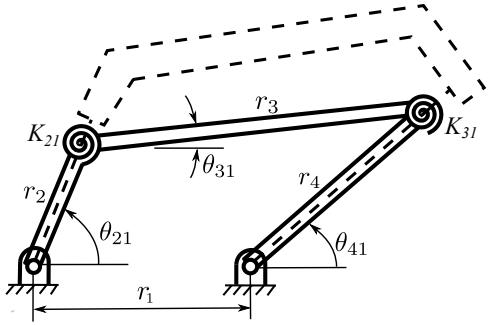


Fig. 3. The pseudo-rigid-body model (PRBM) of the Young mechanism shown in Fig. 2

articulated joints. This enables us to use available knowledge of spherical linkages for modeling their compliant counterparts. To simplify the modeling procedure, we assume the radius of the spherical Young mechanism is unit “1”.

Applying the spherical trigonometry to the spherical triangle of the PRBM shown in Fig. 5 yields

$$\delta = \arccos(\cos \gamma \cos \alpha + \sin \gamma \sin \alpha \cos \theta_2) \quad (7)$$

$$\varphi = \arccos \frac{\cos \alpha - \cos \gamma \cos \delta}{\sin \gamma \sin \delta} \quad (8)$$

$$v = \arccos \frac{\cos \gamma - \cos \alpha \cos \delta}{\sin \alpha \sin \delta} \quad (9)$$

$$\psi = \arccos \frac{\cos \beta - \cos \eta \cos \delta}{\sin \eta \sin \delta} \quad (10)$$

$$\omega = \arccos \frac{\cos \eta - \cos \beta \cos \delta}{\sin \beta \sin \delta} \quad (11)$$

where φ , v , ψ and ω are shown as in Fig. 4. The total potential energy stored in the flexible segments can be estimated using the PRBM as

$$V = \frac{1}{2}(K_2 \psi_2^2 + K_3 \psi_3^2) \quad (12)$$

where

$$\psi_2 = \theta_2 - \theta_{20} - \theta_3 + \theta_{30} \quad (13)$$

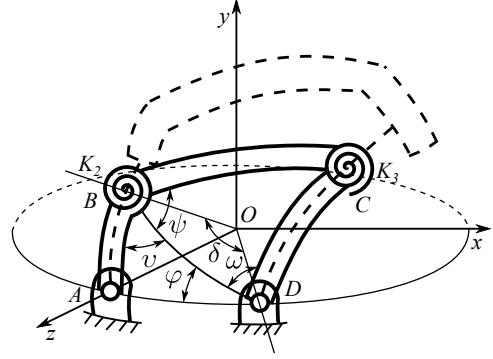


Fig. 4. The pseudo-rigid-body model (PRBM) of a spherical Young mechanism

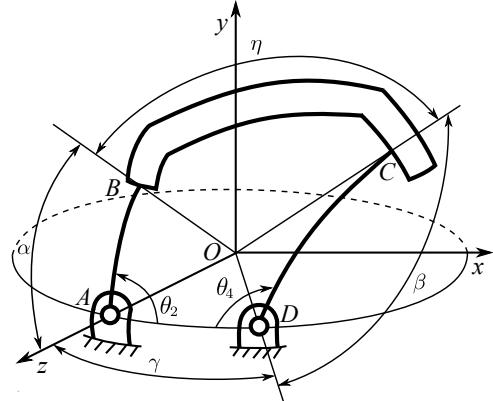


Fig. 5. Spherical Young mechanism

$$\psi_3 = \theta_4 - \theta_{40} - \theta_3 + \theta_{30} \quad (14)$$

$$\theta_3 = \psi + v \quad (15)$$

$$\theta_4 = \omega + \varphi \quad (16)$$

where θ_{20} , θ_{30} , and θ_{40} are the initial angles of the links. Meantime, the spring constants K_2 , and K_3 for the flexible segments are calculated using the PRBM as

$$K_2 = \gamma_\Theta K_\Theta \frac{EI_2}{\alpha} \quad (17)$$

$$K_3 = \gamma_\Theta K_\Theta \frac{EI_3}{\beta} \quad (18)$$

where

$$\gamma_\Theta = 0.85$$

$$K_\Theta = 2.65$$

Taking the first derivative of V with respect to θ_2 yields the input torque M_{in} required to maintain the mechanism

$$\begin{aligned} M_{in} &= \frac{dV}{d\theta_2} = K_2 \psi_2 \frac{d\psi_2}{d\theta_2} + K_3 \psi_3 \frac{d\psi_3}{d\theta_2} \\ &= K_2 \psi_2 (1 - h_{32}) + K_3 \psi_3 (h_{42} - h_{32}) \end{aligned} \quad (19)$$

TABLE I

THE DESIGN PARAMETERS OF A SPHERICAL YOUNG BISTABLE MECHANISM. WE ASSUME POLYPROPYLENE IS USED FOR THE MECHANISM.

E	r	γ	α	η	β	I_2	I_4	θ_{20}
$1.4 \times 10^9 \text{ Pa}$	302.4 mm	24.25°	20.84°	38.08°	32.97°	108 mm^4	13.5 mm^4	10°

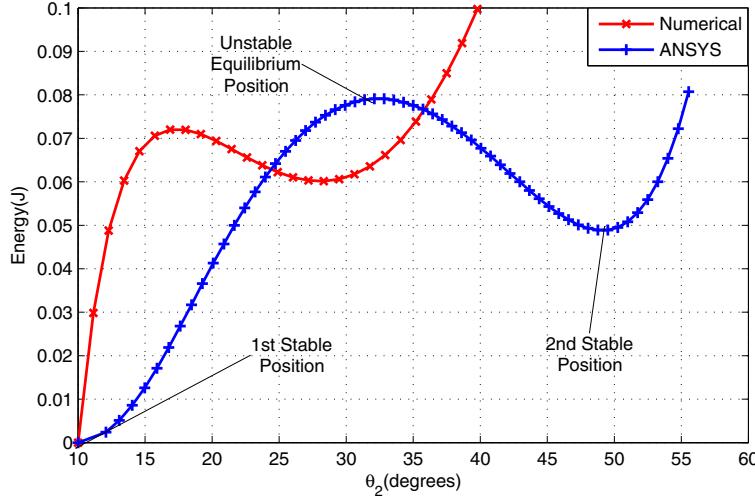


Fig. 6. The strain energy curve of the spherical Young bistable mechanism

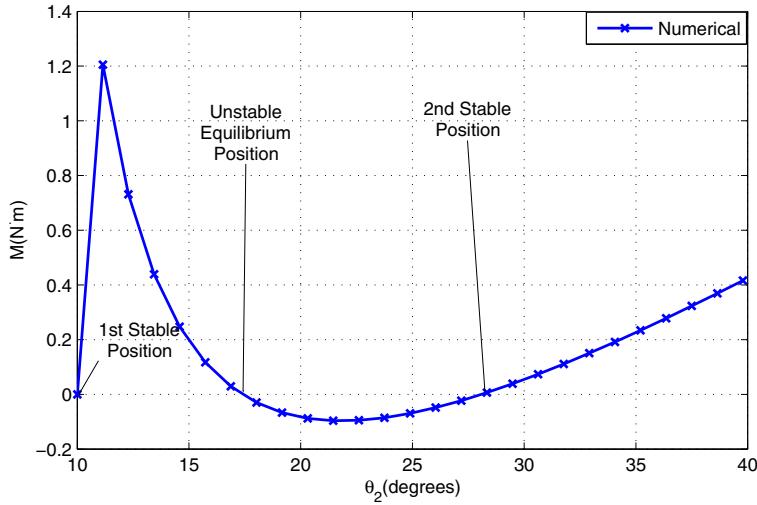


Fig. 7. The driving torque curve of the spherical Young bistable mechanism

where h_{32} and h_{42} are the kinematic coefficients given as

$$h_{32} = \frac{d\theta_3}{d\theta_2} = \frac{d\psi}{d\theta_2} + \frac{dv}{d\theta_2} \quad (20)$$

$$h_{42} = \frac{d\theta_4}{d\theta_2} = \frac{d\omega}{d\theta_2} + \frac{d\phi}{d\theta_2} \quad (21)$$

where

$$\frac{d\phi}{d\theta_2} = \frac{-\cos\gamma + \cos\alpha\cos\delta}{\sin\delta\sqrt{\sin^2\gamma\sin^2\delta - (\cos\alpha - \cos\gamma\cos\delta)^2}} \frac{d\delta}{d\theta_2} \quad (22)$$

$$\frac{d\psi}{d\theta_2} = \frac{-\cos\alpha + \cos\gamma\cos\delta}{\sin\delta\sqrt{\sin^2\alpha\sin^2\delta - (\cos\gamma - \cos\alpha\cos\delta)^2}} \frac{d\delta}{d\theta_2} \quad (23)$$

$$\frac{dv}{d\theta_2} = \frac{-\cos\eta + \cos\beta\cos\delta}{\sin\delta\sqrt{\sin^2\eta\sin^2\delta - (\cos\beta - \cos\eta\cos\delta)^2}} \frac{d\delta}{d\theta_2} \quad (24)$$

$$\frac{d\omega}{d\theta_2} = \frac{-\cos\beta + \cos\eta\cos\delta}{\sin\delta\sqrt{\sin^2\beta\sin^2\delta - (\cos\eta - \cos\beta\cos\delta)^2}} \frac{d\delta}{d\theta_2} \quad (25)$$

$$\frac{d\delta}{d\theta_2} = \frac{-\sin\gamma\sin\alpha\sin\theta_2}{\sqrt{1 - (\cos\gamma\cos\alpha - \sin\gamma\sin\alpha\cos\theta_2)^2}} \quad (26)$$

By taking θ_2 as the independent variable, the total potential energy and the input torque can be calculated using the Eqs.(12) and (19), respectively.

For the design parameters of a spherical Young mechanism listed in Tab. I, the strain energy is plotted as a function of θ_2 in Fig. 6, and the driving torque plotted in Fig. 7. It can be found that both the strain energy curve and the input torque curves both show the bistable behavior of the mechanism. The unstable equilibrium position occurs at $\theta_2 = 17.4^\circ$, and the second stable equilibrium position occurs at $\theta_2 = 28.1^\circ$.

Then, the validity of the spherical Young mechanism, with the design parameters listed in Tab. I, is verified by the finite element analysis package ANSYS. The beam 4 element is used to define the geometry and simulate the kinetostatic behavior. The results are also shown in Fig. 6, which also show the bistable behavior. As the PRBM itself is poor to model the complicated mechanisms, and we can not confirm the compliant beams bending along the spherical surface all the time. Thereof they have a large relative error.

IV. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

This paper introduced the modeling of compliant curved flexible beams by using the PRBM. A spherical Young mechanism, which is varied from the planar Young mechanism by introducing a parameter of spherical radius, was modeled using the PRBM. Finally, a spherical Young mechanism design was presented and analyzed. The results show that the design exhibits bistable behavior.

B. Future Work

The study of compliant spherical linkages promotes the importance of designing new flexible segments that are flexible along the expected spherical locus while relatively stiff in the other directions. Exploring new mechanisms, that combine the characteristics of spherical mechanisms and compliant mechanisms, and their engineering applications are also of our interest.

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