Synthesis of Compliant Multistable Mechanisms Through Use of a Single Bistable Mechanism

Guimin Chen

Yanjie Gou

Aimei Zhang

School of Mechatronics,
Xidian University,
Xi'an,
Shaanxi 710071, China

A compliant multistable mechanism is capable of steadily staying at multiple distinct positions without power input. Many applications including switches, valves, relays, positioners, and reconfigurable robots may benefit from multistability. In this paper, two new approaches for synthesizing compliant multistable mechanisms are proposed, which enable designers to achieve multistability through the use of a single bistable mechanism. The synthesis approaches are described and illustrated by several design examples. Compound use of both approaches is also discussed. The design potential of the synthesis approaches is demonstrated by the successful operation of several instantiations of designs that exhibit three, four, five, and nine stable equilibrium positions, respectively. The equations for determining the actuation force required to move a multistable mechanism are provided. The synthesis approaches enable us to design a compliant mechanism with a desired number of stable positions. [DOI: 10.1115/1.4004543]

Keywords: compliant mechanism, bistability, multistability

1 Introduction

A compliant mechanism is a device which achieves some or all of its motion through the deflection of flexible segments rather than from articulated joints. Due to the reduced number of articulated joints, a compliant mechanism exhibits many advantages over its rigid-body counterpart, such as increased precision and reliability, reduced friction and wear, and decreased manufacturing and maintenance costs. Besides these advantages, compliant mechanisms also offer designers, an economical and effective way to achieve multistability [1]. Here, the term "multistability" for a mechanism implies that the mechanism is capable of steadily staying at multiple distinct positions without power input. Many applications (such as switches, valves, relays, positioners, and reconfigurable robots [2]) may benefit from multistability. Unlike the traditional ways of using locking mechanisms [3–5] and detents [2,6], a compliant multistable mechanism achieves multistability through the storage and release processes of strain energy in its flexible members during the motion of the mechanism.

Compliant multistable mechanisms can be roughly divided into two categories depending on the number of stable positions: compliant bistable mechanisms and compliant mechanisms with more than two stable positions. In the rest of this paper, the term "compliant multistable mechanism" is used to exclusively represent compliant mechanisms with more than two stable positions for convenience. Most of the works on this topic have been focused on developing and studying various bistable mechanism configurations [7–16], including theories for synthesis of compliant bistable four-bar mechanisms [17,18] and methods for designing compliant bistable mechanisms with three specified equilibrium positions (including two stable equilibrium positions and one unstable equilibrium position) [19]. In contrast, less work has been done on compliant multistable mechanisms, although they receive increasing attention. For example, Pendleton et al. [20] and Chen et al. [21,22] presented a few tristable mechanism configurations, Han et al. [23] derived a compliant quadristable mechanism by orthogonally nesting one bistable mechanism into another, and Halverson et al. [24] presented a tension-based compliant rolling-contact element exhibiting multistability. Also, a synthesis approach for compliant multistable mechanisms is developed, which achieves multistability by connecting multiple bistable mechanisms of different load thresholds in series [25]. It should be noted that the accessibilities of the stable positions of a multistable mechanism designed using this approach are path-dependent due to the series structure [25].

In this paper, two new approaches for synthesizing compliant multistable mechanisms are proposed, which enable designers to achieve multistability through use of a single bistable mechanism. The dramatic design potential of these synthesis approaches is illustrated by presenting several novel instantiations of multistable mechanisms (with three, four, five, and nine stable equilibrium positions).

The presentation of the paper is organized as follows: First, a fully compliant bistable mechanism and its force-deflection characteristics are described; second, the two approaches for synthesizing compliant multistable mechanisms are discussed in detail with a subsection on compound use of both approaches and a subsection on analyzing the actuation force required to move a multistable mechanism; finally, some concluding remarks regarding the proposed approaches and our future work are made.

2 Fully Compliant Bistable Mechanism

The structure of the fully compliant bistable mechanism [9] shown in Fig. 1 is used as a building block for synthesizing the multistable mechanisms described in this paper, which can be easily tailored to more complex designs. In addition, the designs can be extended by replacing the bistable mechanism by other bistable mechanisms, such as found in Refs. [13,14]. Figure 2 shows the force-deflection characteristics of a fully compliant bistable mechanism whose design parameters are listed in Table 1, which are achieved using nonlinear finite element analysis. It is understandable that the shuttle of the bistable mechanism is deflectable when pulled in the opposite direction from the as-fabricated position. The corresponding part of the force-deflection curve shown in Fig. 2 is referred to as reverse behavior. Also, when deflected past the second stable equilibrium position, the shuttle of the bistable mechanism can be further deflected before failure, and the

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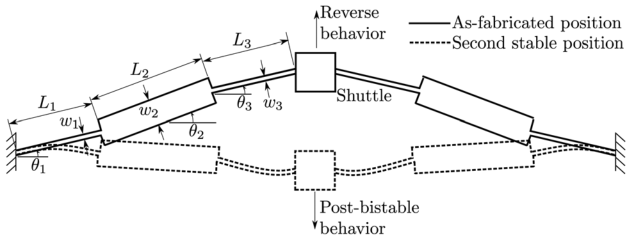

Fig. 1 The structure dimensions of the fully compliant bistable mechanism employed in the designs

corresponding deflection process is called postbistable behavior, which was named by Wilcox and Howell [26]. The whole force-deflection characteristics can be simply denoted as

$$F_b = f(z_0) \tag{1}$$

where z_0 is the travel distance of the shuttle from its first stable equilibrium position (i.e., the as-fabricated position) along the y axis.

Both the reverse behavior and the postbistable behavior are useful in synthesizing compliant multistable mechanisms, as will be further illustrated in this paper.

3 Synthesis Approaches for Compliant Multistable Mechanisms

The synthesis approaches proposed in this paper are based on the general mechanism configuration shown in Fig. 3, which employs a bistable compliant mechanism and several link–slider modules (the compliant counterpart of a link–slider module will be referred to as an end-effector, as will be seen in the fully compliant designs). The bistable mechanism is linked to slider 1 by link 1 (l_1) and the travel direction of slider 1 is orthogonal to the bistable mechanism's motion, slider 1 is linked to slider 2 by link 2 (l_2) and the travel direction of slider 2 is orthogonal to that of slider 1, slider 2 is linked to slider 3 by link 3 (l_3) and the travel direction of slider 3 is orthogonal to that of slider 2, and so forth. Without loss of generality, the shuttle of the bistable mechanism can be considered as slider 0.

Each slider has a certain number of stable positions within its range of motion, and each link–slider module can be considered to interact with a multistable mechanism. Among the stable positions of slider i, the first stable position is defined as the one having the maximum perpendicular distance between the slider and slideway ($i+1$), which distance is denoted as a_{i+1}. Once the first stable position is defined, each of the successive stable positions is numbered in sequence, as illustrated in Fig. 3. The distance a_{i+1} is the defining characteristic for the synthesis approaches, as will be shown later.

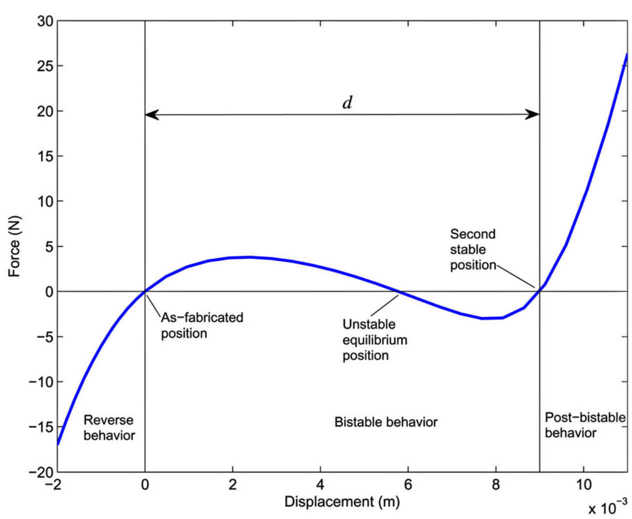

Fig. 2 The force-deflection characteristics of a bistable mechanism

Table 1 Design parameters of a fully compliant bistable mechanism. *H* is the out-of-plane thickness of the mechanism, N_p is the number of the bistable structure sets in parallel, and *E* is the Young's modulus.

Parameter	Value
E (Polypropylene)	2.4×10^9 Pa
N_p	2
H	6 mm
L_1	8 mm
θ_1	0°
w_1	0.8 mm
L_2	20 mm
θ_2	12°
w_2	3 mm
L_3	8 mm
θ_3	0°
w_3	0.8 mm

3.1 Approach 1: $l_i = a_i$. We first observe a partially compliant tristable mechanism design which is composed of a bistable mechanism and a link–slider module, being orthogonally assembled together. A schematic of this design in its three stable equilibrium positions is shown in Fig. 4. The slider travels in the positive and negative x-direction, while the bistable mechanism displaces in the positive y-direction and provides the force required to hold the slider in a stable equilibrium position. Due to the symmetry of the design, link 1 is collinear with the y axis at the second stable position ($l_1 = a_1$), and the travel distance of slider 1 between its stable equilibrium positions can be calculated as

$$S3_{1,2} = S3_{2,3} = \sqrt{l_1^2 - (l_1 - d)^2} \tag{2}$$

where $S3$ indicates the slider has three stable equilibrium positions within its range of motion, the subscript "1,2"specifies the distance traveled is from the first stable equilibrium position to the second, and d is the shuttle travel distance of the bistable mechanism between its two stable positions. An instantiation of this partially compliant tristable design fabricated from polypropylene is shown in Fig. 5. By replacing the link–slider module with a compliant end-effector, a fully compliant tristable mechanism is achieved, as shown in Fig. 6. Detailed design issues of this fully compliant tristable mechanism can be found in Ref. [22].

By adding another link–slider module with $l_2 = a_2$ to the tristable design shown in Fig. 4, we can get a quinquestable compliant mechanism. A schematic of this design in its five stable positions is illustrated in Fig. 7. Several distance parameters are given as

$$S5_{2,3} = S5_{3,5} = \sqrt{l_2^2 - (l_2 - S3_{1,2})^2} \tag{3}$$

$$S5_{1,2} = S5_{4,5} = \sqrt{l_2^2 - (l_2 - 2S3_{1,2})^2} - S5_{2,3} \tag{4}$$

A fully compliant quinquestable mechanism is shown in Fig. 8, which is realized by assembling a compliant end-effector (end-effector 2) to a fully complaint tristable mechanism (similar to the mechanism shown in Fig. 6) at a deflected stable position, as can be seen from Fig. 8(c).

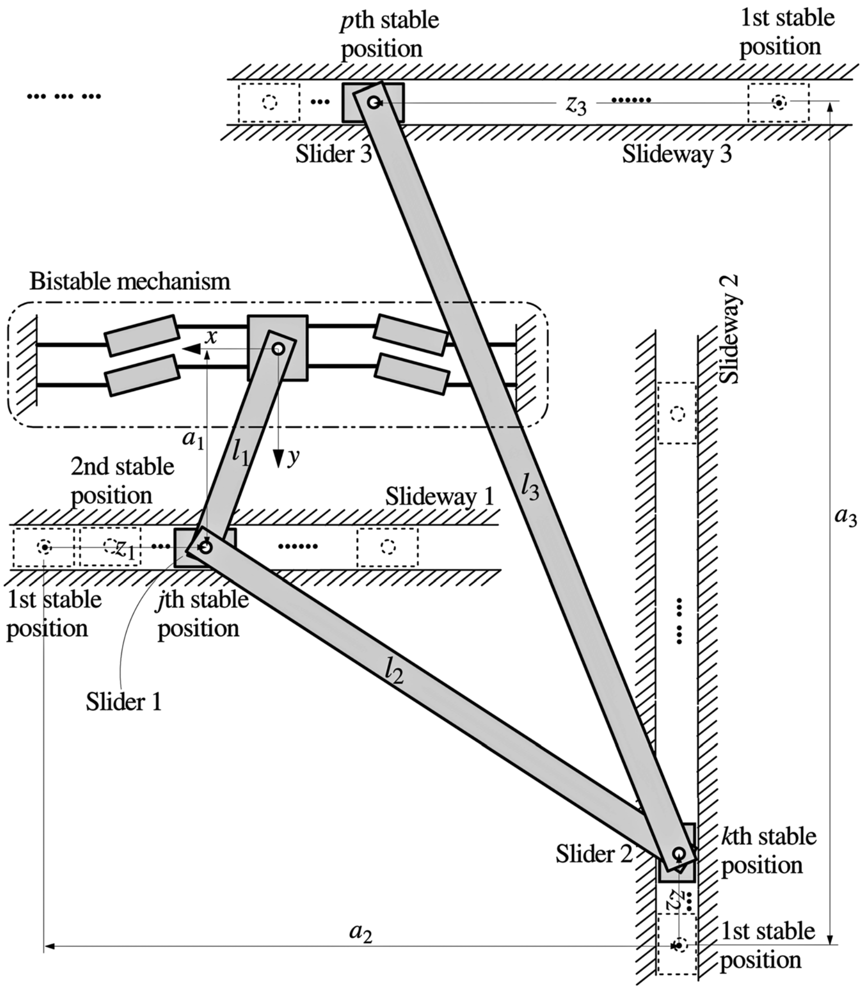

Fig. 3 A general configuration of the compliant multistable mechanisms

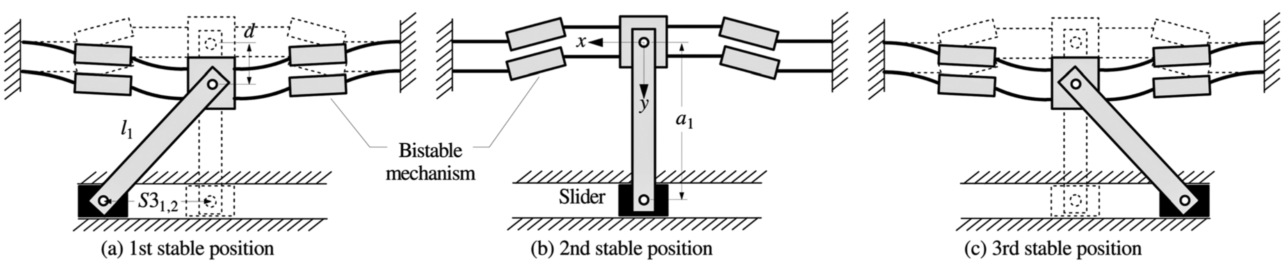

Fig. 4 A schematic of a compliant tristable mechanism illustrated in its three stable equilibrium positions

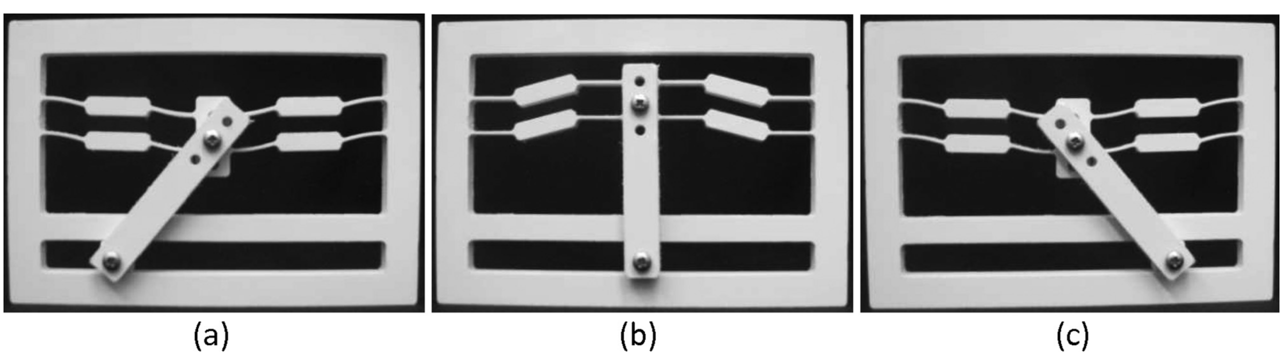

Fig. 5 A partially compliant tristable mechanism. (*a*) The first stable equilibrium position, (*b*) the second stable equilibrium position, and (*c*) the third stable equilibrium position.

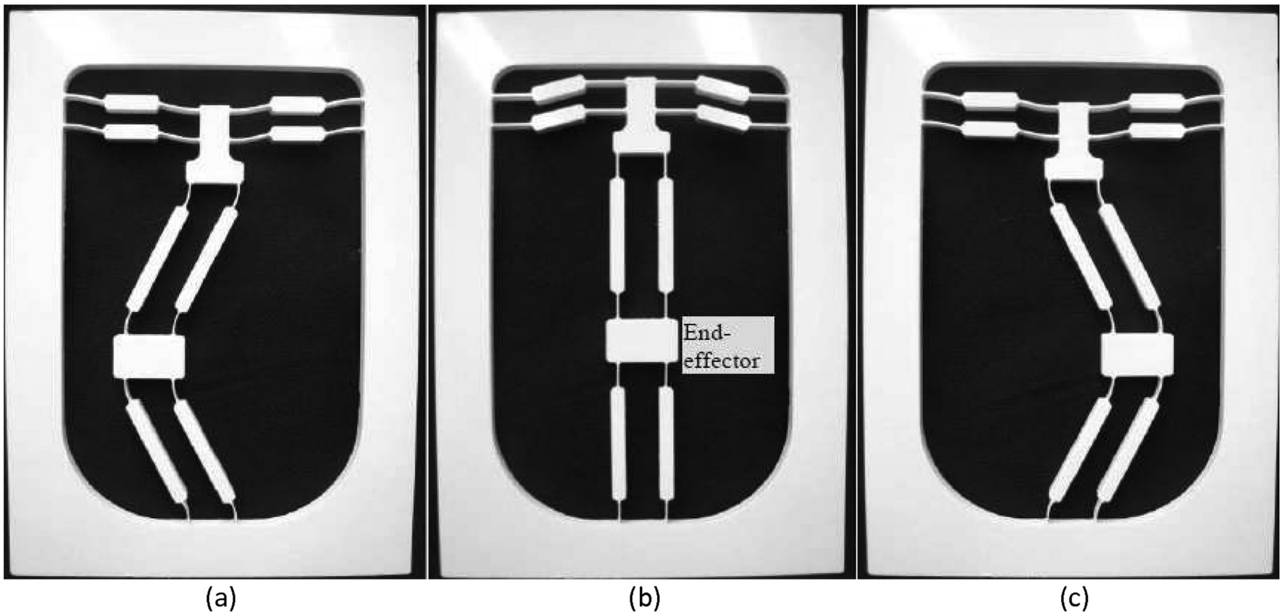

Fig. 6 A fully compliant tristable mechanism [22]. (*a*) The fabricated position (the second stable equilibrium position), (*b*) the first stable equilibrium position, and (*c*) the third stable equilibrium position. The end-effector is the functioning body.

In general, more stable positions can be achieved by employing more link–slider modules, and the number of stable positions can be calculated as

$$M_n = 2^n + 1 \tag{5}$$

where n is the number of the link–slider modules in a mechanism. For example, when $n=3$, we have $M_n=9$, which achieves a compliant novemstable mechanism (having nine stable positions), as shown in Fig. 9.

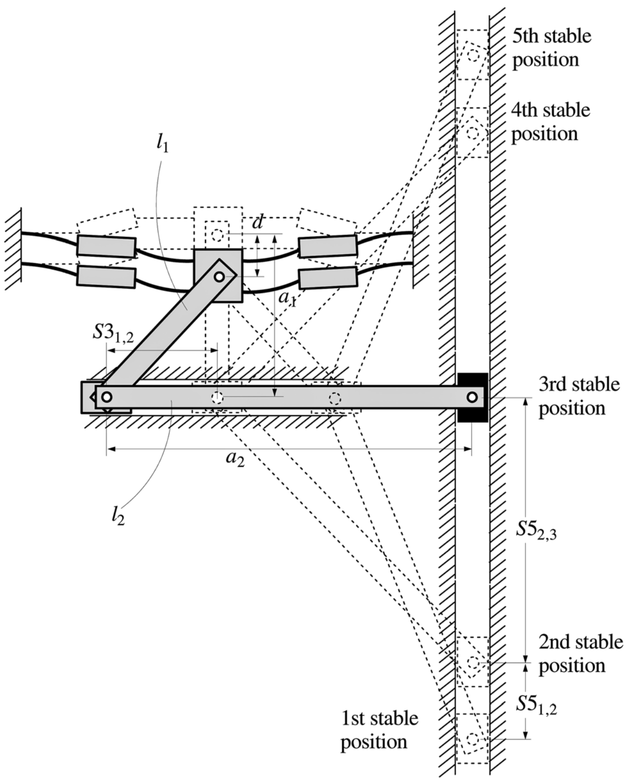

Fig. 7 A schematic of a compliant quinquestable mechanism

3.2 Approach 2: $l_i > a_i$. A simple modification of the tristable mechanism shown in Fig. 4 can make it exhibit quadristablility. When $l_1 > a_1$, slider 1 is assembled at a position (corresponding to the second stable position) that deviates from the symmetric axis (y) with an offset b, as shown in Fig. 10(b). Offset b can be expressed as

$$b = \sqrt{l_1^2 - a_1^2} \tag{6}$$

when slider 1 is moved from the second stable position to the unstable position shown in Fig. 10(c), the bistable mechanism exhibits the reverse behavior, along with which is a process of energy storage in the flexible segments. The stored strain energy reaches its maximum at the unstable equilibrium position, and the corresponding travel distance of the shuttle in the reverse direction is given as

$$r = l_1 - a_1 \tag{7}$$

When slider 1 is moved past the unstable equilibrium position, the bistable mechanism starts to release the stored energy until slider 1 reaches the third stable position. In other words, the reverse behavior causes the jump of slider 1 from the second to the third stable position. The travel distance of slider 1 between its second and third stable positions is given as

$$S4_{2,3} = 2b \tag{8}$$

In addition, the first and the fourth stable positions can be easily identified, as shown in Figs. 10(a) and 10(e). Due to the symmetric configuration of the design, the travel distance between the first and the second stable positions equals that between the third and the fourth stable positions:

$$S4_{1,2} = S4_{3,4} = \sqrt{l_1^2 - (a_1 - d)^2} - b \tag{9}$$

Figure 11 shows a prototype device of a partially compliant quadristable mechanism in its four stable positions, which was tailored from the tristable mechanism shown in Fig. 5. A fully compliant

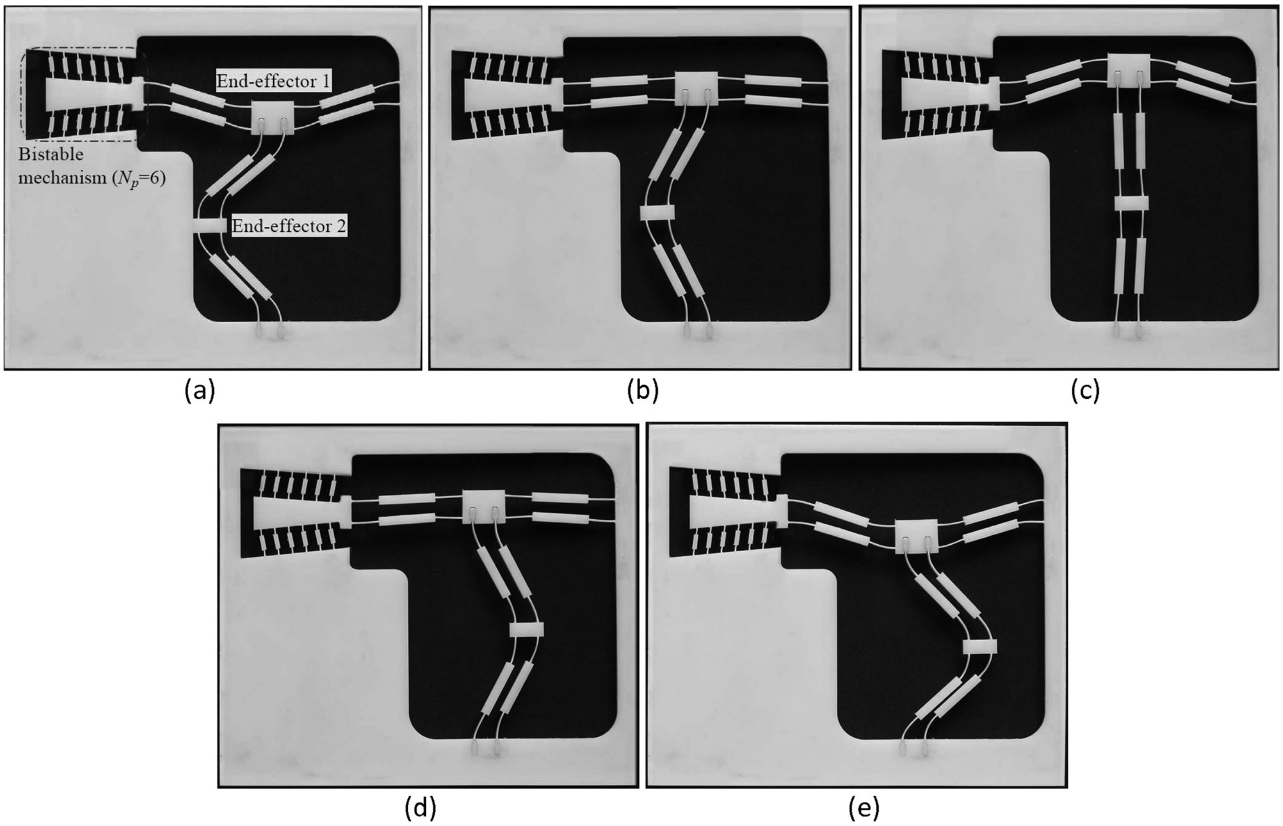

Fig. 8 A fully compliant mechanism with five stable equilibrium positions (end-effector 2 is the functioning body)

counterpart of this quadristable mechanism is illustrated in its four stable equilibrium positions in Fig. 12. This fully compliant quadristable mechanism has promise to be realized in the microregime due to its simplicity and one-piece structure and used as a multiplex optical switch for optical networking applications.

A compliant octostable mechanism can be achieved by adding another link–slider module with $l_2 > a_2$, as illustrated in Fig. 13. This octostable mechanism could be used as a single-pole-eight-throw switch for electrical systems. It is important to note that the reverse behavior causes the mechanism to jump between its second and third stable positions, while the postbistable behavior (which is similar to the functioning process of the reverse behavior described above) contributes to the switch between its fourth and fifth stable positions. A general equation to calculate the number of stable positions is given as

$$M_n = 2^{n+1} \tag{10}$$

3.3 Compound Use of the Two Synthesis Approaches.

It is important to note that the two synthesis approaches can also be used together in a single multistable mechanism design. For example, when $n=2$, $l_1=a_1$, and $l_2>a_2$, we have $M_2=6$, which achieves an sexastable mechanism, as shown in Fig. 14. Moreover, when $n=2$, $l_1>a_1$, and $l_2=a_2$, we have $M_2=7$.

In general, the number of stable positions of a synthesized multistable mechanism can be computed using the following recurrence equation:

$$\begin{cases} M_0 = 2 \\ \quad \cdots \\ M_i = \begin{cases} 2M_{i-1}-1, & l_i = a_i \\ 2M_{i-1}, & l_i > a_i \end{cases} \\ \quad \cdots \\ M_n = \begin{cases} 2M_{n-1}-1, & l_n = a_n \\ 2M_{n-1}, & l_n > a_n \end{cases} \end{cases} \tag{11}$$

where i is the sequence number of each link–slider module and n is the number of the link–slider modules. Table 2 presents a look-up table that can be used as a guide for designing multistable mechanisms with desired number of stable positions. It is shown that up to 16 stable equilibrium positions can be achieved when three link–slider modules are used.

3.4 Actuation Force.

We assume that frictions between sliders and slideways are neglectable. For a partially compliant multistable mechanism to move from its first to its last stable equilibrium positions, only the nth slider (the slider filled with black color in the schematics) needs to be actuated, and the actuation force is transferred to the other sliders due to the existence of slideways. The actuation force is given as

$$F(z_n) = (-1)^n \left(\prod_{i=1}^{n} \cot\theta_i \right) f(z_0) \tag{12}$$

where z_0 is determined from z_n using the recurrence equation given as

$$z_{i-1} = a_i - \sqrt{l_i^2 - (0.5H_i - z_i)^2}, 1 \le i \le n \tag{13}$$

and H_i is the distance between the first and the last stable equilibrium positions of slider i, which can be recursively computed from

$$\begin{cases} H_0 = d \\ H_i = 2\sqrt{l_i^2 - (a_i - H_{i-1})^2}, \quad 1 \le i \le n \end{cases} \tag{14}$$

Figure 15 plots the actuation force and potential energy for the sexastable mechanism shown in Fig. 14. In Fig. 15, $S6_1$, $S6_2$, $S6_3$,

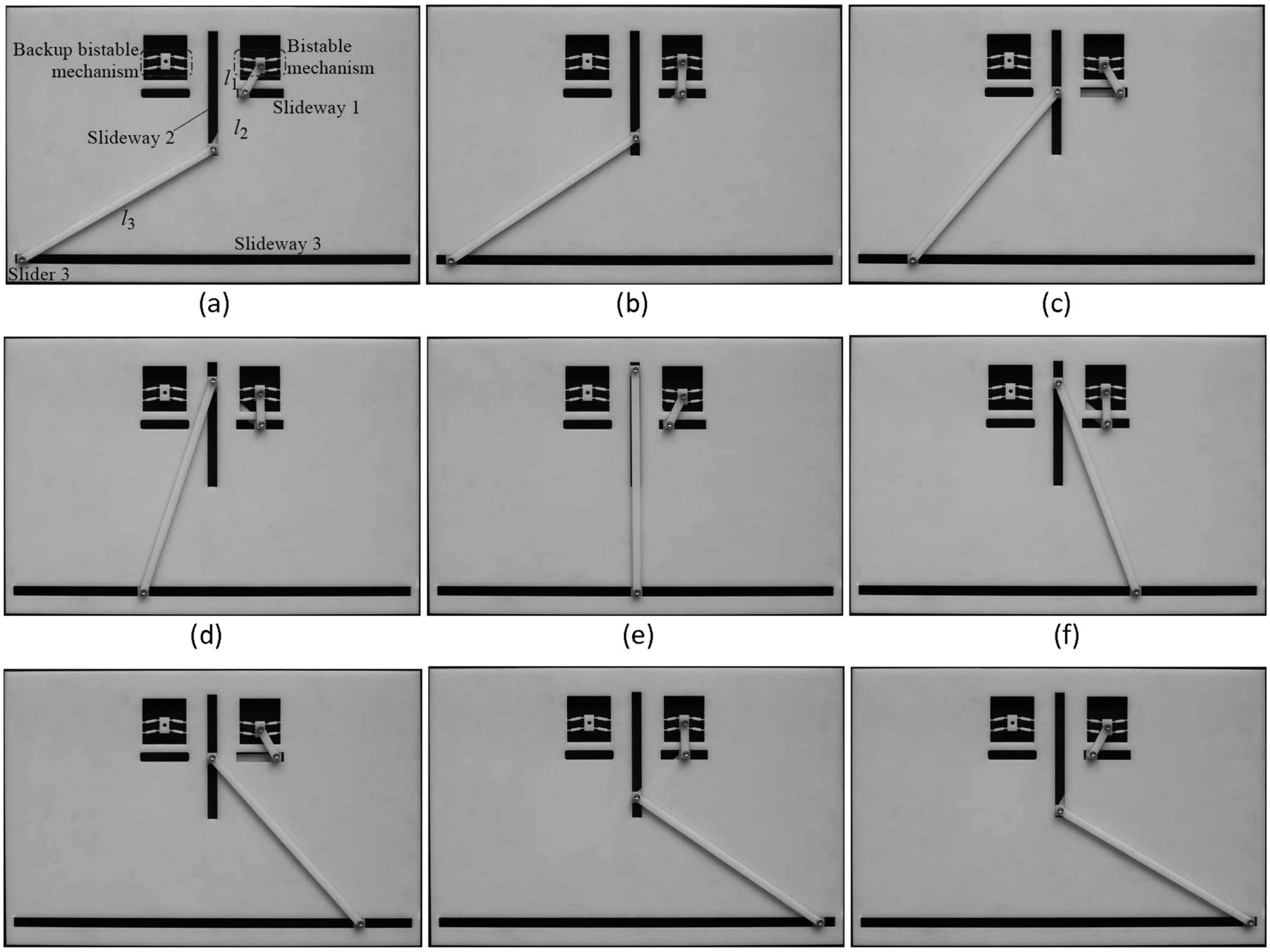

Fig. 9 A partially compliant mechanism with nine stable equilibrium positions (actuation force is applied on slider 3)

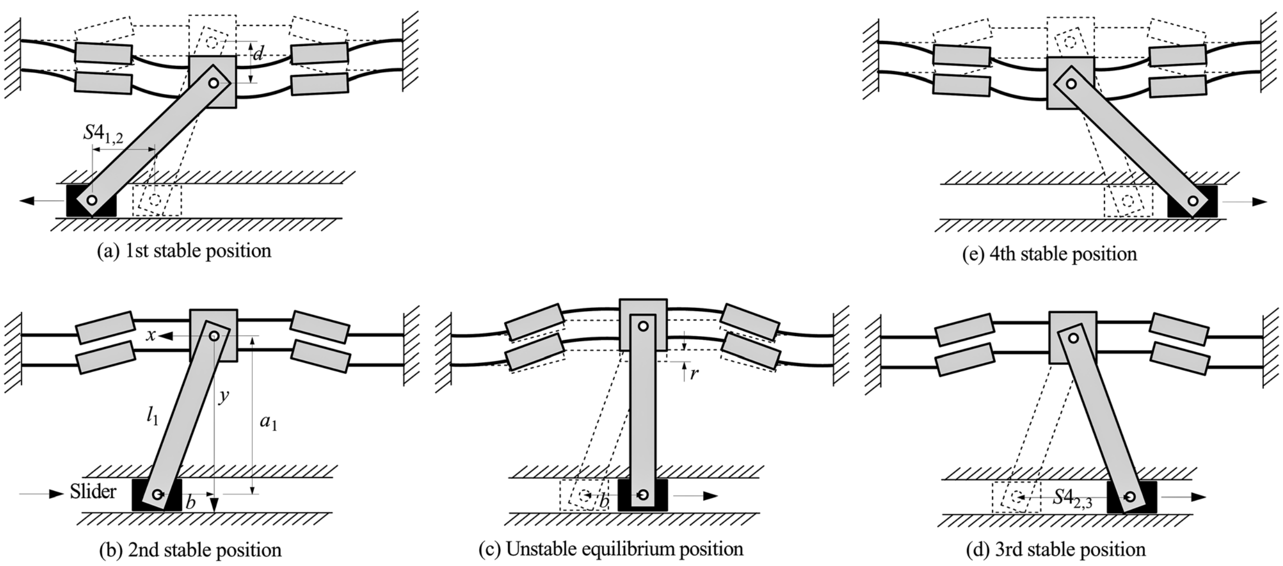

Fig. 10 A schematic of a compliant quadristable mechanism illustrated in its four stable equilibrium positions and one unstable equilibrium position

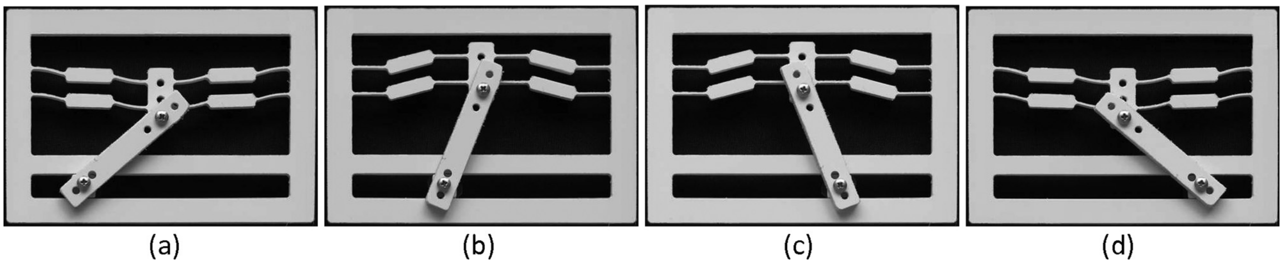

Fig. 11 A partially compliant quadristable mechanism. (*a*) The first stable equilibrium position, (*b*) the as-assembled position (the second stable equilibrium position), (*c*) the third stable equilibrium position, and (*d*) the fourth stable equilibrium position.

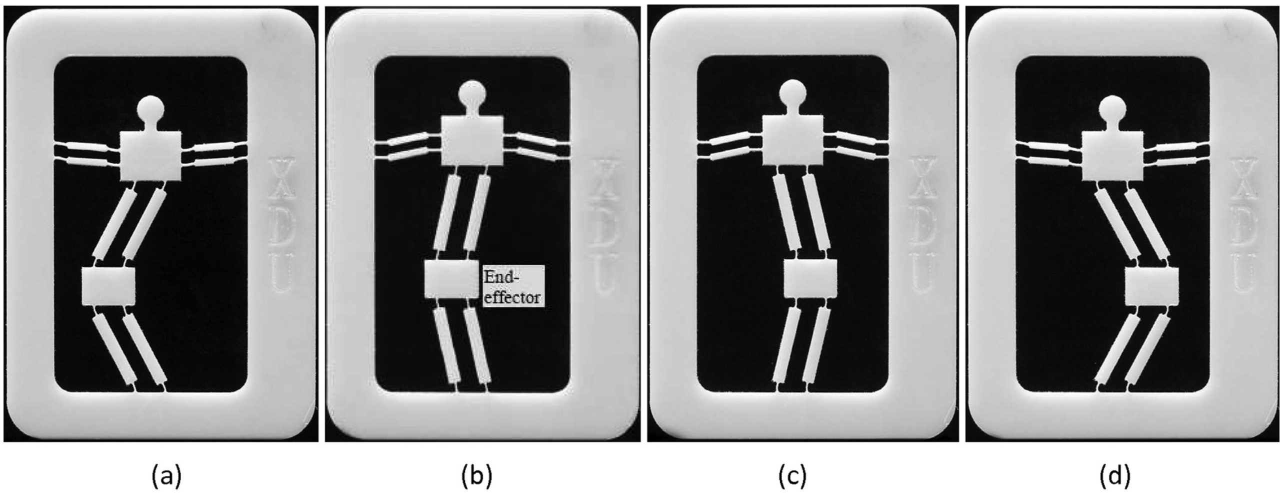

Fig. 12 A fully compliant quadristable mechanism with its as-fabricated position corresponds to its second stable equilibrium position. The end-effector is the functioning body.

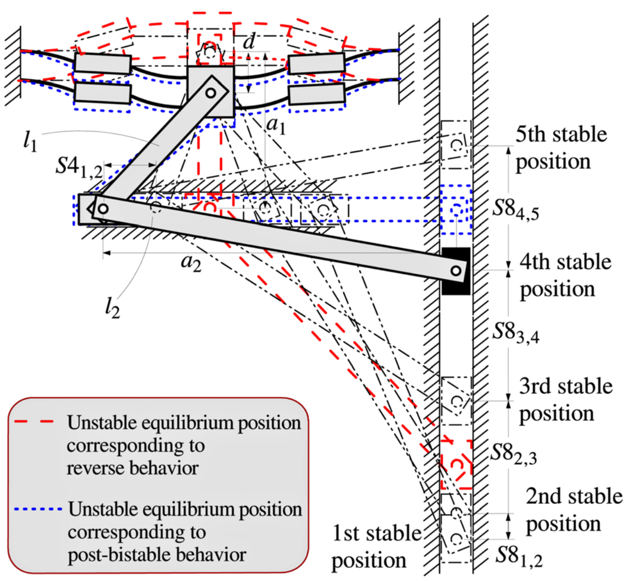

Fig. 13 A schematic of an octostable mechanism (the sixth, seventh, and eighth stable positions are not shown due to the symmetry of the configuration)

$S6_4$, $S6_5$, and $S6_6$ correspond to the six stable equilibrium positions of the sexastabe mechanism.

As to the design of a fully compliant multistable mechanism, the major difficulty lies in the tradeoff between the stiffness of the flexible members in the end-effectors and the force-deflection behavior of the bistable mechanism portion. The energy stored in the flexible members acts to return the bistable mechanism portion to its undeflected position, and to balance the competing forces often causes extremely high stresses. The modeling approach (based on the pseudo-rigid-body model) presented in Ref. [22] can be used to determine the actuation force for fully compliant multistable designs. This is not going to be discussed in the current paper.

4 Conclusion

Two new approaches for synthesizing compliant multistable mechanisms have been presented in this paper, which can enable designers to achieve multistability while only utilizing a single bistable mechanism. The potential of the synthesis approaches is demonstrated by schematic illustrations and the successful operation of several instantiations of designs. These synthesis approaches are intended to facilitate the design of a compliant mechanism with a desired number of stable positions by utilizing the existing knowledge of bistable mechanisms. Future work on this topic will be focused on the design and prototyping of several fully compliant multistable mechanisms and their applications.

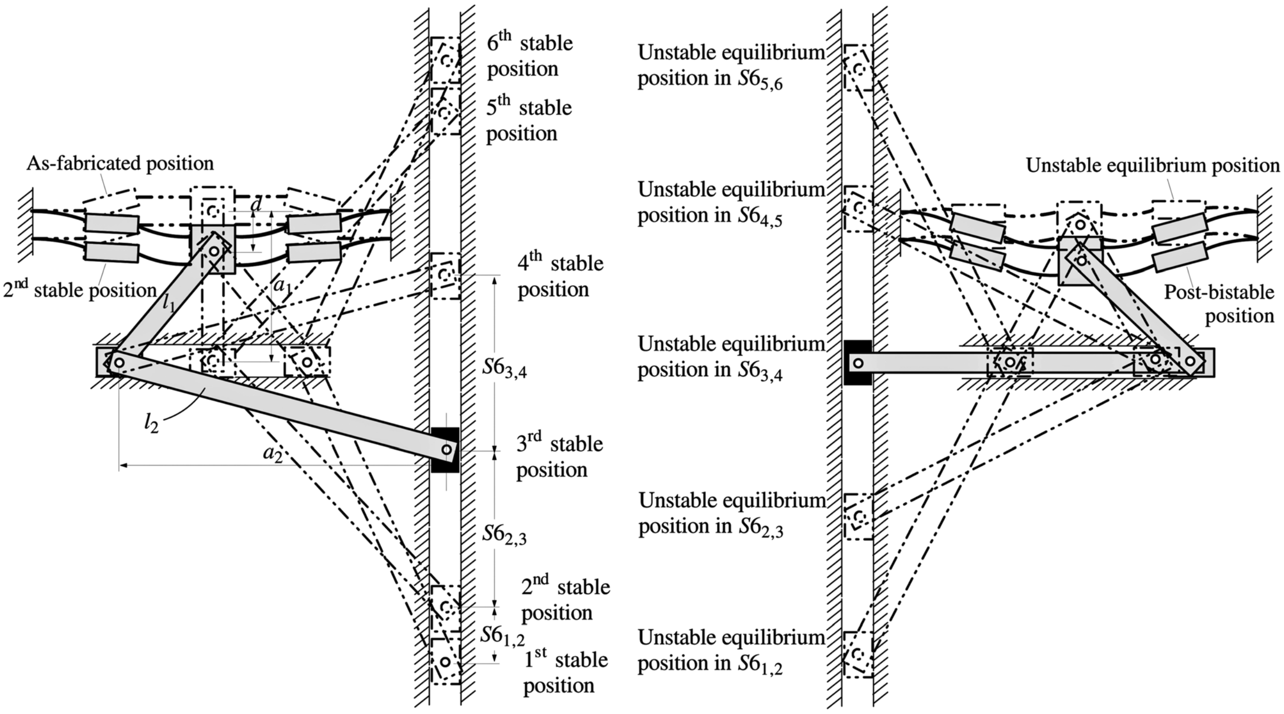

Fig. 14 A schematic of a sexastable mechanism (where $l_1 = a_1$ while $l_2 > a_2$)

Table 2 Total number of stable equilibrium positions of a multistable mechanism design

Number of link–slider modules (n)	Relationships/approaches	Total number of stable equilibrium positions (M_n)
1	$l_1 = a_1$	3
1	$l_1 > a_1$	4
2	$l_1 = a_1, l_2 = a_2$	5
2	$l_1 = a_1, l_2 > a_2$	6
2	$l_1 > a_1, l_2 = a_2$	7
2	$l_1 > a_1, l_2 > a_2$	8
3	$l_1 = a_1, l_2 = a_2, l_3 = a_3$	9
3	$l_1 = a_1, l_2 = a_2, l_3 > a_3$	10
3	$l_1 = a_1, l_2 > a_2, l_3 = a_3$	11
3	$l_1 = a_1, l_2 > a_2, l_3 > a_3$	12
3	$l_1 > a_1, l_2 = a_2, l_3 = a_3$	13
3	$l_1 > a_1, l_2 = a_2, l_3 > a_3$	14
3	$l_1 > a_1, l_2 > a_2, l_3 = a_3$	15
3	$l_1 > a_1, l_2 > a_2, l_3 > a_3$	16

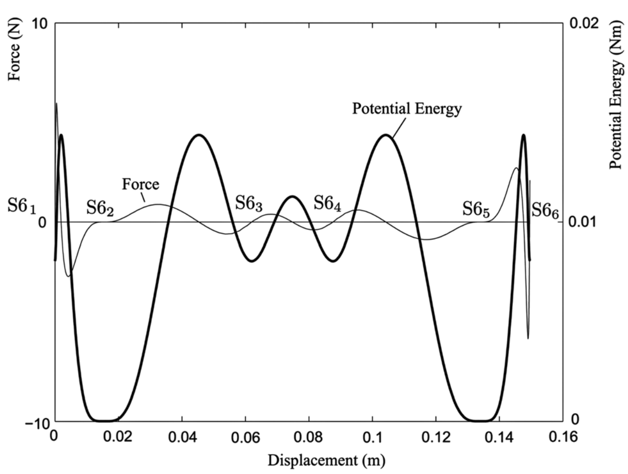

Fig. 15 The actuation force and potential energy versus the slider position

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