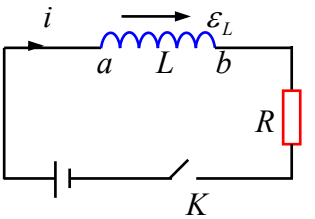
12—6 磁场的能量

一、自感储能



i： $0 \to I$

t， $\varepsilon_L = -L\dfrac{di}{dt}$

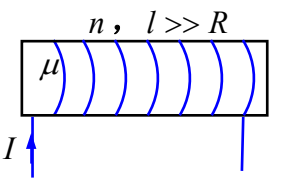
$U_{ab} = -\varepsilon_L = L\dfrac{di}{dt}$

$t \to t+dt$， $dA = U_{ab} i dt = L\dfrac{di}{dt} i dt = Lidi$

$A = \int dA = \int_0^I Lidi = \dfrac{1}{2}LI^2$

$\boxed{W_m = \dfrac{1}{2}LI^2}$：自感储能公式（对任意线圈都成立），$W_e = \dfrac{1}{2}CV^2$

二、磁场能量



磁能分布在磁场中

$L = \mu n^2 l \pi R^2$

$W_m = \dfrac{1}{2}LI^2$

$= \dfrac{1}{2}\mu n^2 l \pi R^2 I^2 = \dfrac{1}{2}\mu nI \cdot nI \cdot l\pi R^2$

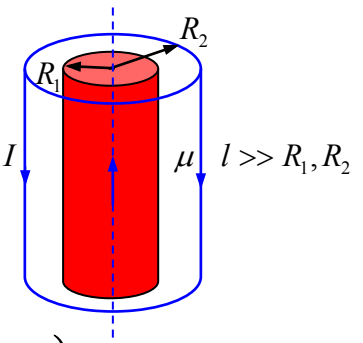
$= \dfrac{1}{2}BHV$

磁场能量密度： $w_m = \dfrac{dW_m}{dV} = \dfrac{1}{2}BH = \dfrac{1}{2}\dfrac{B^2}{\mu} = \dfrac{1}{2}\mu H^2 = \dfrac{1}{2}\vec{B}\cdot\vec{H}$

均匀磁场： $W_m = w_m V$

非均匀磁场： $W_m = \int_V w_m dV = \int_V \dfrac{1}{2}BHdV$

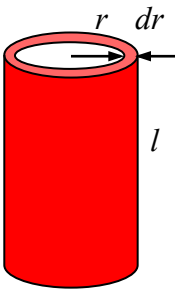
例：一段载流同轴电缆



求：磁能及 L，（$\mu_{导体} \approx \mu_0$）

解：$H=\begin{cases}\dfrac{Ir}{2\pi R_1^2} & r<R_1\\ \dfrac{I}{2\pi r} & R_2<r<R_2\\ 0 & r>R_2\end{cases}$，$w_m=\dfrac{1}{2}\mu H^2=\begin{cases}\dfrac{\mu_0 I^2 r^2}{8\pi^2 R_1^4} & r<R_1\\ \dfrac{\mu I^2}{8\pi^2 r^2} & R_2<r<R_2\\ 0 & r>R_2\end{cases}$

$$W_m=\int_V w_m dV$$



薄圆柱壳，$dV=2\pi r dr l$

$$W_m=\int_0^{R_1}\frac{\mu_0 I^2 r^2}{8\pi^2 R_1^4}2\pi r dr l+\int_{R_1}^{R_2}\frac{\mu I^2}{8\pi^2 r^2}2\pi r dr l$$

$$=\frac{\mu_0 I^2 l}{16\pi}+\frac{\mu I^2 l}{4\pi}\ln\frac{R_2}{R_1}$$

$W_m=\dfrac{1}{2}LI^2$，$L=2W_m/I^2=\dfrac{\mu_0 l}{8\pi}+\dfrac{\mu l}{2\pi}\ln\dfrac{R_2}{R_1}$