11—5 磁场对电流的作用

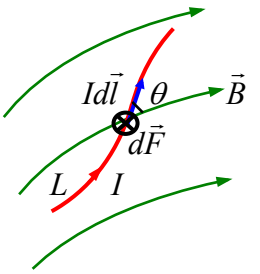
第 1 节　安培定律

$\boxed{d\vec{F} = Id\vec{l} \times \vec{B}}$ ——— 安培定律

$dF = IdlB\sin\theta$

方向：$d\vec{F} \perp \Pi(Id\vec{l}, \vec{B})$

指向：右手螺旋法则



$\vec{F} = \int d\vec{F} = \int_L Id\vec{l} \times \vec{B}$

$Oxyz$，$d\vec{F} = dF_x\vec{i} + dF_y\vec{j} + dF_z\vec{k}$

$F_x = \int dF_x$，$F_y = \int dF_y$，$F_z = \int dF_z$

$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$，一般情况下，$F \neq \int dF$

例：

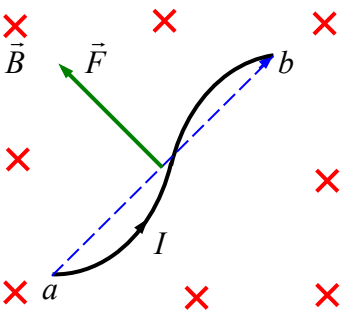
解：$\vec{F} = \int_L Id\vec{l} \times \vec{B}$

$= I(\int_{a(L)}^{b} d\vec{l}) \times \vec{B}$

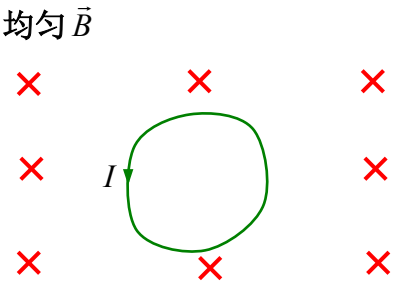
$= I\overrightarrow{ab} \times \vec{B}$

$F = I\overline{ab}B\sin\theta$

θ：$\overrightarrow{ab}$ 与 $\vec{B}$ 的夹角

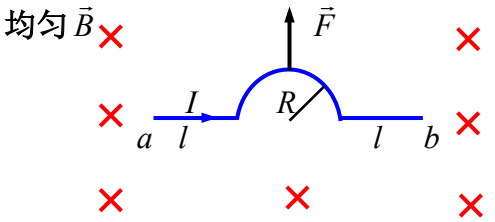


例：



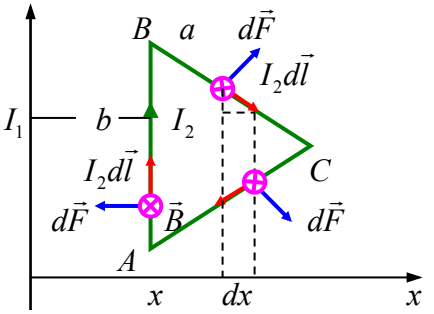
解：$\vec{F} = \oint_L Id\vec{l} \times \vec{B} = I(\oint_L d\vec{l}) \times \vec{B} = \mathbf{0}$

例：计算导线受力



解：$\vec{F} = I\overrightarrow{ab} \times \vec{B}$，$F = I\overline{ab}B\sin\theta = I2(l+R)B$，方向↑

例：正三角形线圈受力



解：**AB**：$dF = I_2 dlB = I_2 dl \dfrac{\mu_0 I_1}{2\pi b}$，方向←

$$F_{AB} = \int dF = \int_A^B \frac{\mu_0 I_1 I_2}{2\pi b} dl = \frac{\mu_0 I_1 I_2 a}{2\pi b}，方向←$$

BC：$dF = I_2 dlB$

$$F_{BC} = \int dF = \int I_2 dlB = \int_b^{b+a\sin 60^\circ} \frac{\mu_0 I_1 I_2}{2\pi x} \frac{dx}{\sin 60^\circ}$$

$$= \frac{\mu_0 I_1 I_2}{\sqrt{3}\pi} \ln \frac{b + \frac{\sqrt{3}}{2} a}{b}，与 x 轴夹角 60^\circ$$

同理，$F_{AC} = F_{BC}$，与 x 轴夹角（-60°）

合力：$F_x = -F_{AB} + F_{BC}\cos 60^\circ + F_{AC}\cos(-60^\circ)$

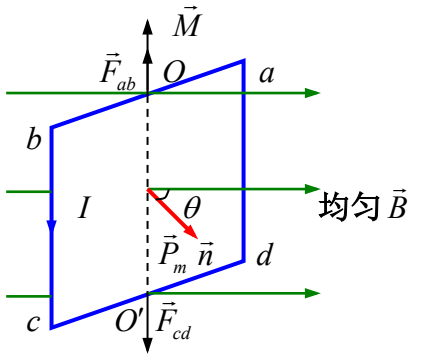
$$= -F_{AB} + F_{BC}$$

$$= -\frac{\mu_0 I_1 I_2 a}{2\pi b} + \frac{\mu_0 I_1 I_2}{\sqrt{3}\pi} \ln \frac{b + \frac{\sqrt{3}}{2} a}{b}$$

$$F_y = 0$$

第 2 节　磁场对载流线圈的作用

一、均匀磁场中的载流线圈



$\vec{P}_m = IS\vec{n}$

$F_{ab} = I\overline{ab}B\sin(\frac{\pi}{2}+\theta)$

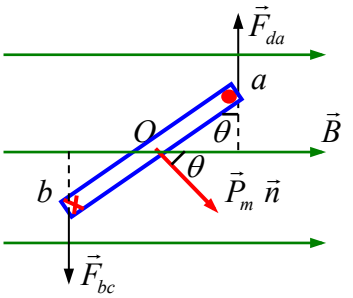
$= I\overline{ab}B\cos\theta$，

$F_{bc} = I\overline{bc}B$

$F_{cd} = I\overline{cd}B\sin(\frac{\pi}{2}-\theta)$

$= I\overline{cd}B\cos\theta$，

$F_{da} = I\overline{da}B$



合力 $\vec{F} = 0$

$M = F_{da}\frac{1}{2}\overline{ab}\sin\theta$

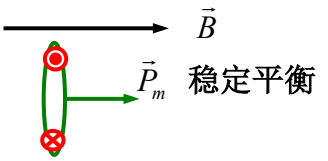
$+F_{bc}\frac{1}{2}\overline{ab}\sin\theta$，

$= F_{da}\overline{ab}\sin\theta = I\overline{da}B\ \overline{ab}\sin\theta = ISB\sin\theta = P_m B\sin\theta$

$$\boxed{\vec{M} = \vec{P}_m \times \vec{B}}$$

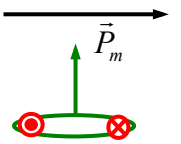
讨论（1）$\vec{P}_m // \vec{B}$

$\vec{F} = 0$，$\vec{M} = 0$



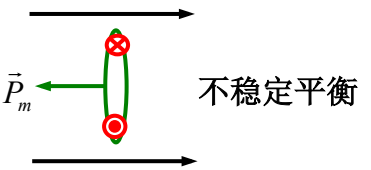
（2）$\vec{P}_m \perp \vec{B}$

$\vec{F} = 0$，$M = P_m B$



（3）$\vec{P}_m // (-\vec{B})$

$\vec{F} = 0$，$\vec{M} = 0$



说明：均匀磁场，$\vec{M} = \vec{P}_m \times \vec{B}$ 适用于任意形状的线圈

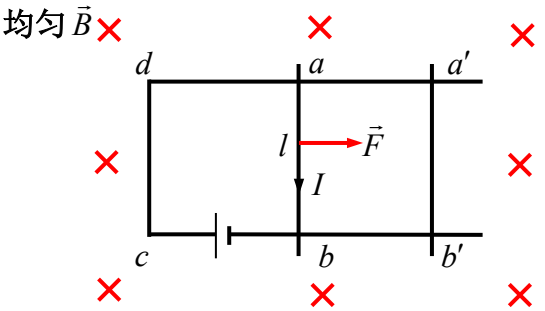
$\vec{M} = \vec{P} \times \vec{E}$

二、非均匀磁场中的线圈

线圈受到的合磁力及力矩不一定为零

第 3 节　磁力的功

一、磁场对载流导线的功

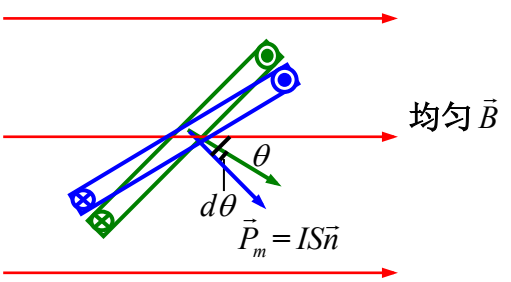


$F = IlB$

$A = F\overline{aa'} = IlB(\overline{da'} - \overline{da}) = I(Bl\overline{da'} - Bl\overline{da})$

$= I(\Phi_{m2} - \Phi_{m1}) = I\Delta\Phi_m$

二、磁场对载流线圈的功



$\vec{M} = \vec{P}_m \times \vec{B}$，　$M = P_m B\sin\theta$

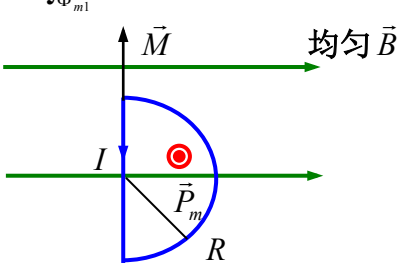
$dA = M(-d\theta) = P_m B\sin\theta(-d\theta) = ISB\sin\theta(-d\theta)$

$= Id(BS\cos\theta) = Id\Phi_m$

$A = \int dA = \int_{\Phi_{m1}}^{\Phi_{m2}} Id\Phi_m$ —— 磁力作功的一般表达式

若 I 恒定，　$A = I\int_{\Phi_{m1}}^{\Phi_{m2}} d\Phi_m = I(\Phi_{m2} - \Phi_{m1}) = I\Delta\Phi_m$

例：

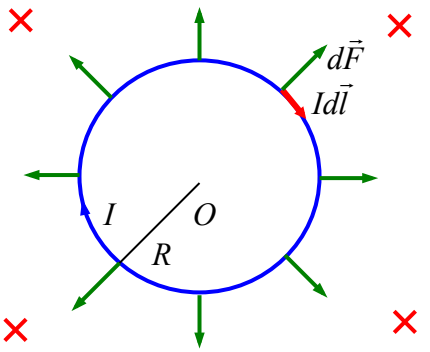


求：（1）$\vec{M}$ =?　（2）线圈转过 90°，　A = ?

解：（1）$\vec{M} = \vec{P}_m \times \vec{B}$，　$M = P_m B\sin\theta = I\dfrac{1}{2}\pi R^2 B$，方向↑

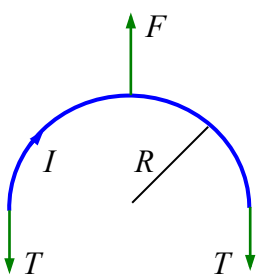
（2）$A = I\Delta\Phi_m = I\dfrac{1}{2}\pi R^2 B$

例：　　均匀 $\vec{B}$



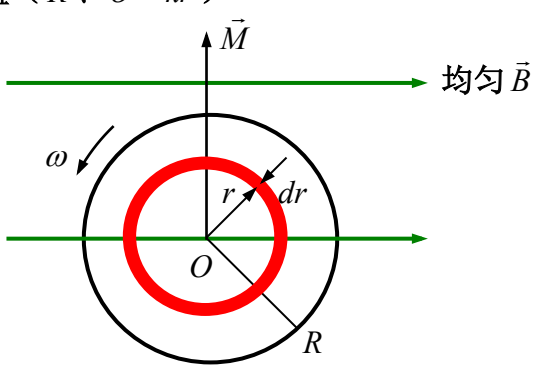
求：张力

解：



$F = I2RB = 2T$，　$T = IRB$

例：带电圆盘（R、$\sigma = kr$）



求：$\vec{M}$

解：$\vec{M} = \vec{P}_m \times \vec{B}$

$dS = 2\pi r dr$，　$dq = \sigma 2\pi r dr = kr2\pi r dr$

$$dI = \frac{\omega}{2\pi} dq = \frac{\omega}{2\pi} kr2\pi r dr = \omega k r^2 dr$$

$$dP_m = dIS = \pi r^2 \omega k r^2 dr = \pi \omega k r^4 dr$$

$$P_m = \int dP_m = \int_0^R \pi \omega k r^4 dr = \frac{1}{5}\pi \omega k R^5$$，⊙

$$M = P_m B \sin\theta = \frac{1}{5}\pi \omega k R^5 B$$，方向↑