

11—3 磁通量 磁高斯定理

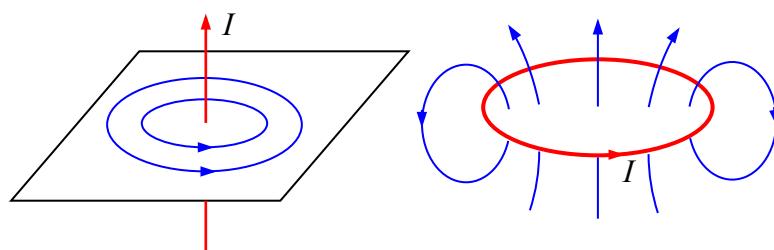
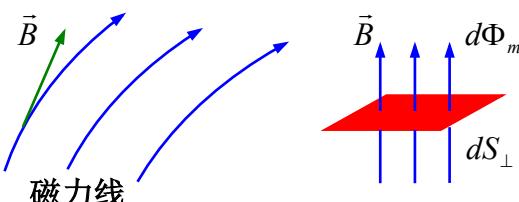
一、磁力线

$$B = \frac{d\Phi_m}{dS_{\perp}}$$

B 等于通过和磁场相垂直的单位面积上的磁力线条数

性质：

“磁力线都是闭合曲线，并与电流相互套合，磁力线的回绕方向与电流方向满足右手定则。”



二、磁通量

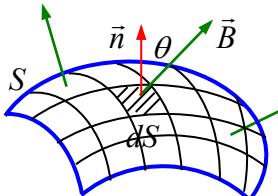
穿过曲面 S 的磁力线

条数 Φ_m ：磁通量（标量）

$$d\Phi_m = BdS_{\perp} = BdS \cos \theta = B_n dS$$

$$d\Phi_m = \vec{B} \cdot d\vec{S}$$

$$\Phi_m = \int d\Phi_m = \int_S \vec{B} \cdot d\vec{S} \quad \text{SI: } Tm^2 = Wb \text{ (韦伯)}$$

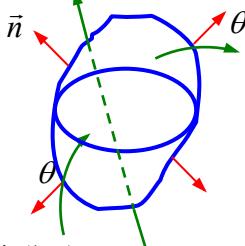


三、磁高斯定理

闭合曲面：

$$\Phi_m = \oint_S \vec{B} \cdot d\vec{S}$$

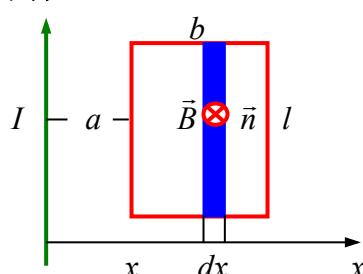
取闭合曲面的外法线方向
为正法线方向



$$\oint_S \vec{B} \cdot d\vec{S} = 0 \quad \text{—— 磁高斯定理}$$

磁场是无源场

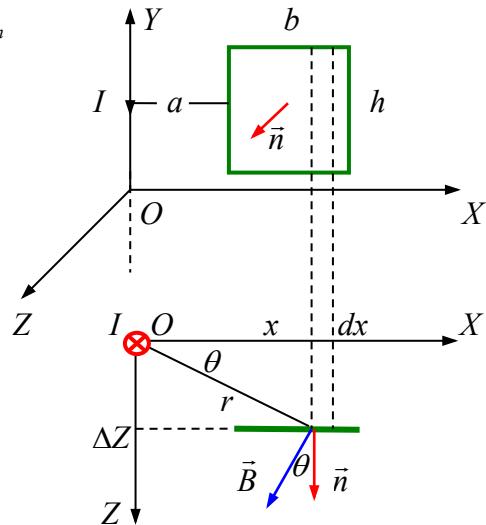
例：求 Φ_m



$$\text{解: } d\Phi_m = \vec{B} \cdot d\vec{S} = B \cos \theta dS = \frac{\mu_0 I}{2\pi x} l dx$$

$$\Phi_m = \int d\Phi_m = \int_a^{a+b} \frac{\mu_0 I}{2\pi x} l dx = \frac{\mu_0 Il}{2\pi} \ln \frac{a+b}{a}$$

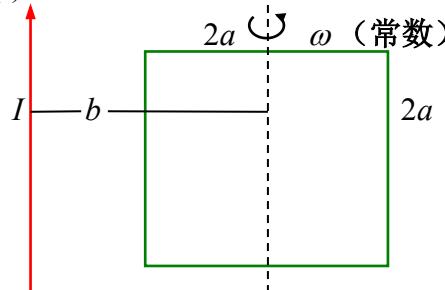
例：求 Φ_m



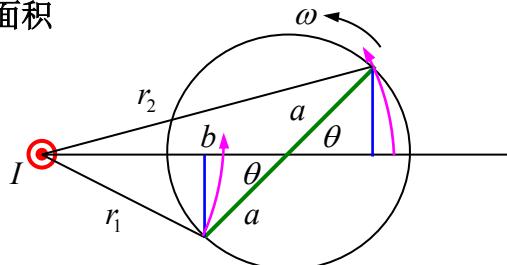
$$\text{解: } d\Phi_m = \vec{B} \cdot d\vec{S} = B \cos \theta dS = \frac{\mu_0 I}{2\pi r} \frac{x}{r} h dx = \frac{\mu_0 I h}{2\pi} \frac{x dx}{x^2 + (\Delta Z)^2}$$

$$\begin{aligned}\Phi_m &= \int d\Phi_m = \int_a^{a+b} \frac{\mu_0 I h}{2\pi} \frac{x dx}{x^2 + (\Delta Z)^2} \\ &= \frac{\mu_0 I h}{4\pi} \ln[x^2 + (\Delta Z)^2] \Big|_a^{a+b} = \frac{\mu_0 I h}{4\pi} \ln \frac{(a+b)^2 + (\Delta Z)^2}{a^2 + (\Delta Z)^2}\end{aligned}$$

例：求 $\Phi_m(t)$



解：等效面积



$$\theta = \omega t, \quad \Phi_m(t) = \frac{\mu_0 I}{2\pi} 2a \ln \frac{r_2}{r_1}$$

$$r_2^2 = a^2 + b^2 + 2ab \cos \omega t, \quad r_1^2 = a^2 + b^2 - 2ab \cos \omega t$$

$$\Phi_m(t) = \frac{\mu_0 I a}{2\pi} \ln \frac{a^2 + b^2 + 2ab \cos \omega t}{a^2 + b^2 - 2ab \cos \omega t}$$