

# 稳恒电流的磁场

## 第1节 磁场与磁感应强度

### 一、磁场

$$\left. \begin{array}{l} \text{运动电荷} \\ \text{载流导线} \\ \text{磁体} \end{array} \right\} \Leftrightarrow \text{磁场} \Leftrightarrow \left. \begin{array}{l} \text{运动电荷} \\ \text{载流导线} \\ \text{磁体} \end{array} \right\}$$

稳恒电流：不随时间变化的电流

### 二、磁感应强度 $\vec{B}$

$\vec{B}$  的方向：静止磁针的 N 极指向

电流元（矢量）： $Id\vec{l}$

大小： $Idl$

方向：沿电流方向

$$dF \propto Idl \sin \theta$$

同一位置： $\frac{dF}{Idl \sin \theta}$  恒定

与电流元无关

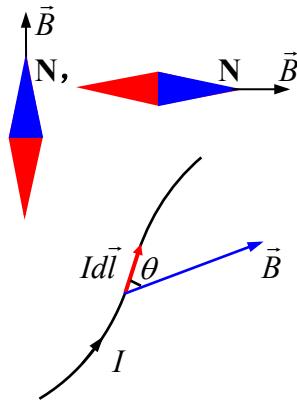
不同位置： $\frac{dF}{Idl \sin \theta}$  一般不同，与磁场中位置有关

$$B = \frac{dF}{Idl \sin \theta}, \quad dF = BIdl \sin \theta$$

$$\theta = 90^\circ, \quad Id\vec{l} \perp \vec{B}, \quad dF_m = BIdl, \quad B = \frac{dF_m}{Idl}$$

“磁场中某点处，磁感应强度  $\vec{B}$  是一个矢量，其方向沿该点处静止磁针的 N 极指向，其大小等于单位电流元受到的最大磁场所力”

SI：N/(Am)=T(特斯拉)



## 第2节 毕奥-萨伐尔定律

### 一、毕奥-萨伐尔定律

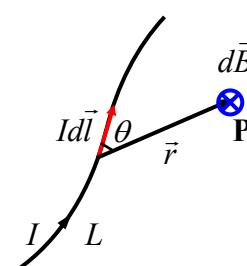
$$dB \propto \frac{Idl \sin \theta}{r^2}, \quad dB = \frac{\mu_0}{4\pi} \frac{Idl \sin \theta}{r^2}$$

$\mu_0$ ：真空磁导率

$$\text{SI: } \mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$$

$$d\vec{B} \perp \Pi(Id\vec{l}, \vec{r})$$

指向：右手螺旋法则



$$dB = \frac{\mu_0}{4\pi} \frac{Idl \times \vec{r}}{r^3}$$

$$dB = \frac{\mu_0}{4\pi} \frac{Idlr \sin \theta}{r^3} = \frac{\mu_0}{4\pi} \frac{Idl \sin \theta}{r^2}$$

方向：沿  $Idl \times \vec{r}$

## 二、稳恒电流的磁场

$$\vec{B} \text{ 满足矢量迭加原理, } \vec{B} = \int d\vec{B} = \int_L \frac{\mu_0}{4\pi} \frac{Idl \times \vec{r}}{r^3}$$

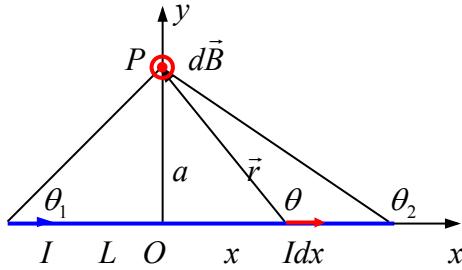
$$Oxyz : d\vec{B} = dB_x \vec{i} + dB_y \vec{j} + dB_z \vec{k}$$

$$B_x = \int dB_x, \quad B_y = \int dB_y, \quad B_z = \int dB_z$$

$$\vec{B} = B_x \vec{i} + B_y \vec{j} + B_z \vec{k}$$

一般,  $B \neq \int dB$ , 除非所有  $d\vec{B}$  方向一致

例: 一段载流直导线的  $\vec{B}$



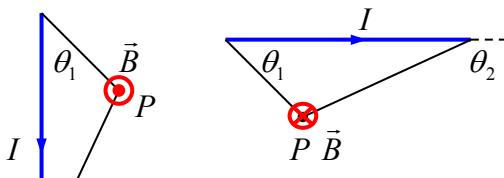
$$\text{解: } dB = \frac{\mu_0}{4\pi} \frac{Idx \sin \theta}{r^2}, \quad B = \int dB = \int_L \frac{\mu_0}{4\pi} \frac{Idx \sin \theta}{r^2}$$

$$a/x = \tan(\pi - \theta) = -\tan \theta, \quad x = -a \tan \theta, \quad dx = a \csc^2 \theta d\theta$$

$$a/r = \sin(\pi - \theta) = \sin \theta, \quad r = a \csc \theta$$

$$B = \int_{\theta_1}^{\theta_2} \frac{\mu_0}{4\pi} \frac{Ia \csc^2 \theta d\theta \sin \theta}{a^2 \csc^2 \theta} = \frac{\mu_0 I}{4\pi a} \int_{\theta_1}^{\theta_2} \sin \theta d\theta = \frac{\mu_0 I}{4\pi a} (\cos \theta_1 - \cos \theta_2)$$

方向



讨论: (1)  $I$  不变,  $L \rightarrow \infty$

$$\text{无限长载流直导线, } \theta_1 = 0, \quad \theta_2 = \pi, \quad B = \frac{\mu_0 I}{2\pi a}$$

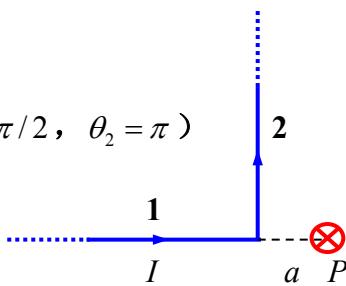
(2)

(3)  $\vec{B} = \vec{B}_1 + \vec{B}_2$

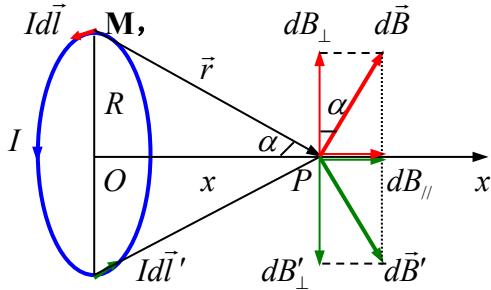
$$\vec{B}_1 = 0$$

$$B_2 = \frac{\mu_0 I}{4\pi a} \quad (\theta_1 = \pi/2, \quad \theta_2 = \pi)$$

$$B = \frac{\mu_0 I}{4\pi a}$$



例：单匝圆线圈（圆电流）轴线上的  $\vec{B}$



$$\text{解: } dB = \frac{\mu_0}{4\pi} \frac{Idl}{r^2}, \quad dB_{\parallel} = dB \sin \alpha, \quad dB_{\perp} = dB \cos \alpha$$

$$\vec{B}_{\perp} = 0$$

$$\begin{aligned} B &= \int dB_{\parallel} = \int dB \sin \alpha = \int \frac{\mu_0}{4\pi} \frac{Idl}{r^2} \sin \alpha = \frac{\mu_0 I \sin \alpha}{4\pi r^2} 2\pi R \\ &= \frac{\mu_0}{2} \frac{RI \sin \alpha}{r^2}, \quad (\sin \alpha = R/r, \quad r = \sqrt{x^2 + R^2}) \\ &= \frac{\mu_0}{2} \frac{R^2 I}{(x^2 + R^2)^{3/2}} \end{aligned}$$

讨论: (1)  $x = 0$ , 圆心处,  $B = \frac{\mu_0 I}{2R}$

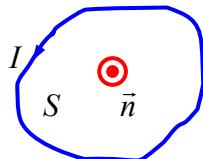
$$(2) \quad x \gg R, \quad B \approx \frac{\mu_0}{2} \frac{R^2 I}{x^3}$$

$$\text{磁矩: } \vec{P}_m = IS\vec{n}$$

$$P_m = IS$$

方向: 沿正法线方向

$$\text{圆线圈 } \vec{P}_m = IS\vec{n} = I\pi R^2 \vec{n}$$



磁偶极子

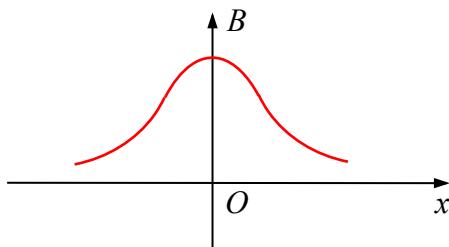
$$\mathbf{N} \text{ 匝线圈 } \vec{P}_m = NIS\vec{n}$$

$$\vec{B} = \frac{\mu_0}{2\pi} \frac{\vec{P}_m}{(x^2 + R^2)^{3/2}}$$

(3)  $\mathbf{N}$  匝圆电流轴线上

$$B = \frac{\mu_0}{2} \frac{R^2 IN}{(x^2 + R^2)^{3/2}}$$

(4)



例：无限长金属薄板，宽度为  $b$

厚度可忽略，电流为  $I$

电流在宽度方向均匀分布

求：薄板中线上方高  $h$  处的  $\vec{B}$

解： $i = I / b$

$$dI = i dx$$

$$dB = \frac{\mu_0 dI}{2\pi r} = \frac{\mu_0 i}{2\pi r} dx$$

$$dB_x = dB \cos \theta$$

$$dB_y = dB \sin \theta$$

$$B_y = \int dB_y = 0$$

$$B = \int dB_x = \int dB \cos \theta ,$$

$$= \int \frac{\mu_0 i}{2\pi r} dx \cos \theta = \frac{\mu_0 i}{2\pi} \int \frac{h dx}{r^2}, \quad (\cos \theta = h/r, \quad r^2 = x^2 + h^2)$$

$$= \frac{\mu_0 i}{2\pi} \int_{-b/2}^{b/2} \frac{h dx}{h^2 + x^2} \quad \left( \int \frac{dx}{a^2 + x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \right)$$

$$= \frac{\mu_0 i}{2\pi} \operatorname{arctg} \frac{x}{h} \Big|_{-b/2}^{b/2} = \frac{\mu_0 i}{\pi} \operatorname{arctg} \frac{b}{2h}, \quad \text{方向} \rightarrow$$

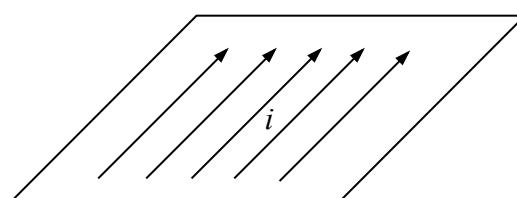
讨论：（1） $b$ 、 $i$ 一定， $h \rightarrow 0$ ，薄板中线上方或下方附近

$$B \rightarrow \frac{\mu_0 i \pi}{\pi 2} = \frac{1}{2} \mu_0 i$$

（2） $h$ 、 $I$ 一定， $b \rightarrow 0$ ，无限长载流直导线

$$B \approx \frac{\mu_0 i}{\pi} \frac{b}{2h} = \frac{\mu_0 I}{2\pi h} \quad (\operatorname{arctg} x \approx x)$$

（3） $h$ 、 $i$ 一定， $b \rightarrow \infty$ ，无限大均匀载流平面



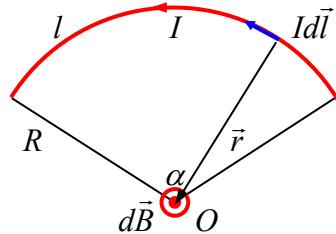
$$B \rightarrow \frac{\mu_0 i \pi}{\pi 2} = \frac{1}{2} \mu_0 i$$

$$\overbrace{\hspace{10cm}}^{\vec{B}}$$

$$\overbrace{\hspace{10cm}}^i$$

$$\overbrace{\hspace{10cm}}^{\text{均匀磁场}}$$

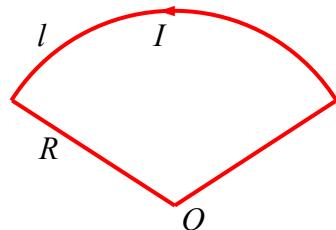
例：载流圆弧线圆心处的  $\vec{B}$



$$\text{解: } dB = \frac{\mu_0}{4\pi} \frac{Idl}{R^2}$$

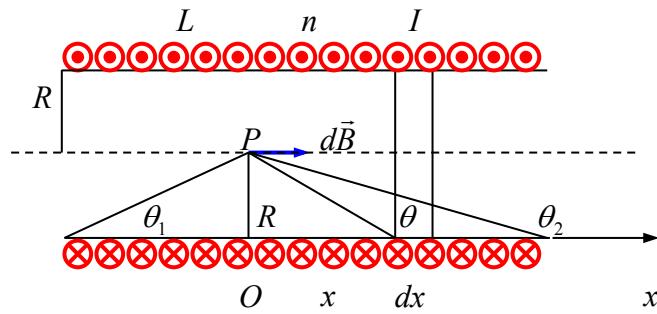
$$B = \int dB = \int \frac{\mu_0}{4\pi} \frac{Idl}{R^2} = \frac{\mu_0 I l}{4\pi R^2} = \frac{\mu_0 I}{4\pi R} \alpha$$

注，  $\alpha$ ：弧度



结果不变

例：载流密绕直螺线管轴线上  $\vec{B}$



$$\text{解: } dN = ndx, \quad dB = \frac{\mu_0}{2} \frac{R^2 IdN}{(x^2 + R^2)^{3/2}} = \frac{\mu_0}{2} \frac{R^2 Indx}{(x^2 + R^2)^{3/2}}$$

$$B = \int dB = \int \frac{\mu_0}{2} \frac{R^2 Indx}{(x^2 + R^2)^{3/2}}$$

$$R/x = \tan(\pi - \theta) = -\tan\theta, \quad x = -R \cot\theta, \quad dx = R \csc^2\theta d\theta$$

$$x^2 + R^2 = R^2 \cot^2\theta + R^2 = R^2 \csc^2\theta$$

$$B = \frac{1}{2} \mu_0 n I R^2 \int_{\theta_1}^{\theta_2} \frac{R \csc^2\theta d\theta}{R^3 \csc^3\theta}$$

$$= \frac{1}{2} \mu_0 n I \int_{\theta_1}^{\theta_2} \sin\theta d\theta = \frac{1}{2} \mu_0 n I (\cos\theta_1 - \cos\theta_2)$$

讨论：（1）  $L \rightarrow \infty$ ，无限长直螺线管，  $\theta_1 = 0, \theta_2 = \pi$

$$B = \mu_0 n I \text{ (常数), 均匀磁场}$$

（2）半无限长直螺线管