静电场习题课 II

四、电荷与外电场的相互作用

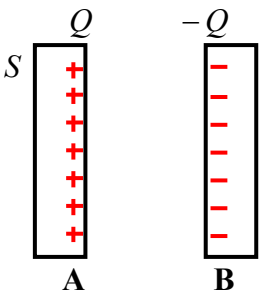
外电场，$\vec{E}_{外}$、$U_{外}$

		受力	电势能
研究对象	点电荷	$\vec{F}=q_0\vec{E}_{外}$	$W=q_0U_{外}$
	点电荷系	$\vec{F}=\sum q_i\vec{E}_{外i}$	$W=\sum q_iU_{外i}$
	带电体	$\vec{F}=\int\vec{E}_{外}dq$	$W=\int U_{外}dq$

注意：$\vec{E}_{外}$、$U_{外}$ 是外电场的场强和电势

不包括研究对象自身产生的电场和电势

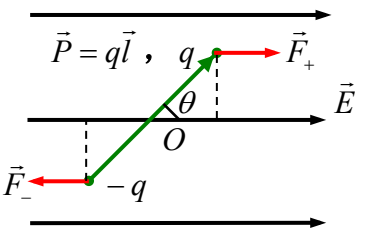
例：两块导体板的相互作用力



解：

$$F=\frac{\sigma}{2\varepsilon_0}Q=\frac{Q^2}{2\varepsilon_0S}$$

例：电偶极子在均匀外电场中受力、力矩、电势能



解：受力：$\vec{F}=\vec{F}_++\vec{F}_-=q\vec{E}-q\vec{E}=0$

力矩：$M=F_+\frac{l}{2}\sin\theta+F_-\frac{l}{2}\sin\theta=F_+l\sin\theta$

$=qEl\sin\theta=PE\sin\theta$

$$\boxed{\vec{M}=\vec{P}\times\vec{E}}$$

电势能：$W=W_-+W_+=-qU_-+qU_+=-q(U_--U_+)$

$=-qEl\cos\theta=-PE\cos\theta=-\vec{P}\cdot\vec{E}$

讨论（1）$\vec{P}//\vec{E}$

$\vec{F}=0$，$\vec{M}=0$

$W=-PE$

（2）$\vec{P}\perp\vec{E}$

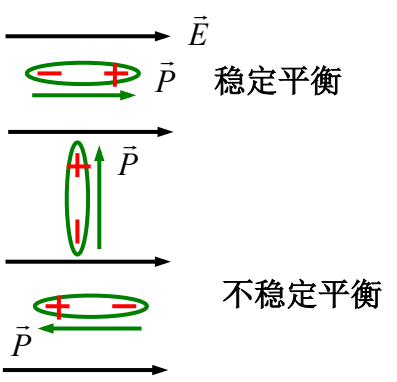
$\vec{F}=0$，$M=PE$

$W=0$

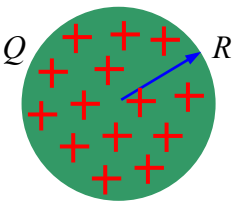
（3）$\vec{P}//(-\vec{E})$

$\vec{F}=0$，$\vec{M}=0$

$W=PE$



例：均匀带电球体的静电能



解：$\rho = Q/(\frac{4}{3}\pi R^3)$

$$E = \begin{cases} \frac{\rho}{3\varepsilon_0} r & r < R \\ \frac{Q}{4\pi\varepsilon_0 r^2} & r > R \end{cases}$$

$$w = \frac{1}{2}\varepsilon_0\varepsilon_r E^2 = \begin{cases} \frac{\rho^2}{18\varepsilon_0} r^2 & r < R \\ \frac{Q^2}{32\pi^2\varepsilon_0 r^4} & r > R \end{cases}$$

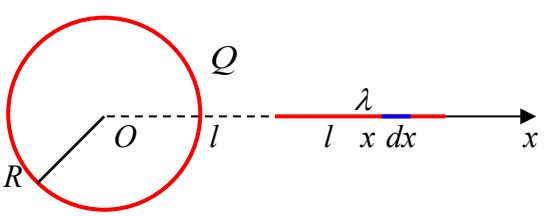
$$W = \int w dV = \int_0^R w dV + \int_R^\infty w dV$$

$$= \int_0^R \frac{\rho^2}{18\varepsilon_0} r^2 4\pi r^2 dr + \int_R^\infty \frac{Q^2}{32\pi^2\varepsilon_0 r^4} 4\pi r^2 dr$$

$$= \frac{2\rho^2\pi}{45\varepsilon_0} R^5 + \frac{Q^2}{8\pi\varepsilon_0 R}$$

$$= \frac{3Q^2}{20\pi\varepsilon_0 R}$$

例：求：均匀带电直线段在均匀带电球面电场中的电势能



解：$dq = \lambda dx$

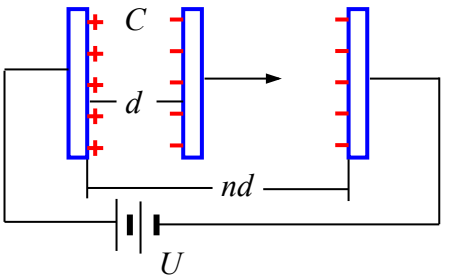
均匀带电球面在 x 处的电势 $U = \frac{Q}{4\pi\varepsilon_0 x}$

$$dW = U dq = \frac{Q}{4\pi\varepsilon_0 x}\lambda dx$$

$$W = \int dW = \int_l^{2l} \frac{Q}{4\pi\varepsilon_0 x}\lambda dx = \frac{\lambda Q}{4\pi\varepsilon_0}\ln 2$$

例：空气平板电容器，电容为C，与电压为U的电源相连

求：两板间距由$d \to nd$过程中，外力作的功



解：方法 I：功能原理

$$A_{电源} + A_{外力} = \Delta W$$

$$A_{电源} = U\Delta Q = U\Delta(CU) = U^2\Delta C$$

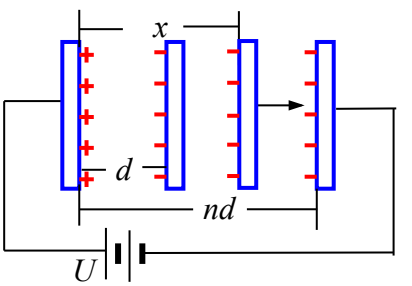
$$\Delta W = \Delta(\frac{1}{2}CU^2) = \frac{1}{2}U^2\Delta C$$

$$A_{外力} = \Delta W - A_{电源} = \frac{1}{2}U^2\Delta C - U^2\Delta C = -\frac{1}{2}U^2\Delta C$$

$$\Delta C = \frac{C}{n} - C = -C(1-\frac{1}{n})$$

$$A_{外力} = -\frac{1}{2}U^2\Delta C = \frac{1}{2}U^2C\ (1-\frac{1}{n}) > 0$$

方法 II：



$$C(x) = \frac{\varepsilon_0 S}{x}$$

$$Q(x) = C(x)U = \frac{\varepsilon_0 SU}{x}$$

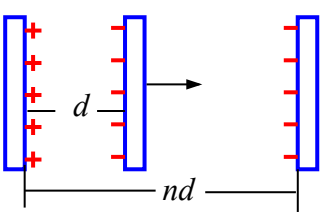
$$\sigma(x) = \frac{Q(x)}{S}$$

$$E = \frac{\sigma(x)}{2\varepsilon_0} = \frac{Q(x)}{2\varepsilon_0 S}，\quad F_{吸引} = EQ(x) = \frac{Q^2(x)}{2\varepsilon_0 S} = \frac{\varepsilon_0 SU^2}{2x^2} = F_{外}$$

$$A = \int F_{外}dx = \int_d^{nd} \frac{\varepsilon_0 SU^2}{2x^2}dx = \frac{1}{2}\varepsilon_0 SU^2(\frac{1}{d} - \frac{1}{nd}) = \frac{1}{2}CU^2(1-\frac{1}{n}) > 0$$

例：接上题，断开电源后，

求：两板间距由$d \to nd$过程中，外力作的功



解：方法 I：功能原理

$$A_{外力} = \Delta W = \frac{Q^2}{2C'} - \frac{Q^2}{2C}$$

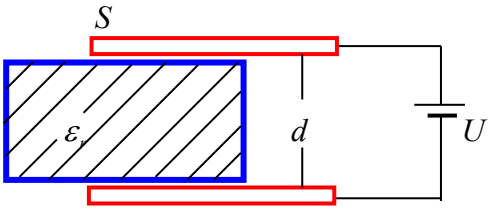
$$= \frac{Q^2}{2C}n - \frac{Q^2}{2C} = \frac{Q^2}{2C}(n-1) = \frac{1}{2}CU^2(n-1) > 0$$

方法 II：$F_{吸引} = Q^2/(2\varepsilon_0 S) = F_{外}$（不变）

$$A = \int F_{外}dx = F_{外}(nd - d) = \frac{Q^2}{2\varepsilon_0 S}d(n-1)$$

$$= \frac{Q^2}{2C}(n-1) = \frac{1}{2}CU^2(n-1) > 0$$

例：平板电容器，U 不变，将一厚 d 、ε_r 的介质板插入电容器



求：（1）电场能量变化（2）电源的功（3）电场对介质板作的功

解：（1）$\Delta W=\Delta(\frac{1}{2}CU^2)=\frac{1}{2}U^2\Delta C=\frac{1}{2}U^2(\varepsilon_r C_0-C_0)$

$$=\frac{1}{2}U^2C_0(\varepsilon_r-1)=\frac{1}{2}\frac{\varepsilon_0 SU^2}{d}(\varepsilon_r-1)$$

（2）$A_{电源}=U\Delta Q=U\Delta(CU)=U^2\Delta C=\frac{\varepsilon_0 SU^2}{d}(\varepsilon_r-1)$

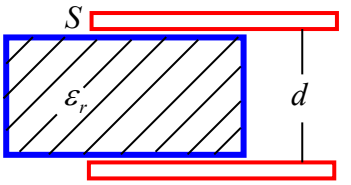
（3）$A_{电场}=-\Delta W$?×

$A_{电源}=\Delta W+A_{电场}$

$A_{电场}=A_{电源}-\Delta W=\frac{1}{2}\frac{\varepsilon_0 SU^2}{d}(\varepsilon_r-1)>0$

例：接上题，断开电源后，再插入介质板

求：（1）电场能量变化（3）电场对介质板作的功



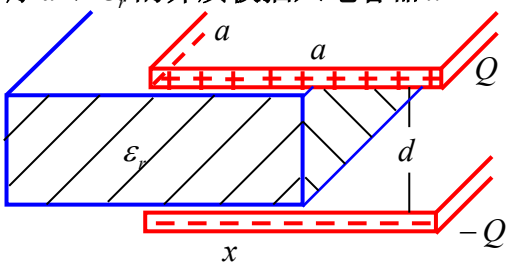
解：（1）$\Delta W=\frac{Q^2}{2C'}-\frac{Q^2}{2C_0}=\frac{Q^2}{2\varepsilon_r C_0}-\frac{Q^2}{2C_0}=\frac{Q^2}{2C_0}(\frac{1}{\varepsilon_r}-1)$

$$=-\frac{1}{2}C_0U^2(1-\frac{1}{\varepsilon_r})=-\frac{1}{2}\frac{\varepsilon_0 S}{d}U^2(1-\frac{1}{\varepsilon_r})<0$$

（3）$A_{电场}=-\Delta W=\frac{1}{2}\frac{\varepsilon_0 S}{d}U^2(1-\frac{1}{\varepsilon_r})>0$

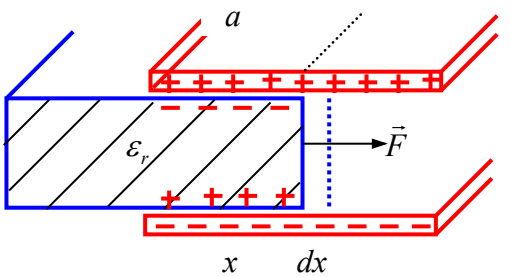
例：平板电容器，板面为正方形，边长 a，板间距 d，带电 $\pm Q$

把一厚 d 、ε_r 的介质板插入电容器 x



求：介质板受电场作用力的大小和方向

解：



$$dA = Fdx = -dW$$

$$F = -\frac{dW}{dx}，\quad W = \frac{Q^2}{2C}$$

$$C(x) = \frac{\varepsilon_0\varepsilon_r ax}{d} + \frac{\varepsilon_0 a(a-x)}{d}$$

$$= \frac{\varepsilon_0 a}{d}(\varepsilon_r x + a - x) = \frac{\varepsilon_0 a}{d}[a + (\varepsilon_r - 1)x]$$

$$W = \frac{Q^2 d}{2\varepsilon_0 a[a + (\varepsilon_r - 1)x]}$$

$$F = -\frac{dW}{dx} = \frac{Q^2 d(\varepsilon_r - 1)}{2\varepsilon_0 a[a + (\varepsilon_r - 1)x]^2}$$

若电容器与电压为U的电源保持连接，再做本题。

$$dA_{电源} = dW + dA_{电场}$$

$$dA_{电源} = UdQ = Ud(CU) = U^2 dC$$

$$dW = d(\frac{1}{2}CU^2) = \frac{1}{2}U^2 dC$$

$$dA_{电场} = Fdx$$

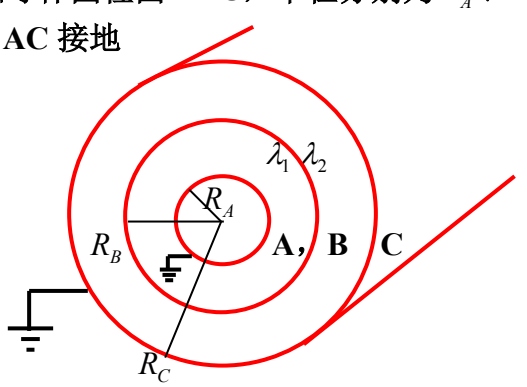
$$U^2 dC = \frac{1}{2}U^2 dC + Fdx$$

$$Fdx = \frac{1}{2}U^2 dC$$

$$F = \frac{1}{2}U^2\frac{dC}{dx} = \frac{1}{2}U^2\frac{\varepsilon_0 a}{d}(\varepsilon_r - 1)$$

例：三个同轴导体圆柱面 **ABC**，半径分别为R_A、R_B、R_C

B 带电，**AC** 接地



求：**B** 内外表面上电荷线密度λ_1/λ_2=?