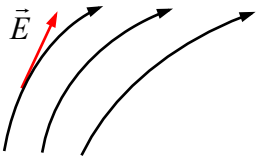
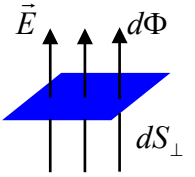
第 3 节　电通量　高斯定理

一、电力线

$$E=\frac{d\Phi}{dS_{\perp}}$$

E 等于通过和电场相垂直的单位面积上的电力线条数





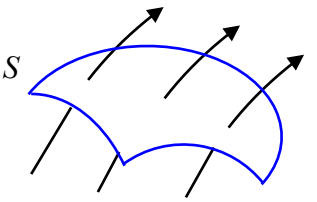
静电场电力线的性质：

（1）起自正电荷，终止于负电荷，在无电荷区域不能中断

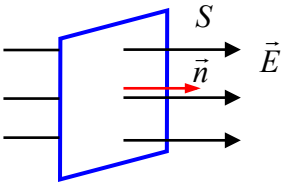
（2）不能形成闭合曲线，任意两条电力线不能相交

二、电通量

穿过曲面 S 的电力线条数 Φ：电通量（标量）

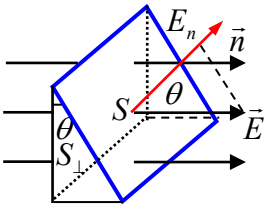


1、均匀电场



$$\Phi=ES$$

2、均匀电场



$$\Phi=ES_{\perp}=ES\cos\theta=E_nS$$

3、任意电场中的任意曲面

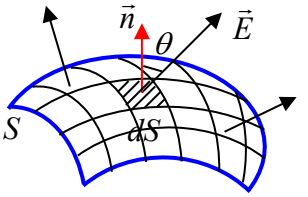
$$d\Phi=EdS_{\perp}=EdS\cos\theta=E_ndS$$

定义：面元矢量 $d\vec{S}=dS\cdot\vec{n}$

$$\vec{E}\cdot d\vec{S}=EdS\cos\theta$$

$$d\Phi=\vec{E}\cdot d\vec{S}$$

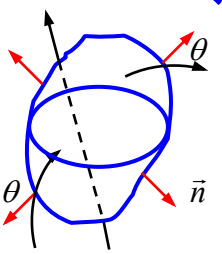
$$\Phi=\int d\Phi=\int_S\vec{E}\cdot d\vec{S}$$



闭合曲面：

$$\Phi=\oint_S\vec{E}\cdot d\vec{S}$$

取闭合曲面的外法线方向为正法线方向

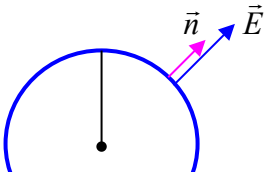


高斯面

三、高斯定理

$$\Phi=\oint_S\vec{E}\cdot d\vec{S}=\frac{1}{\varepsilon_0}\sum_{内}q_i$$

证明：1、$\Phi=\oint_S\vec{E}\cdot d\vec{S}$



$=\oint_S E\cos\theta dS$， S r dS

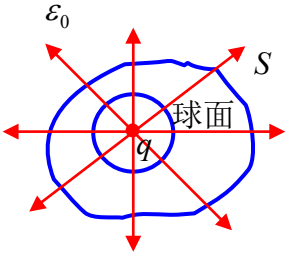
$=\oint_S \frac{q}{4\pi\varepsilon_0 r^2}dS$， q O

$=\frac{q}{4\pi\varepsilon_0 r^2}4\pi r^2=\frac{q}{\varepsilon_0}$，与 r 无关

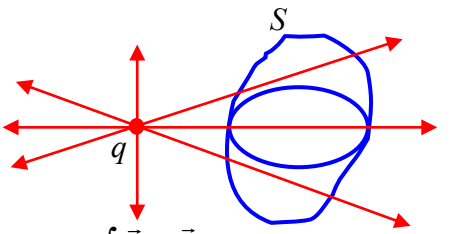
2、 $\Phi=\oint_S \vec{E}\cdot d\vec{S}$

$=\oint_{球面} \vec{E}\cdot d\vec{S}$

$=\frac{q}{\varepsilon_0}$

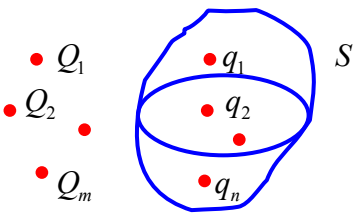


3、



$\Phi=\oint_S \vec{E}\cdot d\vec{S}=\mathbf{0}$

4、任意电场，任意闭曲面 S



$$\vec{E}=\vec{E}_{q1}+\vec{E}_{q2}+\cdots+\vec{E}_{qn}+\vec{E}_{Q1}+\vec{E}_{Q2}+\cdots+\vec{E}_{Qm}$$

$$\Phi=\oint_S \vec{E}\cdot d\vec{S}=\oint_S(\vec{E}_{q1}+\cdots+\vec{E}_{qn}+\vec{E}_{Q1}+\cdots+\vec{E}_{Qm})\cdot d\vec{S}$$

$$=\oint_S \vec{E}_{q1}\cdot d\vec{S}+\cdots+\oint_S \vec{E}_{qn}\cdot d\vec{S}+\oint_S \vec{E}_{Q1}\cdot d\vec{S}+\cdots+\oint_S \vec{E}_{Qm}\cdot d\vec{S}$$

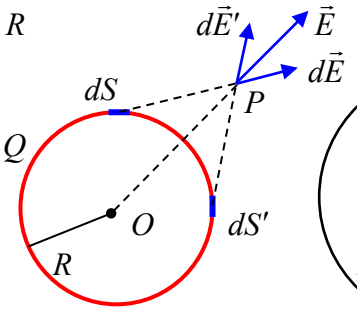
$$=\frac{q_1}{\varepsilon_0}+\cdots+\frac{q_n}{\varepsilon_0}+0+\cdots+0=\frac{1}{\varepsilon_0}\sum_{内}q_i$$

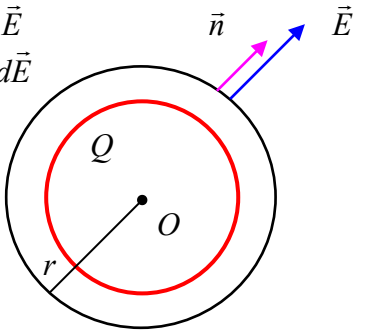
Φ 仅与 $\sum_{内}q_i$ 有关，$\vec{E}$ 与所有电荷及其分布有关

四、高斯定理的应用

例：均匀带电球面

解：1、$r > R$





$$\Phi = \oint_S \vec{E} \cdot d\vec{S} = \oint_S E\cos\theta dS$$

$$= E\oint_S dS = E4\pi r^2 = \frac{Q}{\varepsilon_0}, \quad E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

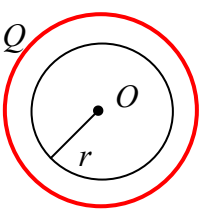
2、$r < R$

$$\Phi = \oint_S \vec{E} \cdot d\vec{S}$$

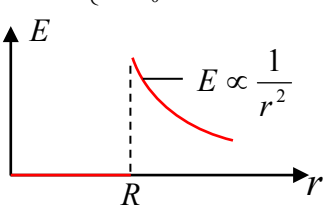
$$= \oint_S E\cos\theta dS,$$

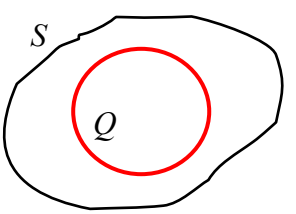
$$= E4\pi r^2 = 0,$$

$$E = 0$$



$$E = \begin{cases} 0 & r < R \\ \dfrac{Q}{4\pi\varepsilon_0 r^2} & r > R \end{cases}$$





$$\Phi = \oint_S \vec{E} \cdot d\vec{S} = \frac{Q}{\varepsilon_0}$$

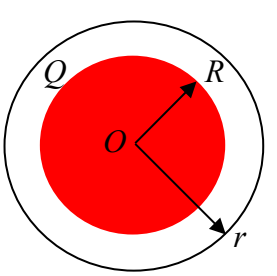
例：均匀带电球体，$\rho = \dfrac{Q}{\frac{4}{3}\pi R^3}$

解：1、$r > R$

$$\Phi = \oint_S \vec{E} \cdot d\vec{S}$$

$$= E4\pi r^2 = \frac{Q}{\varepsilon_0}$$

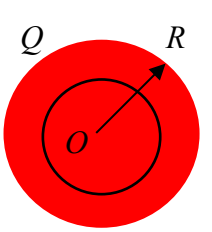
$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$



2、$r < R$

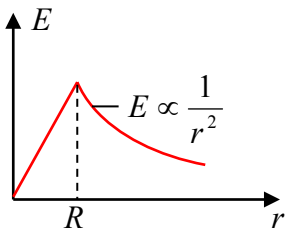
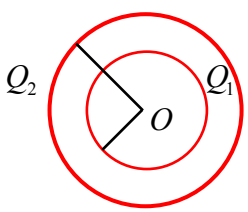
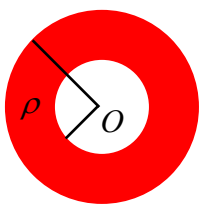
$$\Phi = \oint_S \vec{E} \cdot d\vec{S}$$

$$= E4\pi r^2$$



$$= \rho \cdot \frac{4}{3}\pi r^3 / \varepsilon_0$$

$r \searrow$

$$E = \frac{\rho}{3\varepsilon_0} r \text{ ,} \quad \vec{E} = \frac{\rho}{3\varepsilon_0}\vec{r}$$

$$E = \begin{cases} \dfrac{\rho}{3\varepsilon_0} r & r < R \\ \dfrac{Q}{4\pi\varepsilon_0 r^2} & r > R \end{cases}$$