Principles of Communications

Chapter V: Representation and Transmission of Baseband Digital Signal

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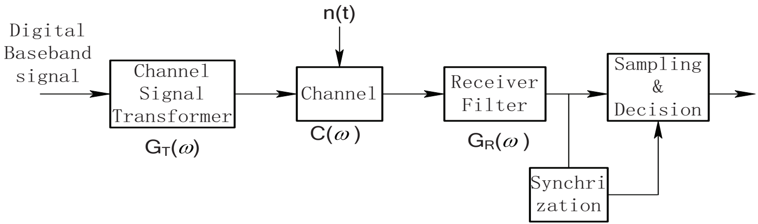
5.1 Introduction

- Definition: digital signal, if its energy mainly locates in $[0, f_H]$, is named digital baseband signal. Related to this, if its energy locates in $[f_L, f_H]$ and $f_L >> 0$, is named passband signal.
- Why we study digital baseband signal?
  - Widely used in the short distance communication.
  - Many basic problems are similar in either baseband or passband.
  - All passband systems based on linear modulations can be equivalent to passband.

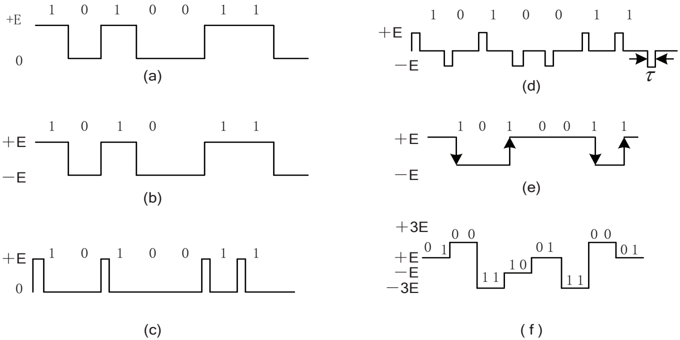


- Channel signal transformer: the input is the pulse sequence produced by the terminal equipment or encoder; Usually, it is not suitable to be transmitted in the channel. This transformation is primarily implemented by coder and waveform transformation. It aims to match the channel, reduce the inter-symbol interference, and make it favorable to get synchronization, sample and decision.
- Channel: it is the medium for baseband signal's transmission. The transmission properties of the channel usually don't satisfy the transmission conditions without distortion, they even change randomly. In the analysis of communications system, we often equal the noise and import it in the channel.
- Receiver filter: Remove the out-band noise and make the output waveform easily to sample and decide.
- Sampling-decision: it samples and decides the output waveform of the receiver filter at the certain time. The sampling time is gotten from the received signal by the synchronization.

5.2 Waveform of Baseband Digital Signal

- Unipolar waveform: "1"– $+E$, "0"– zero level; Duty ratio: 100%.
- Bipolar waveform: "1"– $+E$, "0"– $-E$; Duty ratio: 100%.
- Unipolar return-to-zero waveform:
  - "1"— $+E$, "0"— zero level; But signal voltage return to the value 0 in the duration of symbol "1".
  - Duty ratio: $< 100\%$.
- Bipolar return-to-zero waveform:
  - "1"— $+E$, "0"—$-E$; Signal voltage return to the value 0 in the duration of each symbol.
  - Duty ratio: $< 100\%$.
- Differential waveform:
  - "1" and "0" are denoted by the change of voltage.
- Multi-level waveform: 4-ary signals
  - "00"— $+3E$, "01"— "+E";
  - "10"— $-E$; "11"— "-3E".

5.3 Temporal Characteristics of Baseband Signal $s(t)$



- Assume that $g(t) = \begin{cases} g_1(t), & \text{“}0\text{''}, \; p \\ g_2(t), & \text{“}1\text{''}, 1-p \end{cases}$. Baseband signal can be expressed as $s(t) = \sum_{n=-\infty}^{\infty} g(t - nT_s)$.
- $s(t)$ consists of stationary wave $v(t)$ and alternating wave $u(t)$, i.e., $s(t) = u(t) + v(t)$ and $v(t) = E[s(t)]$.

5.4 Frequency Characteristics of Baseband Signal

It can be verified that power spectral density of $s(t)$

$$\begin{aligned} P_s(f) =& P_u(f) + P_v(f) \\ =& f_s p(1-p)|G_1(f) - G_2(f)|^2 \\ &+ \sum_{m=-\infty}^{\infty} |f_s[pG_1(mf_s) + (1-p)G_2(mf_s)]|^2 \delta(f - mf_s). \end{aligned}$$

Discussions

- Power spectrum $P_s(f)$ of the digital baseband signal consists of the continuous spectrum $P_u(f)$ and the discrete spectrum $P_v(f)$, the continuous spectrum carrying digital information must exist, while the discrete spectrum could be zero.
- The existence of the discrete spectrum of the baseband signal is determined by the $G_1(f) \leftrightarrow g_1(t)$, $G_2(f) \leftrightarrow g_2(t)$, and the probability p.

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- For the case of the unipolar binary waveform: $g_1(t) = 0, g_2(t) = g(t)$, and $p = 1/2$,

$$P_s(f) = \frac{1}{4} f_s |G(f)|^2 + \frac{1}{4} \sum_{m=-\infty}^{\infty} |f_s G(mf_s)|^2 \delta(f - mf_s).$$

- For the case of the bipolar binary waveform: $g_1(t) = -g_2(t) = g(t)$, and $p = 1/2$,

$$P_s(f) = f_s |G(f)|^2.$$

  The discrete spectrum doesn't exist.

- How to determine the existence of discrete spectrum? Fact: If p, $g_1(t)$, and $g_2(t)$ satisfy that $p = \frac{1}{1 - g_1(t)/g_2(t)}$, the discrete spectrum of the digital baseband signal doesn't exist.
- Frequency bandwidth: it depends on the $g_1(t)$ and $g_2(t)$. If they are rectangular pulse with 100% duty ratio, $G(f) = T_s Sa(\pi f T_s)$ the frequency bandwidth is $B = 1/T_s$.

5.5 Symbol Code Types of Baseband Digital Signals for Transmission

- Purpose: the converted waveform are more suitable for transmission.
- Requirements:
  - No D.C. component and very small low frequency components.
  - Timing information, high transmission efficiency, error-detecting ability, suitable for various information sources.
- 1. AMI code: Alternative Mark Inverse code.
  - Coding rule:

Digital bit	AMI code
0	0
1	+1, -1 alternatively

  - Advantages: no DC component; decoder is simple.
  - Drawback: there are long "0" sequence and not good for synchronization. To overcome this, HDB3 code is proposed.

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2. HDB3 Code

- 3^{rd} Order High Density Bipolar code.
- Coding Rule
  - Let N denote the number of continuous 0s. If $N \leq 3$, AMI coding rule is applied.
  - If $N > 3$, convert the 4th "0" to non-zero, denoted by $+V$ or $-V$. The polarity of the adjacent V code is alternative.
  - The polarity of V code should be same to the previous non-zero code and the following non-zero codes should be alterative.
  - If the above three properties cannot be guaranteed, convert the first "0" of the continuous 0 string into "+B" or "-B". Its polarity is same to the following "V" code.
- Examples:
  - Exa 1: $1\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 1.$
  - Exa 2: $0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0.$
- Decode: "10001"—"10000" and "1001"—"0000".

3. PST Code

- Paired Selected Ternary code.
- Coding Rule
  - Grouping

Digital bits	"+" Mode	"-" Mode
00	−+	−+
01	0+	0−
10	+0	−0
11	+−	+−

  - While there is one "1" in each group, "+" mode and "–" mode are changed alternatively.
- Advantages: no DC component and enough timing information.
- Drawbacks: Frame synchronization information is needed.
- Example: $01\ 00\ 11\ 10\ 10\ 11\ 00.$

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4. Other Code Types

- Bi-phase code: also called Manchester code.
  - Code rules: “1”—“10”, “0”—“01”
- Miller code
  - Code rules: “1”— Jump in the symbol duration, “0” — other wise.
- CMI code
  - Code rules: “1”— “11” and “00” alternatively; “0” — “01”.
- nBmB code: nB to mB.

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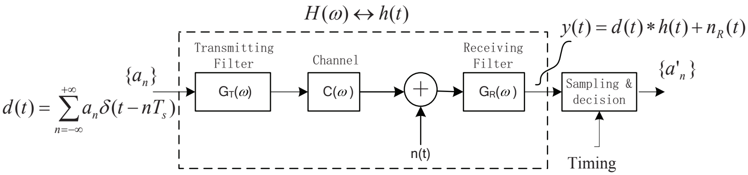
5.6 Signal Transmission and Inter-symbol Interference

1. Model of Baseband Digital Signal Transmission



- The input: digital sequence $\{a_n\}$ – $d(t) = \sum_{n=-\infty}^{\infty} a_n \delta(t - nT_s)$.
- System characteristics: signal pass through the cascade system of $G_T(\omega)$, $C(\omega)$, and $G_R(\omega)$, denoted by $H(\omega) = G_T(\omega)C(\omega)G_R(\omega)$. The noise is only through $G_R(\omega)$ and the output is $n_R(t)$.
- The output of the receiving filter:

$$y(t) = d(t) * h(t) + n_R(t) = \sum_{n=-\infty}^{\infty} a_n h(t - nT_s) + n_R(t).$$

2. Sampling and Decision

(1). Sampling

- 1. Sampling: the sampling time $t = kT_s$. The sampled signal

$$y(kT_s) = \sum_{n=-\infty}^{\infty} a_n h(kT_s - nT_s) + n_R(kT_s)$$
$$= a_k h(0) + \sum_{\substack{n=-\infty \\ n\neq k}}^{\infty} a_n h(kT_s - nT_s) + n_R(kT_s).$$

- Discussions
  - $a_k h(0)$ – the sample point of the kth symbol (useful signal);
  - $n_R(kT_s)$ – additional noise;
  - $\sum_{\substack{n=-\infty \\ n\neq k}}^{\infty} a_n h(kT_s - nT_s)$ – the inter-symbol interference (ISI).

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(2). Decision

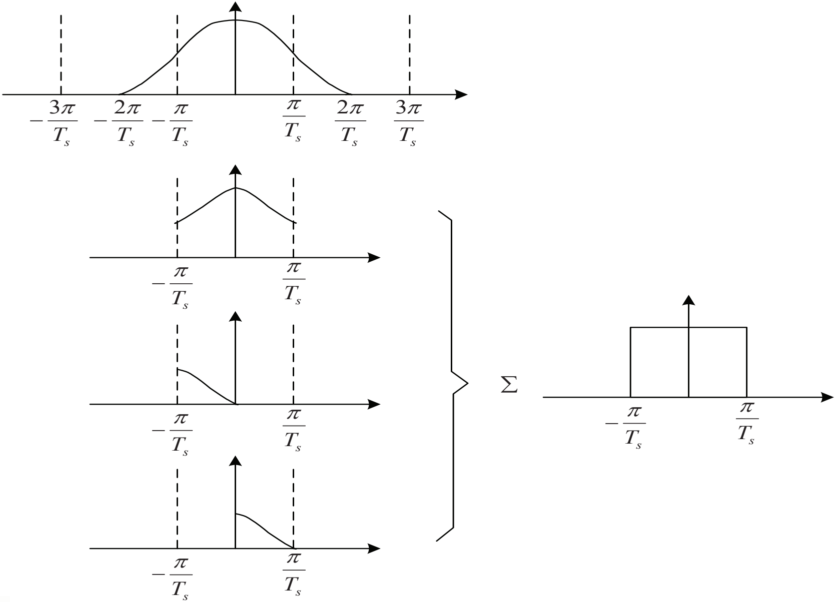
- Set the decision threshold V_0. Design the decision rules:
  - If $y(kT_s) > V_0$, $a_k = 1$;
  - If $y(kT_s) < V_0$, $a_k = 0$;
- The decision rules should be consistent with the mapping of the transmitting signal. For example, if the mapping rule is that: "1"– low; "0"– high, then the above rules do not hold and the opposition rules should be adopted;
- Discussion: Noise and ISI can affect the correct decision of the signal. Noise is introduced by the channel, and can't be completely eliminated. while the ISI is determined by the system characteristics, it should be eliminated through reasonable design of communication systems.

3. ISI Free System Characteristics

- Time domain condition: $h(kT_s) = \begin{cases} 1, & k = 0 \\ 0, & k \neq 0 \end{cases}$.
- Frequency domain condition: (derived according to the above time domain condition)

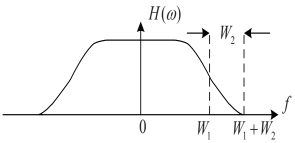
$$\frac{1}{T_s} \sum_i H(\omega + \frac{2\pi i}{T_s}) = 1, \quad |\omega| \leq \frac{\pi}{T_s}.$$

- ISI-free frequency domain condition is whether the overlay of $H(\omega)$ shifted by an interval $\frac{2\pi}{T_s}$ is constant in the range $[-\frac{\pi}{T_s}, \frac{\pi}{T_s}]$ which is equivalent to the ideal low-pass characteristics;


$-\frac{3\pi}{T_s}$
$-\frac{2\pi}{T_s}$
$-\frac{\pi}{T_s}$
$\frac{\pi}{T_s}$
$\frac{2\pi}{T_s}$
$\frac{3\pi}{T_s}$
Σ

4. Several Important Concepts

- Roll-off factor: $\alpha = \frac{W_2}{W_1}$.
  - W_1 — the center freq. of the roll-off part;
  - W_2 — half of the roll-off bandwidth;
  - $0 < \alpha < 1$.



- Frequency bandwidth efficiency:

$$\eta = \frac{R_s}{B} \rightarrow \eta = \frac{2W_1}{W_1 + W_2} = \frac{2}{1+\alpha}$$

.

- If $\alpha = 0$, $\eta_{\max} = 2Baud/Hz$, $h(t) = \frac{\sin \pi t/T_s}{\pi t/T_s}$.

5. Nyquist Rate

- Nyquist Rate: If the transfer function is a real function and odd symmetry at $f = W_1$, we can obtain free inter-symbol transmission rate $R_s = 2W_1$.
- Raised Cosine Transfer Function:
  - Impulse Response:

$$h(t) = \frac{\sin \pi t/T_s}{\pi t/T_s} \frac{\cos \alpha \pi t/T_s}{1 - 4\alpha^2 t^2/T_s^2}$$

.

  - Two special cases

$\alpha = 0$, ideal low-pass characteristic, $\eta = 2Baud/Hz$;

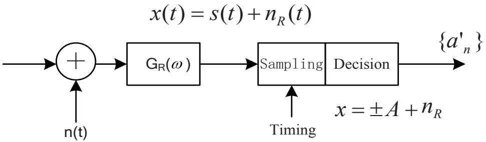
$\alpha = 1$,

$$H(\omega) = \begin{cases} \frac{T_s}{2}\left(1 + \cos \frac{\omega T_s}{2}\right), & |\omega| \leq \frac{2\pi}{T_s} \\ 0, & |\omega| \leq \frac{2\pi}{T_s} \end{cases}$$

  - In GSM system, $\alpha = 0.22$.

6. Anti-noise Performance

(1) Review



- Noise: $n(t)$, Gaussian white noise. Assume the $n_R(t)$ is with zero mean and variance σ_n^2.
- Signal: Bipolar waveform, $\pm A$, "1" — "+A" and "0"—"-A".
- Two steps to recover digital bits
  - Sampling for signal: $+A$ or $-A$.
  - Sampling for noise: n_R — Gaussian variable $f(v) = \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left(-\frac{v^2}{2\sigma_n^2}\right)$.
  - Decision according to threshold V_d: If $x > V_d$, we obtain "1", else if $x < V_d$, we obtain "0".

(2) Bit Error Ratio — Derivations

- Assume the transmitted bit is "1". The input signal of the decision block: $x = A + n_R$. Then $x \in N(A, \sigma_n^2)$ and its pdf is $f(x) = \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left(-\frac{(x-A)^2}{2\sigma_n^2}\right)$
- The error probability of symbol "1" (Draw a picture)
$$P_{e_1} = P(x < V_d) = \int_{-\infty}^{V_d} \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left(-\frac{(x-A)^2}{2\sigma_n^2}\right) dx.$$ Let $V = \frac{x-A}{\sqrt{2}\sigma_n}$. Then
$P_{e_1} = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\frac{V_d - A}{\sqrt{2}\sigma_n}} e^{-V^2} dV = \frac{1}{2} + \frac{1}{2}\mathrm{erf}\left(\frac{V_d - A}{\sqrt{2}\sigma_n}\right)$. (error function $\mathrm{erf}(x) = \int_0^x e^{-t^2} dt$)
- Similar, we derive $P_{e1} = \frac{1}{2} - \frac{1}{2}\mathrm{erf}\left(\frac{V_d + A}{\sqrt{2}\sigma_n}\right)$
- The total BER: $\mathrm{P_e} = \mathrm{P(0)P_{e_0}} + \mathrm{P(1)P_{e_1}}$.

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3. Best Decision Threshold

- $\mathrm{P_e} \sim (\mathrm{P(0)}, \mathrm{P(1)}, \mathrm{V_d})$. P(0) and P(1) are determined by the source but V_d can be adjusted. $\min_{V_d} \ P_e$
- The best decision threshold should satisfy $\frac{\partial P_e}{\partial V_d} = 0$, i.e.,

$$\frac{P(1)}{\sqrt{2\pi}\sigma_n}\exp\left(-\frac{(V_d^* - A)^2}{2\sigma_n^2}\right) - \frac{P(0)}{\sqrt{2\pi}\sigma_n}\exp\left(-\frac{(V_d^* + A)^2}{2\sigma_n^2}\right) = 0$$

$$\frac{1}{2\sigma_n^2}\left[-(V_d^* - A)^2 + (V_d^* + A)^2\right] = \ln\frac{P(0)}{P(1)}$$

$$V_d^* = \frac{\sigma_n^2}{2A}\ln\frac{P(0)}{P(1)}$$

- Specially, if $P(0) = P(1) = \frac{1}{2}$, $V_d^* = 0$. BER is $P_e = \frac{1}{2}\mathrm{erfc}\left(\frac{A}{\sqrt{2}\sigma_n}\right)$
- $P_e \sim \frac{A}{\sigma_n}$. While $\frac{A}{\sigma_n} \nearrow$, $P_e \searrow$.
- How about Unipolar waveform?

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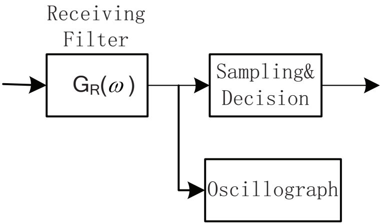
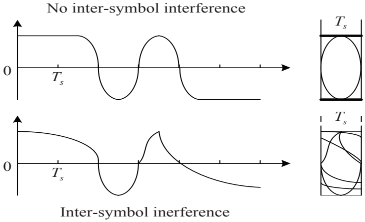
5.7 Eye Pattern

1. Introduction

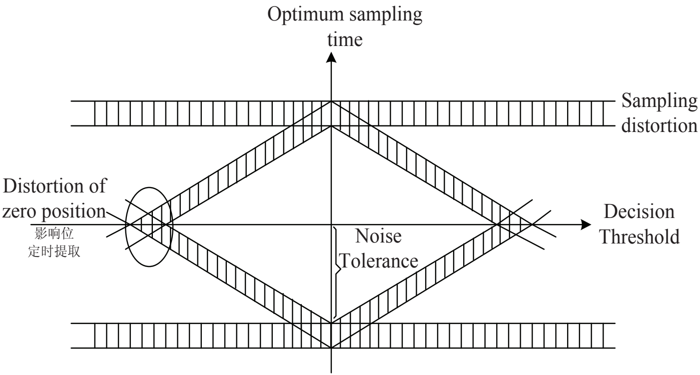




- Let an oscilloscope be connected to the output of the receiving filter, and then adjust the oscilloscope horizontal scanning period so that it is synchronized with the period of the received symbol. In this case, a pattern similar to the human eye on the oscilloscope screen can be observed, which is called "eye pattern".

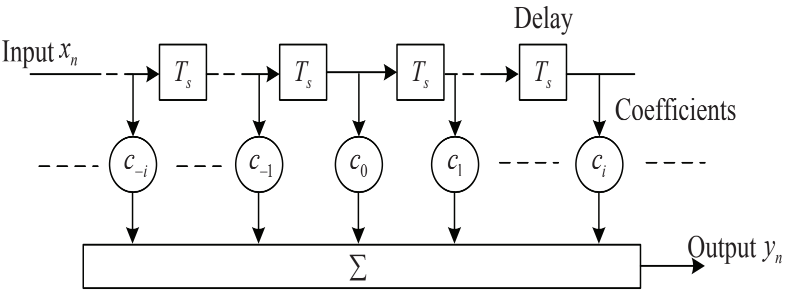
2. Parameters of Eye Pattern

- “multi-eyelids” are presented if the ISI serious; “single-edged eyelid” indicates that the ISI is relatively small.



5.8 Time-domain Equalizer

1. Introduction to the Equalizer



- Design $G_T(\omega)$ and $G_R(\omega)$ carefully, the ISI-free conditions for $H(\omega) = G_T(\omega)C(\omega)G_R(\omega)$ can be met for fixed the channel characteristics $C(\omega)$. However, the $C(\omega)$ may change with time and lead to serious ISI.

2. Time Domain Equalization

- Is there an adoptable filter, $T(\omega)$, to realize the free-ISI?
- Assume that the total system characteristic $H'(\omega) = H(\omega)T(\omega)$. When $H'(\omega) = \sum_i H'(\omega + \frac{2\pi i}{T_s}) = T_s$, $|\omega| \leq \frac{\pi}{T_s}$, there is no ISI. So we have

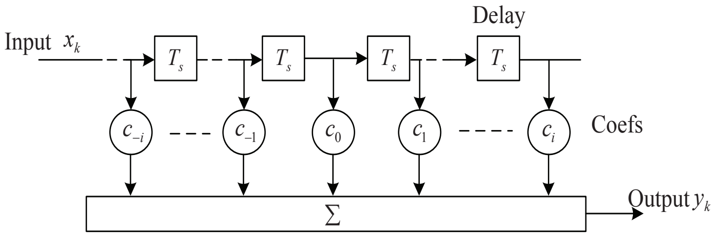
$$\sum_i H(\omega + \frac{2\pi i}{T_s})T(\omega + \frac{2\pi i}{T_s}) = T_s, \quad |\omega| \leq \frac{\pi}{T_s}.$$

- If $T(\omega)$ is periodic function with period $\frac{2\pi}{T_s}$, i.e., $T(\omega + \frac{2\pi i}{T_s}) = T(\omega)$, we obtain

$$T(\omega) = \frac{T_s}{\sum_i H(\omega + \frac{2\pi i}{T_s})}, \quad |\omega| \leq \frac{\pi}{T_s}.$$

- We draw a conclusion that above $T(\omega)$ can eliminate ISI.

3. How to realize $T(\omega)$



- If $T(\omega)$ is periodic function with period $\frac{2\pi}{T_s}$, we can expand $T(\omega)$ as Flourier series, i.e.,

$$T(\omega) = \sum_{k=-\infty}^{\infty} c_n e^{-j\omega k T_s}, \quad c_k = \frac{T_s}{2\pi} \int_{-\pi/T_s}^{\pi/T_s} T(\omega) e^{j\omega k T_s} d\omega.$$

- Transversal filter, as shown in the above figure, can realize the function. In practice, the order of the transversal filter is finite.
- There are many algorithms to adopt the filter coefficients, such as zero forcing and MMSE.

4. Measurement of Equalization Effects

- The ISI can not be completely eliminated using the finite-order transversal filter. There are several criterions to evaluate the remained ISI. On the other hand, according to different criterions, different adaptive equalization algorithm can be designed.

- Peak-distortion criterion:

$$D = \frac{1}{y_0} \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} |y_n|.$$

- Mean square distortion criterion:

$$\varepsilon^2 = \frac{1}{y_0^2} \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} |y_n|^2.$$