Principles of Communications

Chapter III: Analog Modulation System

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Introduction

Purposes and Classification

- Frequency spectrum of baseband signal can be moved near the desired frequency and we can combine several signals for multichannel transmission.
- The anti-jamming ability for the signal transferred through channels can be improved by modulations.
- Modulation mode affects the utilization of transmission bandwidth.
- A cosine waveform is mathematically expressed as $c(t) = A\cos(\omega_c t + \varphi)$. Modulation enable parameter A, ω_c, and φ vary with the baseband signal.
  - Amplitude modulation: Linear Modulation.
  - Frequency/phase modulation: Nonlinear Modulation.
  - Question: What is the difference between the Linear Modulation and the Nonlinear Modulation?

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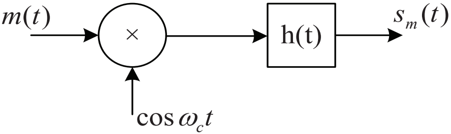
Linear Modulation

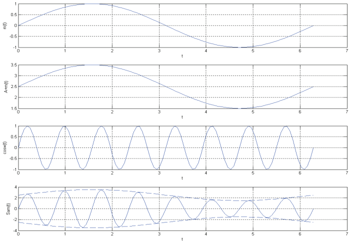


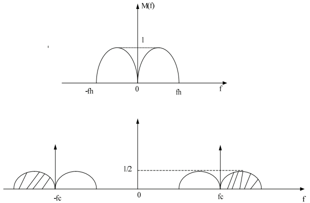
- Temporal expression: $s_m(t) = [m(t)\cos\omega_c t] * h(t)$.
- Frequency domain expression:
  $S_m(f) = \frac{1}{2}[M(f - f_c) + M(f + f_c)]H(f)$.
- According to the setting $H(f)$ and the characteristic, linear modulation can be divided as: AM, DSB-SC, SSB, and VSB.

AM

Signal $m(t), [0, f_H]$ consists of direct component A and alternating component $m'(t)$. And $H(f) = 1$ allows the signal pass through without distortion.

- Temporal expression: $s_m(t) = [A + m'(t)] \cos \omega_c t$ and $A > \max_t |m'(t)—$.
- Frequency domain expression: $S(f) = \frac{1}{2}[M(f - f_c) + M(f + f_c)] + A\pi[\delta(f - f_c) + \delta(f + f_c)]$.
- $B = 2f_H$. $P = \frac{A^2}{2} + \frac{|m'(t)|^2}{2}$. DC component doesn't carry any information and power efficiency is low.

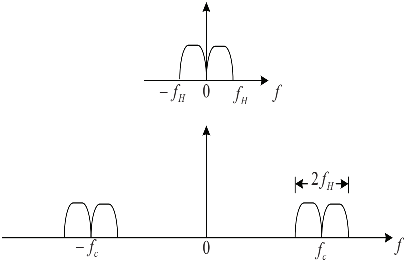




DSB-SC: Double-sideband Modulation - Suppressed Carrier

Signal $m(t)$ is only with alternating component. And $H(f) = 1$ allows the signal pass through without distortion.

- Temporal expression: $s_m(t) = m(t)\cos\omega_c t$.
- Frequency domain expression: $S_m(f) = \frac{1}{2}[M(f - f_c) + M(f + f_c)]$.
- $B = 2f_H$. Two sidebands (Upper-side and Lower-side) in DSB modulation contain the same information. (Frequency band efficiency is low.)



SSB/VSB

Signal $m(t)$ is only with alternating component. And $H(f)$ are low-pass filter or high-pass filter to allow only the upper-side or the lower-side signal pass through.

- Temporal expression: $s_m(t) = m(t)\cos\omega_c t \mp \hat{m}(t)\sin\omega_c t$.
- $\hat{m}(t)$ is the Hilbert transform of $m(t)$. Its definition is $\hat{M}(\omega) = -j\text{sgn}(\omega)M(\omega)$ and $\text{sgn}(\omega) = \begin{cases} 1, & \omega > 0 \\ -1, & \omega < 0 \end{cases}$.

$$
\begin{aligned}
2S_m(f) &= \left[M(\omega+\omega_c) + M(\omega-\omega_c)\right] - j\left[\hat{M}(\omega+\omega_c) - \hat{M}(\omega-\omega_c)\right] \\
&= \left[M(\omega+\omega_c) + M(\omega-\omega_c)\right] \\
&\quad - \left[\text{sgn}(\omega+\omega_c)M(\omega+\omega_c) - \text{sgn}(\omega-\omega_c)M(\omega-\omega_c)\right] \\
&= M(\omega+\omega_c)\left[1-\text{sgn}(\omega+\omega_c)\right] \\
&\quad + M(\omega-\omega_c)\left[1+\text{sgn}(\omega-\omega_c)\right] \\
&= S(\omega)H(\omega) - -\text{high}-\text{pass filter}
\end{aligned}
$$

Similar derivations for lower-side case.

$(\sin\omega_c t \leftrightarrow j\pi[\delta(\omega+\omega_c) - \delta(\omega-\omega_c)])$.

- $B = f_H$.
- Generate SSB signal: Filtering or Hilbert transform. (not easy)

VSB

If $H(f+f_c)+f(f-f_c)=c, \quad |f|<f_H$, the receiver can recover the baseband signal without distortion.

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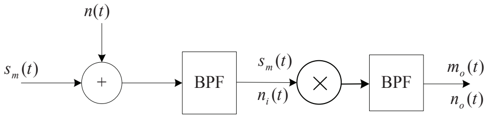
Coherent Demodulation



Assume that the modulation is AM.

- Step 1. Signal: $s_m(t) = (A + m(t))\cos\omega_c t$;
  Noise: $n_i(t) = n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t$.
- Step 2. Signal:

$$s_m(t)\cos\omega_c t = (A + m(t))\cos^2\omega_c t$$

$$= \frac{1}{2}A^2 + \frac{1}{2}m(t) + \frac{1}{2}(A + m(t))\cos 2\omega_c t \xrightarrow{LPF} \frac{1}{2}A^2 + \frac{1}{2}m(t).$$

Noise:

$$n_i(t)\cos\omega_c t = \frac{1}{2}n_c(t) + \frac{1}{2}n_c(t)\cos 2\omega_c t - \frac{1}{2}n_s(t)\cos 2\omega_c t$$

$$\xrightarrow{LPF} \frac{1}{2}n_c(t).$$

- Output SNR (Signal Noise Ratio):

$$\frac{S_o}{N_o} = \frac{\overline{m^2(t)}}{\sigma^2} = \frac{\overline{m^2(t)}}{n_0 B}.$$

- Modulation Gain: $G = \frac{S_o/N_o}{S_i/N_i}$

$$\frac{S_i}{N_i} = \frac{\frac{1}{2}(A^2 + \overline{m^2(t)})}{\sigma^2} = \frac{\frac{1}{2}(A^2 + \overline{m^2(t)})}{n_0 B}$$

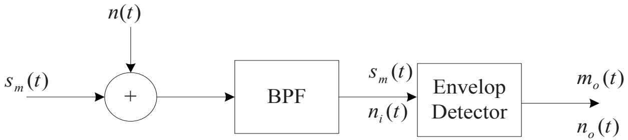
$$G = \frac{S_o/N_o}{S_i/N_i} = \frac{2\overline{m^2(t)}}{A^2 + \overline{m^2(t)}},$$

$$G_{\max} = \frac{2}{3} \quad (m(t) = A\cos\omega_c t)$$

.

- Can we apply the coherent scheme to DSB and SSB?

Non-coherent Demodulation



Assume that $s_m(t) = [A + m(t)]\cos\omega_c t$, $A \geq |m(t)|_{\max}$, and noise $n_i(t) = n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t$. We have

$$\begin{aligned} s_m(t) + n_i(t) &= [A + m(t) + n_c(t)]\cos\omega_c t - n_s(t)\sin\omega_c t \\ &= E(t)\cos[\omega_c t + \varphi(t)]. \end{aligned}$$

where

$$\begin{aligned} E(t) &= \sqrt{[A + m(t) + n_c(t)]^2 + n_s(t)^2} \\ &= \sqrt{[A + m(t)]^2 + 2n_c(t)[A + m(t)] + n_c(t)^2 + n_s(t)^2}. \end{aligned}$$

Hint: $(1 + x)^{\frac{1}{2}} \approx 1 + \frac{1}{2}x$.

Non-coherent Demodulation

- For the case of $(A + m(t))^2 >> n_c(t)^2 + n_s(t)^2$,
  - $E(t) \approx A + m(t) + n_c(t)$
  - $\frac{S_o}{N_o} = \frac{\overline{m^2(t)}}{\overline{n_c^2(t)}} = \frac{\overline{m^2(t)}}{n_o B}$.
  - $G_{AM} = \frac{\overline{m^2(t)}}{A^2 + \overline{m^2(t)}}$. It is the same to coherent demodulation
- For the case of $(A + m(t))^2 >> n_c(t)^2 + n_s(t)^2$,

$$E(t) \approx R(t) + [A + m(t)] \cos\theta$$

, where $R(t) = \sqrt{n_c^2(t) + n_s^2(t)}$ and $\cos\theta = \frac{n_c(t)}{R(t)}$.

- Threshold effect: when the input $\frac{S_i}{N_i}$ decreases to some special value (threshold), the output $\frac{S_o}{N_o}$ decreases dramatically. The phenomenon is named as "threshold effect".
- Can we apply the non-coherent scheme to the DSB or SSB?

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Nonlinear Modulation – Basic Concepts

Parameters: $S_m(t) = A\cos[\omega_c t + \varphi(t)]$

- $\omega_c t + \varphi(t)$ – instant phase.
- $\varphi(t)$ – instant phase shift with respect to $\omega_c t$.
- $\frac{d[\omega_c t + \varphi(t)]}{dt}$ – instant angular frequency.
- $\frac{d\varphi(t)}{dt}$ – instant angular frequency shift with respect to ω_c.

- Phase modulation (PM): For any t, $\varphi(t)$ is proportional to the signal $m(t)$, i.e., $\varphi(t) = K_p m(t)$ and

$$s_{PM}(t) = A\cos[\omega_c t + K_p m(t)].$$

- Frequency modulation (FM): $\frac{d\varphi(t)}{dt}$ is proportional to the signal $m(t)$, i.e., $\frac{d\varphi(t)}{dt} = K_f m(t)$. So we have

$$s_{PM}(t) = A\cos\left[\omega_c t + \int_{-\infty}^{t} K_f m(\tau)d\tau\right].$$

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Narrowband FM and Wideband FM

Assume that modulating signal $m(t) = A_m \cos \omega_H t$. The corresponding FM signal is

$$s_{FM}(t) = A\cos\left[\omega_c t + \int_{-\infty}^{t} K_f A_m \cos \omega_H \tau d\tau\right]$$
$$\approx A\cos[\omega_c t + \frac{K_f A_m}{\omega_H}\sin \omega_H t]$$

- The maximum instant angular frequency shift Δf with respect to ω_c is $K_f A_m$.
- Define the **modulation index** as $m_f = \frac{K_f A_m}{\omega_H} = \frac{\Delta f}{f_H}$.
- For signal $m(t)$ $[0, f_H]$, its corresponding bandwidth could be roughly regarded as $B_{FM} = 2(m_f + 1)f_H$. (Every frequency component f_H could be shifted to $f_H + \Delta f$)
  - Narrowband FM ($m_f << 1$): $B_{FM} \approx 2f_H$.
  - Wideband FM ($m_f >> 1$): $B_{FM} \approx 2m_f f_H = 2\Delta f$.
- Generally, one can find that the anti-noise performance of the system is better while m_f, or B_{FM}, is bigger.

Comparisons

Table: Comparisons of the Different Analog Modulation

Mods	Frequency Band	Anti-noise	Complexity	Applications
AM	$2f_H$	not good	simple	Medium/short wave broadcasting
DSB	$2f_H$	medium	medium	—-
SSB	f_H	medium	complex	Short wave broadcasting, audio
VSB	$[f_H, 2f_H]$	medium	medium	TV broadcasting
FM	$2(m_f+1)f_H$	good	medium	Audio broadcasting

$G_{AM} = \frac{2}{3}$, $G_{DSB} = 2$, $G_{SSB} = 1$, $G_{FM} = 3m_f^2(m_f+1)$.