Principles of Communications

Chapter II: Signals – Homework

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2.1 Assume a random process $X(t)$ can be expressed as

$$X(t) = 2\cos(2\pi t + \theta) \qquad -\infty < t < \infty$$

where θ is a discrete random variable, its probability distribution is as follows.

$$P(\theta = 0) = 0.5, \quad P(\theta = \pi/2) = 0.5$$

Find $E[X(t)]$ and $R_X(0, 1)$.

2.2 Assume a random process $X(t)$ can be expressed as

$$X(t) = 2\cos(2\pi t + \theta) \qquad -\infty < t < \infty$$

Judge it is a power signal or energy signal. And find its power spectral density or energy spectral density.

2.3 Assume a signal can be expressed as

$$x(t) = \begin{cases} 4\exp(-t) & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Is it a power signal or energy signal? And find its power spectral density or energy spectral density.

2.4 Assume $X(t) = x_1 \cos 2\pi t - x_2 \sin 2\pi t$ is a random process,where x_1 and x_2 are statistically independent Gaussian random variables, and their mathematical expectations are 0,variances are σ^2. Find:

(1)$E[X(t)], E[X^2(t)]$;

(2) The probability distribution density of $X(t)$;

(3)$R_X(t_1, t_2)$.

2.5 Find the autocorrelation function of $X(t) = A\cos\omega t$, and find its power from its autocorrelation function.

2.6 The autocorrelation function of a stationary random process $X(t)$ is given to be a periodic function with period 2:

$$R(\tau) = 1- \mid \tau \mid \quad -1 \le \tau < 1$$

Find the power spectral density $P_X(f)$ of $X(t)$, and draw its curve.

2.7 A period signal $x(t)$ is applied on the input of a linear system, and the output signal is

$$y(t) = \tau[dx(t)/dt]$$

where τ is constant .Find the transfer function $H(f)$ of the linear system.

2.8 If a Gaussian white noise passes the filter shown in Fig.2.10.4 Its mean is 0, and double-side power spectral density is $n_0/2$. Find the probability density of the output noise.

2.9 均值为零的高斯随机变量，其方差$\sigma_x^2 = 4$,求$x > 2$的概率。

2.10 随机过程$X(t)$ 的均值为a，自相关函数为$R_x(\tau)$，随机过程$Y(t) = X(t) - X(t-T)$，T为常数，求证$Y(t)$是否为平稳随机过程。

2.11 随机过程$z(t) = x_1 \cos\omega_0 t - x_2 \sin\omega_0 t$，若$x_1$和$x_2$是彼此独立且均值为0，方差为$\sigma^2$的正态随机变量，试求：

(1)$E[z(t)], E[z^2(t)]$；
(2)$z(t)$ 的一维分布密度函数$f(z)$；
(3) $B(t_1, t_2)$与$R(t_1, t_2)$。

2.12 已知一随机过程$z(t) = m(t)\cos(\omega_0 t + \theta)$，它是广义平稳随机过程$m(t)$ 对一载频进行振幅调制的结果。此载频的相位θ在$(0, 2\pi)$上为均匀分布，设$m(t)$ 与θ是统计独立的，且$m(t)$ 的自相关函数$R_m(\tau)$ 为

$$R_m(\tau) = \begin{cases} 1+\tau, & -1 < \tau < 0 \\ 1-\tau, & 0 \le \tau < 1 \\ 0, & others \end{cases}$$

(1) 证明$z(t)$ 是广义平稳的；
(2) 绘出自相关函数$R_z(\tau)$ 的波形；
(3) 求功率谱密度$P_z(\omega)$及功率S。

2.13 将均值为0，自相关函数为$\frac{n_0}{2}\delta(t)$ 的高斯白噪声加到一个中心角频率为ω_c 带宽为B的理想带通滤波器上，如图P-1所示。
(1)求滤波器输出噪声的自相关函数;
(2)写出输出噪声的一维概率密度函数。
2.14 随机过程$X(t) = A\cos(\omega t + \theta)$ ，式中，A, ω, θ 是相互独立的随机变量，其中A 的均值为2，方差为4，θ 在区间$(-\pi, \pi)$上均匀分布，ω 在区间$(-5, 5)$ 上均匀分布。
(1)随机过程$X(t)$ 是否平稳？是否各态历经？
(2)求出自相关函数。
2.15 若$\xi(t)$是平稳随机过程，自相关函数为$R_\xi(\tau)$ ，试求它通过如图P-2系统后的自相关函数及功率谱密度。
2.16 设$x_1(t)$ 与$x_2(t)$ 为零均值且互不相关的平稳过程，经过线性时不变系统，其输出分别为$z_1(t)$与$z_2(t)$，试证明$z_1(t)$ 与$z_2(t)$ 也是互不相关的。

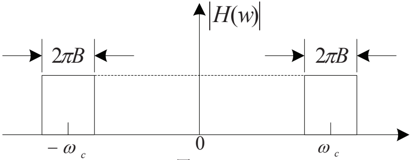

图 P-1

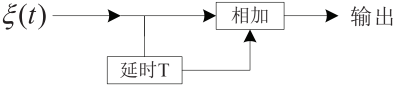

图 P-2