Abstract—This paper presents a study of the microwave backscattering from 2-D time-evolving nonlinear surfaces of a sea with finite depth by using the second-order small-slope approximation. According to the shallow-water dispersion relation, the revised nonlinear hydrodynamic choppy wave model in connection with an experiment-verified sea spectrum for finite-depth water is employed to construct the wave profiles in the finite-depth sea. The numerical results show that the discrepancy between the choppy surfaces of the infinite-depth sea and their finite-depth counterparts for monostatic normalized radar cross section is much smaller than that between the linear surfaces and the nonlinear choppy surfaces. Furthermore, the comparison of the Doppler spectra of the backscattered echoes from the linear and nonlinear choppy sea surfaces shows that the nonlinear hydrodynamic features significantly impact the Doppler spectrum. In particular, the Doppler spectrum for nonlinear finite-depth sea presents much higher second-order peaks and increased spectral amplitudes in the frequency range around the Doppler peak frequency, which reiterates the importance of the role that the nonlinear hydrodynamic effect of waves played in the interpretation of backscattering from finite-depth nearshore seas from the qualitative point of view.

Index Terms—Doppler spectrum, finite-depth sea, microwave scattering, nonlinear sea surface, second-order small-slope approximation (SSA) (SSA-II).

I. INTRODUCTION

The study of microwave scattering from oceanic surfaces has received great attention for its myriad applications in areas such as oceanic surveillance, target detection, and remote sensing in marine environment [11–6]. The appropriate description of the wave sea structure is prerequisite to investigate scattering from sea surfaces. The linear surface model remains in common use, which is a linear superposition of gravity harmonic waves with Gaussian random phases. However, in some complicated circumstance that the hydrodynamic nonlinearities of the sea waves should be considered, such as scattering analysis and Doppler spectrum interpretation for shallow sea, the simplest linear model seems incompetent.

Nonlinear hydrodynamics links the motion of waves with different scales, which lead to transformation of the wave shape. Based on this, some nonlinear models came into being and had been widely used. Rino et al. [7] comparatively studied the backscattering and the Doppler spectra from time-varying linear and nonlinear Creamer [8] sea surfaces. Toporkov and Brown [9], [10] made a comprehensive study of the Creamer nonlinear surface scattering characteristics. For other nonlinear sea-surface models such as the West model [11], Johnson et al. [12] and Hayslip et al. [13] carried out related research studies. Recently, Soriano et al. [14] have extended the Doppler spectral analysis to the 2-D nonlinear surfaces (3-D microwave scattering issues) from 1-D ones. Nouguier et al. [15] resorted to the so-called nonlinear “choppy wave model” (CWM) [16] probing the impact of nonlinear-wave profiles in scattering from sea surfaces. They [17], [18] also combined the CWM with the weighted curvature approximation (WCA) and derived related statistical expression to simulate ocean Doppler features at microwave frequencies. Thus, compared with the Creamer model and West model, the CWM has proven to enjoy some desirable properties such as analytical simplicity and numerical efficiency. Most of these studies are limited to infinite-depth sea, however.

In recent decades, Bitner [19] studied the nonlinear effects of shallow-water wind waves and drew the conclusion that the shallow-water wind waves can be treated like a quasi-normal random process, and the nonlinear effects may slightly decrease the mean wave height. Barrick and Lipa [20], [21] presented details of the analytical techniques for the modeling and inversion of second-order high-frequency (HF) radar Doppler spectra of sea echo, and they stressed that the energy in the second-order spectrum increases as the water depth decreases and the hydrodynamic contribution is more important when sea waves move into shallow water. Holden and Wyatt [22] discussed modifications necessary to account for the effects of shallow water in the simulation and inversion of radar Doppler spectra. Moreover, the related research on fluid mechanics [23] found that the energy transportation in shallow water is more efficient than that in deep water and the nonlinear–wave–current interactions and wave–wave interactions occur more frequently, which mainly control the shape and evolution of the wind-wave spectrum. Therefore, to simulate the structure of the finite-depth shallow-sea waves, the corresponding finite-depth sea spectrum should be considered. In general, as a wave propagates from an...
infinite-depth sea to a finite-depth region, water depth decreases and the wave slows down, although its frequency remains the same. As a result, it changes shape: Its crest becomes shorter and steepens, while its trough lengths and flattens out. Thus, the nonlinear CWM is much appropriate for constructing this kind of surface profiles. In this paper, we revised the CWM to take the infinite-depth effect into account. Moreover, we just consider sea surfaces of constantly finite depth, and the extra effects of wave energy dissipation such as breaking are certainly not taken into account. Based on the preceding analysis, the revised CWM combining with the finite-depth sea spectrum is more suitable to accurately describe the profiles of a finite-depth sea.

The small-slope approximation (SSA) theory comprises a basic approximation of the theory (SSA-I) and second-order corrections to it (SSA-II), which has been widely applied to evaluate microwave scattering from sea surfaces [24], [25]. It is a competent candidate to bridge the gap between the Kirchhoff approximation (KA) and the small-perturbation method (SPM). Compared with the KA and SSA-I, the SSA-II is capable of predicting polarization sensitivity (both copolarization and cross-polarization) for Doppler spectra, which is indispensable for studying Doppler characteristics of sea surfaces. The more recent WCA [17], [18] is also a promising analytical model, whose scattering amplitude is a correction to SSA-I and based on a combination of SPM and KA kernels that are evaluated at local angles; thus, it is a local model that cannot predict correct cross-polarization in the plane of incidence. Based on the consideration of the continuity of the future work for evaluation of the cross-polarization, thus, the SSA-II is employed in this study.

This paper is organized as follows. In Section II, the revised nonlinear CWM combining with the experiment-verified finite-depth sea spectrum is used to simulate 2-D surfaces of finite-depth seas. Section III presents the SSA-II model for evaluating the microwave backscattering from finite-depth sea in detail. Comparison of numerical results of the monostatic normalized radar cross section (NRCS) and Doppler spectra for infinite-depth sea is used in this paper, which was proposed in a relatively comprehensive study of finite-depth wave spectra of Bouws et al. [28]. This study comprised measurements made at coastal sites during three field experiments (TEXEL in the Dutch North Sea, MARSEN in the German Bight, and ARSLOE in the U.S. east coast). The spectrum of actively growing wind waves in finite-depth sea can be analytically expressed as

\[ S_{TMA}(f) = S_J(f)\Phi(d) \]

with

\[ S_J(f) = \frac{\alpha g^2}{(2\pi)^4 f^5} \exp \left[ -1.25 \left( \frac{f_m}{f} \right)^4 \right] \exp \left[ -\frac{(f - f_m)^2}{2\sigma^2 f_m^2} \right] \]

and

\[ \Phi(d) = \frac{\left[ k(f, d) \right]^{-3} \frac{\partial k(f, d)}{\partial f}}{\left[ k(f, \infty) \right]^{-3} \frac{\partial k(f, \infty)}{\partial f}} = \frac{\tanh^3(kd)}{\tanh(kd) + kd - kd \tanh^2(kd)} \]

where \( S_J(f) \) is the JONSWAP spectrum, \( f \) is the frequency of the sea wave, \( f_m \) is the spectrum peak frequency, and \( g \) is the acceleration of gravity. The Phillips parameter \( \alpha \) changes as the wind speed \( u \) at a height of 10 m. The factors \( \gamma \) and \( \sigma \) exhibit no trend as functions of \( u \). The exact value of the aforementioned parameters can be referred to [29]. \( k(f, d) \) is the wavenumber of the sea wave, which is related to \( f \) and water depth \( d \) according to the shallow-water gravity-capillarity dispersion relation as \( f = \sqrt{gk(1 + k^2/k_m^2)\tanh(kd)/2\pi} \), where \( k_m = 363.2 \text{ rad/m} \) is the wavenumber with minimum phase speed. The function \( \Phi(d) \) is applied to describe the upper bound of the equilibrium range of the spectrum, meanwhile. When \( d \) is chosen to be infinite, the spectrum is just the JONSWAP spectrum for infinite-depth sea.

Fig. 1 just shows the influence of the water depth to the TMA spectrum. From Fig. 1, it is evident that the slope of
the HF part of the spectrum is much more flat as the seawater depth decreases. Compared with deep-water sea spectrum, the TMA spectrum is more competent in the subsequent surface realizations of the finite-depth sea.

## B. Sea-Surface Realizations

To study the time-evolving scattering characteristics of the sea surface correctly, the geometric property of the sea surface should be captured in detail. For the fully developed infinite-depth sea, the spectral method is applied to yield a linear superposition of harmonic waves whose amplitudes are independent normal-distributed random values times the square root of the position of harmonic waves whose amplitudes are independent.

The point of the sea surface is now as before. Compared with the classical linear wave, this particular warping in the nonlinear model sharpens the wave peaks and broadens the wave valleys, which is a nonlinear hydrodynamic behavior that makes the sea surface generated more consistent with the real sea surface of the finite-depth sea.

Fig. 2 shows the comparison of the 2-D bird’s-eye view of the linear infinite-depth sea and the nonlinear finite-depth choppy sea, the wind speed is 6 m/s, and the depth of the shallow sea is 5 m. From the comparison of the circles at the corresponding same position in Fig. 2(a) and (b), it is found that the crests of the finite-depth choppy sea are sharpened while those of the linear sea are smoother.

To clearly show the influences of nonlinear hydrodynamics and finite-depth factor on the geometric appearance of the surface, a comparison of 1-D sea-surface profiles is shown in Fig. 3. In Fig. 3(a), it is much easier to observe that, for the nonlinear choppy surface wave, their peaks become much steeper than that of the linear wave but their troughs flatten out. This consists of the conclusion drawn from Fig. 2. From the comparison in Fig. 3(b), it is found that the finite-depth factor also influences the profile shape. To a certain extent, as the water depth decreases, the wave crests steepen while wave troughs become much gentler. This property of the nonlinear hydrodynamic finite-depth wave model is more consistent with the actual shallow-sea waves.

## III. SSA-II Model for Backscattering From 2-D Surfaces of Finite-Depth Seas

Consider a tapered plane wave illuminating upon a 2-D rectangle sea surface \( L_x \times L_y \) to eliminate the edge effect caused
by choosing limited size surface. The geometry of the scattering problem is shown in Fig. 4. \( \theta_i \) and \( \phi_i \) denote the incident angle and incident azimuth angle, respectively, while \( \theta_s \) and \( \phi_s \) represent the scattering angle and scattering azimuth angle, respectively. The incident wave vector \( \mathbf{k}_i \) and scattering wave vector \( \mathbf{k}_s \) can be decomposed into their horizontal components and vertical components, respectively (see Fig. 4)

\[
\mathbf{k}_i = k_0 - q_0 z \quad \mathbf{k}_s = k_1 + q_1 z
\]

where \( k_0^2 + q_0^2 = k_1^2 + q_1^2 = k_s^2 \); meanwhile, \( q \) and \( q_0 \) both should be larger than zero. The tapered incident field can be expressed as

\[
\mathbf{E}_i(\mathbf{r}) = \mathbf{G}(\mathbf{r}, h) \exp(-j\mathbf{k}_i \cdot \mathbf{r})
\]

with the 2-D taper function \( \mathbf{G}(\mathbf{r}) \) detailedly defined in [34]. Thus, the scattering amplitude of the SSA-II model for linear sea is expressed as

\[
\mathbf{S}(k_i, k_s; t) = \frac{2\sqrt{q_0 q_1}}{(q_0 + q_1)\sqrt{P}} \int \frac{dr}{(2\pi)^2} \mathbf{G}(\mathbf{r}, h) \times \exp\left[-j(k_1 - k_0) \cdot \mathbf{r} + j(q_1 + q_0)h(r, t)\right] \\
\times \left(B(k_1, k_0) - \frac{j}{4} \int M(k_1, k_0; \xi) \mathbf{H}(\xi, t) \times \exp(j\xi \cdot \mathbf{r}) d\xi\right) \tag{10}
\]

where

\[
\mathbf{H}(\xi, t) = \frac{1}{(2\pi)^2} \int h(r, t) \exp(-j\xi \cdot \mathbf{r}) dr \tag{11}
\]

is the Fourier transform of the surface elevation. \( P \) is the incident wave power captured by the sea surface. The kernel functions of the integral \( B \) and \( M \) are \( 2 \times 2 \) matrices which are mainly dependent on the configuration angles, the polarization, and the complex permittivity of the lower medium. The corresponding details can be found in [34]; here, we safely omit them for the sake of brevity.

For revised CWM, the integral variables \( \mathbf{r} \) in (10) should be replaced by \( \mathbf{r} = \mathbf{r} + C_f(r, t) \); thus, the Jacobian \( J \) of the transformation from \( \mathbf{r} \) to \( \tilde{\mathbf{r}} \) is utilized to accomplish this change of integral variables. Equation (10) should be rewritten as

\[
\mathbf{S}(k_i, k_s; t) = \frac{2\sqrt{q_0 q_1}}{(q_0 + q_1)\sqrt{P}} \int \frac{dr}{(2\pi)^2} \mathbf{G}(\tilde{\mathbf{r}}, h) \times \exp\left[-j(k_1 - k_0) \cdot (\mathbf{r} + C_f(r, t)) \right] \\
+ j(q_1 + q_0)h(r, t)] \\
\times J(r, t)\left(B(k_1, k_0) - \frac{j}{4} \int M(k_1, k_0; \xi) \mathbf{H}(\xi, t) \times \exp(j\xi \cdot \tilde{\mathbf{r}}) d\xi\right) \tag{12}
\]

where \( J(r, t) = J_{xx} J_{yy} - J_{xy} J_{yx} \), with individual terms

\[
J_{xx} = 1 + \partial C_{fx}(r, t)/\partial x, \quad J_{yy} = 1 + \partial C_{fy}(r, t)/\partial y, \quad J_{xy} = \partial C_{fx}(r, t)/\partial y = J_{yx}, \quad C_f = (C_{fx}, C_{fy}).
\]

The symbol \( \partial \) denotes the calculation of corresponding partial derivatives. Thus, the average NRCS can be expressed as

\[
\sigma_{\text{SSA-II}} = 4\pi q_0 q_1 \left\langle |\mathbf{S}(k_i, k_s; t)|^2 \right\rangle \tag{13}
\]

where the angle brackets denote the ensemble average.

Compared with NRCS, the Doppler spectrum of the sea backscattered echoes is a more refined tool to detect the subtle changes in the fluid motion. Furthermore, the hydrodynamic nonlinearities have a dramatic impact on the Doppler spectrum. Thus, the expression of the Doppler spectrum based on the periodogram method [10] is given by

\[
S_{\text{Dop}}(f) = \left\langle \left| \frac{1}{T} \int_0^T \mathbf{S}(k_i, k_s; t) \exp(-j2\pi ft) dt \right|^2 \right\rangle. \tag{14}
\]

Like in (13), the angle brackets also denote the ensemble average over much surface realizations. In the following spectra analysis, each Doppler spectrum is evaluated over 120 samples...
of the surface realizations involving time-varying sea surfaces with 256 time steps.

IV. Numerical Results and Analysis

It is instructive to compare the monostatic NRCS for the nonlinear hydrodynamic choppy sea surfaces and that for the linear sea surfaces with the experimental measured data. In Fig. 5, the experimental data from the airborne four-frequency radar system of the Naval Research Laboratory [36] for L-band (1.228 GHz) are chosen as a benchmark. Thus, the water depth is set to be infinite. The wind speed is 5 m/s in upwind direction, the size of the sea surface is \( L_x = L_y = 128 \lambda \), in which \( \lambda \) denotes the incident microwave wavelength, and the surface sample interval is set as \( \lambda/8 \). The taper wave beam waist is chosen to be \( L_x/6 \), and the final NRCS is averaged over 40 surface realizations.

It is evident that, in small-incident-angle region, the NRCSs for the choppy surfaces and the linear sea surfaces are nearly the same for both HH and VV polarizations, while for larger angles, it is evident that the results for CWM are larger than that for linear surfaces; this can be explained by the reason that the nonlinear hydrodynamic CWM takes the wave–wave interactions into account. Meanwhile, it is not difficult to see that the results for choppy sea surfaces are more consistent with the experimental data. Thus, the conclusion can be drawn that the validity of the nonlinear hydrodynamic CWM is obtained for infinite-depth sea.

Fig. 6 shows the comparison of the monostatic NRCS versus incident angles for infinite-depth sea and 5-m-deep sea. The wind speed is 10 m/s. (a) VV polarization. (b) HH polarization.

is not great for most of the incident angles, which prompts us to investigate the differences of the Doppler characteristics between these two kinds of surfaces.

Fig. 7 shows the normalized Doppler spectra of the linear surfaces and the nonlinear choppy surfaces of infinite-depth seas. The surface area and corresponding sample interval are the same as those in the aforementioned NRCS calculation. The wind speed is 5 m/s, the incident wave frequency is 1.228 GHz, and the incident angles are fixed at 35° and 70°, respectively. The corresponding Bragg frequencies \( f_{\text{Bragg}} = \sqrt{g \sin \theta_i/(\pi \lambda)} \) are noted in the plots by the vertical black solid lines. First, as the incident angle increases, it can be seen that the spectral peaks move closer to the corresponding Bragg frequencies for both linear and nonlinear surfaces. It attributes to the fact that the Bragg scattering component accounts for a larger proportion when the incident angle increases. From 35° to 70°, the Doppler spectra of signals backscattered from linear surfaces become narrower than those from the choppy surfaces. This is due to the fact that the nonlinear-wave components propagate faster than the linear-wave components. Second, for linear surfaces, the spectra for both HH and VV polarizations are centered near the Bragg frequency, while in the case of choppy surfaces, the spectrum for HH polarization is centered at much higher frequencies than that for VV polarization; moreover, both of them are farther away from the Bragg frequency. These are attributed to the fact that the CWM corrects the horizontal component of particle velocities by adding a displacement related to the surface elevation to the horizontal position of the particles. It is known to all that these velocities remarkably affect the Doppler spectrum; thus, the shape of the Doppler spectrum is impacted by the modulation of Bragg waves by longer waves. These conclusions are also supported by those found in [37] and [38].
To investigate the corresponding characteristics of Doppler spectra for other wind directions aside from upwind, we have performed related simulations to illustrate the influence of the wind direction to the Doppler spectra features in Fig. 8. $\phi_w$ is designated as the angle between the radar looking direction and the upwind direction. The incident angle is set as $70^\circ$, the incident frequency is also $1.228$ GHz, and the wind speed is $3$ m/s. Three wind directions of $\phi_w = 0^\circ$, $60^\circ$, and $90^\circ$ are presented for comparison. As $\phi_w$ changes from $0^\circ$ to $90^\circ$, the negative peak appears more and more prominent for both types of sea surface and finally comparable with the positive peak; moreover, compared with linear surfaces, the Doppler spectral widths for nonlinear choppy surfaces are much greater for both polarizations and all three wind directions.

After investigating the Doppler spectrum for deep-sea surfaces, now, we focus on the study of its finite-depth sea counterpart. In [20] and [21], based on the HF radar experiment measured data and Weber and Barrick theory in [39],
some characteristics of the Doppler spectrum for echoes from shallow-water waves have been observed and verified, which could serve as qualitative arguments to validate our model. Thus, Fig. 9 shows the comparison of the Doppler spectra for backscattered echoes from infinite-depth sea and 5-m-finite-depth sea in the similar HF band circumstance. The incident angles are set as 35°, and the radar frequency is 30 MHz. The wind speed is 5 m/s. It can be found that, when the water depth decreases, the second-order peaks of the Doppler spectrum rise and the amplitude of the second-order spectrum also increases, even if the first-order peaks are nearly at the same level, which are similar to the corresponding features presented in [20] and [21]. Although the origin of this phenomenon has not yet been understood explicitly, it probably implies that strong nonlinear effects are inherent in the finite-depth water wave field.
The incident angle is 70°, and the wind speed is 10 m/s. First, when the wind direction changes from 0° to 90°, it can be seen that the peaks at negative frequency gradually grow in the spectra for both types of surfaces and polarizations. Second, it is not difficult to find that the discrepancy of Doppler spectra attributed to infinite-depth effect is less pronounced when the wind direction changes from upwind direction to crosswind. This observation is similar as in Fig. 8. Finally, it is also noticeable that the spectral amplitudes for finite-depth sea surfaces are much higher than those for infinite-depth surfaces in the frequency range just around the Doppler peak frequency. It is probably a significant sign that the nonlinear-wave–wave interactions are strengthened as the water depth decreases. As waves propagate into finite-depth water from the deep water, the depth effect will work after the waves “touch” the bottom. Due to shoaling, the nonlinear interactions between waves become stronger. The features of the Doppler spectra shown in Figs. 9 and 10 consist of the corresponding simulations verified by experimental measurements in finite-depth sea in [21] and [22] in the qualitative sense. Moreover, from the runtime involved for the Doppler spectra simulation that is shown in Table I, it is indicated that simulations for finite-depth sea surface consume a little more time than those for infinite-depth sea surface, and it is also the same situation when the wind direction changes from upwind to crosswind.

V. CONCLUSION

This paper has investigated the characteristics of the microwave backscattering from the 2-D time-evolving nonlinear surfaces of finite-depth seas using SSA-II in connection with the CWM. A comparative study has been presented to highlight the features of both the NRCS and the Doppler spectra arising from nonlinear hydrodynamic effects and shoaling effects. In infinite-depth sea circumstance, the comparison of the monostatic NRCS for the linear sea model and nonlinear CWM with the experimental measured data shows that the nonlinear CWM has better performance. When both are based on the same nonlinear CWM, the discrepancy between the infinite-depth sea and finite-depth sea for monostatic NRCS is not great. On the other hand, the characteristics of the Doppler spectra for nonlinear choppy sea surfaces are distinct from their counterpart of linear sea surfaces probably due to the nonlinear-wave–wave interactions, which shows that nonlinear hydrodynamics should be given adequate consideration in the Doppler spectra analysis. Furthermore, from the Doppler spectra analysis for nonlinear finite-depth sea in different wind directions, it is demonstrated that the discrepancy of Doppler spectra attributed to infinite-depth effect is less pronounced when the wind direction changes from upwind direction to crosswind. When the water depth decreases, it is also observed that the Doppler spectra have higher second-order peaks and increased spectral amplitudes in the frequency range just around the Doppler peak frequency, which reiterates the importance of the role that the nonlinear hydrodynamics played in the interpretation of backscattering from finite-depth seas. The analysis presented in this paper will help to better investigate the backscattering from the surfaces of finite-depth nearshore seas from the qualitative point of view.

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