A Novel User Scheduling for Multiuser MIMO Systems with Block Diagonalization

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Abstract—**In order to exploit multiuser diversity in multiuser MIMO systems with Block Diagonalization, A novel multi-user scheduling algorithm, guarantee both the fairness between users and higher capacity of the system, is proposed in this paper. We provide a number of different scheduling schemes and compare them in the terms of average capacity and the fairness as a function of the number of users. Numerical results show that our algorithm can make perfect tradeoff between fairness and capacity.**

Keywords- fairness; Block Diagonalization; multiuser MIMO systems

I. INTRODUCTION

Multiple-input multiple-output (MIMO) communication systems [1] have the ability to offer high spectral efficiency as well as link reliability. In point-to-point multiple antenna systems[2],[3],[4], it is well known that the capacity increases linearly with the minimum of the number of transmit or receive antennas, irrespective of the availability of channel state information at the transmitter. In MIMO broadcast channels [5], [6], it was shown that the capacity can be boosted by exploiting the spatial multiplexing capacity of transmit antennas and transmit to multiple users simultaneously with the help of Space Division Multiple Access (SDMA) [7], rather than trying to maximize the capacity of a single-user link.

As we all know that the sum capacity in a multiuser broadcast channel is defined to be the maximum aggregation of all users' data rates. Dirty paper coding (DPC) [8], an optimal solution, has been proven that can achieve the capacity region perfectly. The fundamental idea of DPC is that when a transmitter has perfect knowledge of the interference in a channel, a code can be designed to compensate for it availably, and the capacity of the channel is the same as if there were no interference. Unfortunately, DPC approach cannot be implemented into practice in terms of its complexity. An alternative and more practical precoding technique for downlink broadcast MIMO channels is block diagonalization (BD) [9], [10]. With the help of BD, each user's data is multiplied by a linear precoding matrix before transmission and the precoding matrix of every user is restricted to be in the null space of all other users' channels. Hence BD has made a great contribution in the field of interference alignment (IA) in the case of knowing the CSI of cell and adjacent cell [15]. On the other hand, BD is inferior in terms of sum capacity relative to DPC. Due to the fact that each user's precoding matrix must lie in the null space of all other user's channels and the number of users that can be simultaneously supported with BD is limited by the number of transmit and receive antennas. It is necessary that a subset of all the users in the multiuser MIMO system should be selected with BD. An optimal user set that maximizes the sum capacity requires an exhaustive search method which can exploit the full multiuser diversity that the sum capacity grows like $\log\log K$ in a large user region, K represents the number of user in the systems. Unfortunately, exhaustive search method is not computation- ally feasible especially when the number of users in the system is large enough and does not consider the fairness among users [11], [12],[13].

In this paper, we proposed a novel multi-user scheduling algorithm for BD in the multiuser MIMO downlink system with the aim of maximizing the sum capacity while keeping both the low computational complexity and relatively fairness among users. The main idea of the algorithm is that it iteratively selects users until the maximum number of simultaneously supportable user is achieved. In each user selection step, the algorithm selects only one user who provides the maximum combine measurement, which contains capacity and fairness at the same time, with those already selected users. Simulation results show that the proposed algorithm can make a best tradeoff between fairness and the sum capacity by adjusting the forgetting factor and its complexity increases only linearly, rather than exponentially, with the total number of users in the multiuser MIMO downlink systems.

The paper is organized as follows. Section Ⅱ introduces the system model. The proposed multi-user scheduling algorithm is discussed in section Ⅲ. Simulation results are given in section Ⅳ. Finally, the conclusions are drawn in section Ⅴ.

II. SYSTEM MODEL

In this section, we describe the system model and briefly depict the block diagonalization scheme for multiuser MIMO systems. A multiuser MIMO downlink channel is considered in this paper, where M_T transmit antennas are located at the

The work in the paper was partly funded by the 111 Project (B08038), EU FP7 Project MONICA (PIRSES-GA-2011-295222), National High Technology Research and Development Program of China (863 Program) (2012AA 120604), and the Open Research Fund of National Mobile Communications Research Laboratory, Southeast University (2012D01).

base station and $M_{R,u}$ receive antennas are located at the u-th user terminal, $u = 1,2,\ldots,K$. K denotes the total number of users in the system. Naturally, the total number of receive antennas can be expressed as

$$M_R = \sum_{i=1}^{\hat{K}} M_{R,u} \tag{1}$$

A system model for such a system is depicted in Fig.1.

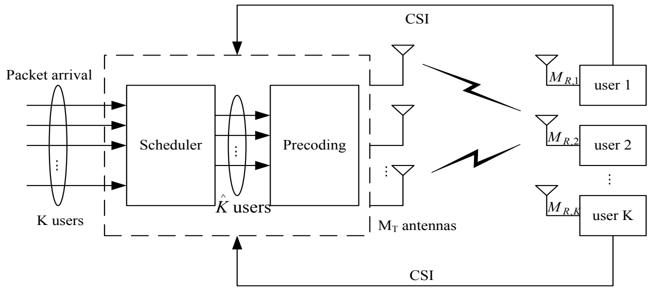

Fig.1 System model for a multiuser MIMO downlink

For the u-th user signal $x_u \in C^{M_{R,u}\times 1}$, the received signal $y_u \in C^{M_{R,u}\times 1}$ is given as

$$\begin{aligned} y_u &= H_u \sum_{k=1}^{\hat{K}} W_k x_k + z_u \\ &= H_u W_u x_u + \sum_{k=1,k\neq u}^{\hat{K}} H_u W_k x_k + z_u \end{aligned} \tag{2}$$

where $H_u \in C^{M_{R,u}\times M_T}$ is the channel matrix between BS and the u-th user, $W_u \in C^{M_T \times M_{R,u}}$ is the precoding matrix for the u-th user which can be obtained by BD in this paper, $\hat{K}$ is the number of users who can be supported simultaneously with BD, and z_u is the noise vector. Note that $\{H_u W_k\}_{u\neq k}$ incurs interference to the u-th user unless the below equation satisfied in Equation (2):

$$H_u W_k = \mathbf{0}_{M_{R,u}\times M_{R,u}}, \forall u \neq k \tag{3}$$

where $\mathbf{0}_{M_{R,u}\times M_{R,u}}$ is a zero matrix.

In order to meet the total transmit power constraint, the precoders $W_u \in C^{M_T\times M_{R,u}}$ must be unitary, $u = 1,2,\ldots,\hat{K}$ .Under the condition of Equation (3), the received signal in Equation (2) is now interference-free, that is

$$y_u = H_u W_u x_u + z_u, \quad u = 1,2,\ldots,\hat{K} \tag{4}$$

We now discuss how to obtain $\{W_k\}_{k=1}^{\hat{K}}$ that satisfy the condition in Equation (3). Let us construct the following channel matrix that contains the channel gains of all users except the u-th user

$$\tilde{H}_u = [(H_1)^H \ldots (H_{u-1})^H (H_{u+1})^H \ldots (H_K)^H]^H \tag{5}$$

When $M_R = M_T$, that is, the total numbers of antennas used by all active users are the same as the number of BS antennas, Equation (3) is equivalent to:

$$\tilde{H}_u W_u = \mathbf{0}_{(M_R - M_{R,u})\times M_{R,u}}, \quad u = 1,2,\ldots,\hat{K} \tag{6}$$

This implies that the precoder matrix $W_u \in C^{M_T\times M_{R,u}}$ must be designed to lie in the null space of $\tilde{H}_u$ .We now discuss how to design the precoders that satisfy Equation (6).Note that the dimension of the matrix $\tilde{H}_u \in C^{(M_R - M_{R,u})\times M_T}$ is less than min（M_R - $M_{R,u}, M_T$）.Under the condition of $M_R = M_T$, min（M_R - $M_{R,u}, M_T$）= M_R - $M_{R,u}$ .Then the singular value decomposition(SVD) of $\tilde{H}_u$ can be expressed as

$$\tilde{H}_u = \tilde{U}_u \tilde{\Lambda}_u [\tilde{V}_u^{non-zero} \tilde{V}_u^{zero}]^H \tag{7}$$

where $\tilde{V}_u^{non-zero} \in C^{(M_R - M_{R,u})\times M_T}$ and $\tilde{V}_u^{zero} \in C^{M_{R,u}\times M_T}$ are composed of right singular vectors that correspond to non-zero singular values and zeros singular values, respectively. Multiplying $\tilde{H}_u^{DL}$ with $\tilde{V}_u^{zero}$, we have the following relationship

$$\tilde{H}_u \tilde{V}_u^{zero} = \mathbf{0} \tag{8}$$

From Equation (8), it can be seen that $\tilde{V}_u^{zero}$ is in the null space of $\tilde{H}_u$, that is to say namely, when a signal is transmitted in the direction of $\tilde{V}_u^{zero}$, all but the u-th user receives no signal at all. Thus, the precoding matrix of the u-th user can be represented by

$$W_u = \tilde{V}_u^{zero} \tag{9}$$

Then, the equivalent channel of the u-th user is expressed by

$$\bar{H}_u = H_u W_u \tag{10}$$

Now the capacity of the u-th user is

$$R(u) = \log | I + \frac{1}{\sigma^2} \bar{H}_u Q_u \bar{H}_u^H | \tag{11}$$

where $Q_u = E[\mathrm{x_u x_u^H}]$ is user u's input covariance matrix and σ^2 is the energy of noise.

III. FAIRNESS BASED LOW-COMPLEXITY USER SCHEDULING ALGORITHM

BD can be thought of as a generalization of channel inversion for situations with multiple antennas per user. The BD algorithm is being applied more broadly with its low complexity but is restricted to channels where the number of transmit antennas (M_T) is no smaller than the total number of receive antennas (M_R) in the network. Hence, we have to select a subset of users which is equal to the number of simultaneously supportable users with BD under the condition that $M_R \le M_T$.

In this section, we first introduce a fairness based low-complexity user scheduling algorithm, and then analyze the computational complexity of the proposed algorithm in the end.

A. Fairness based low-complexity user scheduling algorithm

In order to achieve an optimal user set, all possible user set must be searched which does not consider the fairness and cannot be implemented into practice with its complexity. In this subsection, a fairness based low-complexity multi-user scheduling algorithm is proposed. The algorithm iteratively selects users until the maximum number of simultaneously supportable user is achieved. In each user selection step, the algorithm select only one user who provides the maximum combine measurement, which contains capacity and fairness at the same time, with those already selected users.

Let s_i denotes the user index selected in the *i*-th iteration, i.e., $s_i \in \{1,2,\ldots K\}$ and $1 \le i \le \hat{K}$ .Let Ω denote the set of unselected users and γ denote the set of selected users. Let V_k be the basis for the row vector space of H_k after applying the Gram-Schmidt orthogonalization (GSO) [14] procedure to the rows of H_k . T_k is the average capacity of user k . The flow chart of the fairness based low-complexity user scheduling algorithm is described in fig.2. Before the proposed algorithm begins, we should set $T_n^k = 1$, $k \in \{1,2,\ldots K\}$ .The detailed steps are shown in the following:

1) Ready to schedule user at a certain timeslot

Set $\Omega = \{1,2,\ldots K\}$, $\gamma = \Phi$ and $i = 1$.

2) Select the first user

The first user we select should satisfy the equation

$$s_1 = \arg\max_{k\in\Omega} \frac{||\mathrm{H_k}||^2}{T_k} . \tag{12}$$

Now let $V = V_{s_1}$, $\Omega = \Omega - \{\mathrm{s_1}\}$ and $\gamma = \gamma + \{\mathrm{s_1}\}$.

3) Select the remaining $\hat{K}$-1 users

For each $k \in \Omega$, let $\tilde{H}_k = H_k - H_k V^H V$ .Then $\tilde{H}_k$ is in the null space of V .We set $i = i+1$ .Now we estimate the value of i . If the condition $i > \hat{K}$ is satisfied, the process jumps to step 4.

for $j = 1 : i-1$

a) Let $\hat{H}_{s_j,k} = [\mathrm{H}_{s_1}^T \ldots \mathrm{H}_{s_{j-1}}^T \mathrm{H}_{s_{j+1}}^T \ldots \mathrm{H}_{s_{i-1}}^T \mathrm{H_k^T}]^\mathrm{T}$;

b) Let $W_{s_j,k}$ be the row basis for $\hat{H}_{s_j,k}$;

end-for

For each $s \in \gamma$, let $\tilde{H}_s = H_s - H_s W_{s,k}^H W_{s,k}$. Then, the i-th user we select in this step must satisfy the equation

$$s_i = \arg\max_{k\in\Omega} \left(\sum_{s\in\gamma} \frac{||\tilde{\mathrm{H}}_\mathrm{s}||_\mathrm{F}^2}{T_s} + \frac{||\tilde{\mathrm{H}}_\mathrm{k}||_\mathrm{F}^2}{T_k} \right) \tag{13}$$

Set $\Omega = \Omega - \{\mathrm{s_i}\}$, $\gamma = \gamma + \{\mathrm{s_i}\}$ and $V = [\mathrm{V^T}, \tilde{V}_{s_i}^\mathrm{T}]^\mathrm{T}$. Then the process is back to step 3.

4) Transmit data and update average capacity

Until now, the maximum number of simultaneously supportable user is reached. That is to say, the size of γ is equal to $\hat{K}$. The algorithm is terminated at current timeslot and the BS transmits data x_k , $k \in \gamma$ to the user k with the help of BD. In order to keep fairness between users and prepare to the next schedule timeslot in the systems, the average capacity for all users should be updated in time as follows

$$T_k = \begin{cases} (1 - \frac{1}{\mathrm{t_c}})T_k + \frac{1}{\mathrm{t_c}} R(\mathrm{k}), & \mathrm{k} \in \gamma \\ (1 - \frac{1}{\mathrm{t_c}})T_k, & k \notin \gamma 且 k \in \Omega \end{cases} \tag{14}$$

where R(k) can be obtained by Equation (11) and different fairness can be achieved by tuning the forgetting factor f which is the inverse of the time constant t_c .

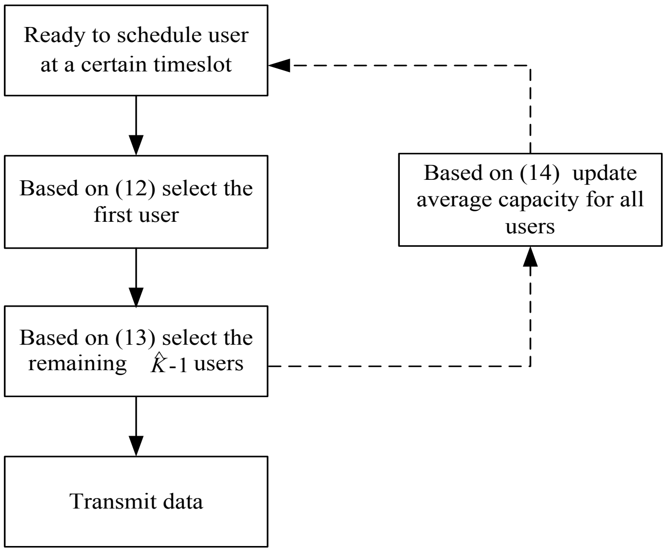

Fig.2 Flow chart for user scheduling

B. Computational complexity analysis

In this subsection we quantify the complexity of the proposed algorithm and compare with the brute-force approach in detail. The complexity we considered in this paper is counted as the number of flops. A flop can be regarded as a real floating point operation. A real addition, multiplication, or division is counted as one flop. A complex addition and multiplication require two flops and six flops respectively. Although flop counting cannot characterize the true computational complexity, the order of the computation load is captured completely, so it is the reason that the computational complexity can be obtained with the aid of flops in this correspondence.

For an $m \times n$ complex-valued matrix $H \in C^{m \times n}$, we first provide the flop count of several matrix operations that are frequently used in the following analysis. We assume $K >> \hat{K}$, $\hat{K} M_{R,u} \approx M_T$, and $m \le n$ in this section.

- $\|\mathrm{H}\|_F^2$ takes $2mn$ real multiplications and $2mn$ real additions. Hence, the flop count for $\|\mathrm{H}\|_F^2$ is $4mn$.
- GSO (H) takes $4m^2n - 2mn$ real multiplications, $4m^2n - 2mn$ real additions, and $2mn$ real divisions. The flop count for GSO is $8m^2n - 2mn$.
- The flops count for SVD of complex-valued $m \times n$ ($m \le n$) matrices, we approximate the flop count as $24mn^2 + 48m^2n + 53m^3$ by treating every operation as complex multiplex multiplication.
- In order to compute the equation $\tilde{H}_k = H_k - H_k V^* V$, we need $18\mathrm{m}^2\mathrm{n}$ flops.

The detailed complexity analysis of the proposed algorithm in this paper is following:

1) $i = 1$: The Frobenius norm of K users needs $4KM_{R,u}M_T$ flops counts.

2) $i \ge 2$: For each $k \in \Omega$, we need $18(\mathrm{i}-1)\mathrm{M}_{\mathrm{R,u}}^2 M_T$ flops for $\tilde{H}_k = H_k - H_k V^* V$, $8(\mathrm{i}-1)^2\mathrm{M}_{\mathrm{R,u}}^2 M_T - 2(\mathrm{i}-1)M_{R,u}M_T$ flops for $W_{s_j,k}$; $18(\mathrm{i}-1)\mathrm{M}_{\mathrm{R,u}}^2 M_T + 4(\mathrm{i}-1)$ $M_{R,u}M_T$ flops for $\|H_s - H_s W_{s,k}{}^* W_{s,k}\|_F^2$; and $4(\mathrm{i}-1)$ $M_{R,u}M_T$ flops for $\|\tilde{\mathrm{H}}_k\|_F^2$.

Therefore, the total flops φ_n of the proposed algorithm is

$$\varphi_n \approx \sum_{i=2}^{\hat{K}} \{[36(\mathrm{i}-1) + 8(\mathrm{i}-1)^2]\mathrm{M}_{\mathrm{R,u}}^2 M_T + 6(\mathrm{i}-1)M_{R,u}M_T\} \times(\mathrm{K}-\mathrm{i}+1) + 4\mathrm{K}M_{R,u}M_T \approx O(\mathrm{K}) \tag{15}$$

In the optimal user scheduling algorithm, an exhaustive search over the $\sum_{i=1}^{\hat{K}} C_K^i$ possible user sets should be executed. The complexity of this brute-force search method is

$$\begin{aligned}\varphi_n &\approx \sum_{i=2}^{\hat{K}} C_K^i \mathrm{i}[48(\mathrm{i}-1)^2\mathrm{M}_{\mathrm{R,u}}^2\mathrm{M_T} + 24(\mathrm{i}-1)\mathrm{M}_{\mathrm{R,u}}\mathrm{M_T}^2 + 54(\mathrm{i}-1)^3\mathrm{M}_{\mathrm{R,u}}^3] \\ &> C_K^{\hat{K}}\hat{K}[48(\hat{K}-1)^2\mathrm{M}_{\mathrm{R,u}}^2\mathrm{M_T} + 24(\hat{K}-1)\mathrm{M}_{\mathrm{R,u}}\mathrm{M_T}^2 + 54(\hat{K}-1)^3\mathrm{M}_{\mathrm{R,u}}^3] \\ &\approx O(C_K^{\hat{K}}) = O(\mathrm{K}^{\hat{\mathrm{K}}})\end{aligned} \tag{16}$$

Compared (15) with (16), we can get the result that the proposed algorithm has the linear complexity with K, because no more than $\hat{K}K$ user sets need to be searched over. However, the complexity of the brute-force method is $O(\mathrm{K}^{\hat{\mathrm{K}}})$.

IV. Simulation Results

In this section, we compare the performance of different user scheduling algorithms including Frobenius norm-based user selection algorithm (BD n-algorithm), the use selection algorithm proposed in this paper (BD proposed) and the user selection algorithm with First-In First-Out (BD FIFO).

For simplicity, we assume that the number of receive antenna with every user is the same in the system. The maximum number of simultaneously supportable user $\hat{K}$ is equal to $\left\lceil M_T / M_{R,u} \right\rceil$, where $\lceil \bullet \rceil$ is the ceiling operation.

We set $M_T = 4$ and $M_{R,i} = 2$, $i \in \{1,2,\ldots\mathrm{K}\}$. The channel model we choose is Rayleigh Fading Channel. The users are divided into three parts in terms of the channel power, the channel power of the first part is 1, the second is 1/2 and the third is 1/4 respectively. The total number of channel realizations is 10,000.The SNR in the simulation we set is 20dB all the time.

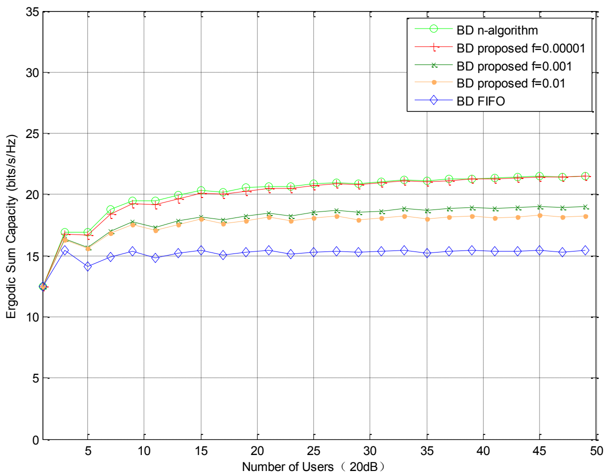

Fig. 3 the comparison of ergodic sum capacity

In Fig.3 we compare the ergodic sum capacity with the different user scheduling schemes. We observed that the ergodic sum capacity of BD proposed increases as the number of user increases due to the multiuser diversity obtained as well as BD n-algorithm. However, the capacity of BD FIFO keeps constant all the time because of no multiuser diversity achieved. Looking at the BD proposed in different forgetting factor f .When f increases, the capacity will be absolutely decreases. That is true because when the forgetting factor approaches 1, the algorithm approaches the round robin scheduler (RRS) and no multiuser diversity can be exploited with this setting mode, whereas fairness should be boosted naturally as f approaches 1 which can be seen in Fig.4.

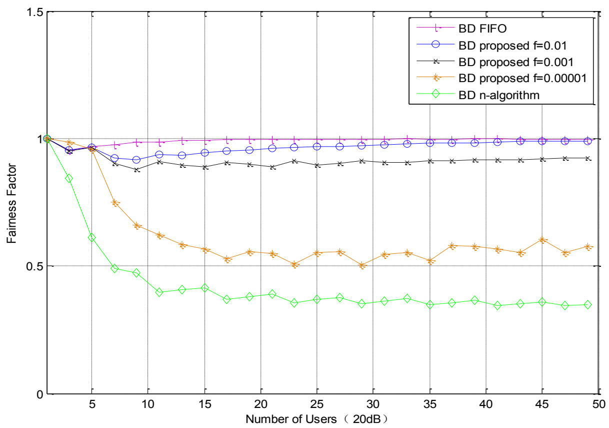

Fig. 4 the comparison of fairness

As can be seen from the fairness factor versus the number of user in Fig.4, the fairness of BD proposed in forgetting factor $f = 0.01$ and BD FIFO are almost optimal with f approximately equals to one. As the forgetting factor f decreases, the fairness of BD proposed also decreases. The reason is that when f approaches 0, the algorithm now approaches the greedy scheduler (GS), thus achieving the maximum multiuser diversity of the system which can be seen in Fig. 2 but at the expense of the fairness. Compared to BD FIFO with which the users have almost the same opportunity to be selected, BD n-algorithm only selects the users whose channel condition is good, so its fairness is very poor.

V. CONCLUSION

In this paper, the fairness based multi-user scheduling algorithm is proposed. The goal is to select a subset of users to maximize combine measurement, which contains throughput and fairness, with low complexity. The main idea of the proposed algorithm in this paper is to select the set of users such that the sum of the combine measurement that considering both fairness and effective channel energy of these selected users is as large as possible.

It is well known that the optimal multi-user scheduling algorithm is complete search. Unfortunately, the complexity of the complete search is roughly $O(K^{\hat{K}})$, where K is total number of users and $\hat{K}$ is the maximum number of simultaneous users. BD n-algorithm can obtain a higher capacity at low complexity, but no fairness is considered as well as complete search. Compared to BD n-algorithm, BD FIFO can only keep the optimal fairness with no multiuser diversity achieved. Simulation results show that the proposed multi-user scheduling algorithm can make a best tradeoff between fairness and throughput by adjusting the forgetting factor f and its complexity increases only linearly, rather than exponentially, with the total number of users.

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