Efficiency Metrics for Wireless Communications

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***Abstract*-Green wireless communications have received increasing attention from government, academia, and industry. However, how to define a green wireless communication system has been an open topic. In this paper, firstly, we present the definition of green efficiency of wireless links and carry out a comprehensive analysis of the efficiency metrics, e.g., the b/s/Hz spectrum efficiency, the b/s/Hz/W energy efficiency, the b/TENU power efficiency and our proposed (b·m)/s/Hz/W green efficiency of a wireless link, in particular we can prove that green efficiency is a strict convex function with respect to transmission power and distance. Secondly, by tuning the cell size with users' QoS (quality of service) guarantee, we introduce green efficiency into a cellular network and achieve the theoretical upper bound of green efficiency for a cellular network. Finally, we present a green efficiency-based network planning method for practical environment. Simulation results show that the proposed network planning mechanism can approach the upper bound of green efficiency.**

***Keywords*- Green Wireless Communications, Efficiency, Network Planning**

I. INTRODUCTION

Given that there is a worldwide growth in the number of mobile customers, with the majority of the tele-traffic evolving from low data rate speech and modest-rate text messages towards high data rate multimedia services (e.g., online music and video), an increasing contribution of the information and communication technology (ICT) industry to the overall energy consumption of the world is observed. Therefore, energy consumption will become a more important constraint in the design of future mobile communications systems. Therewith, green wireless communications have received much attention from government, academia, and industry [1]. Up to now, the continuous investment in green communications brings about a wealth of theoretical knowledge and practical engineering solutions, e.g., high efficient class F amplifiers [2], network topology, routing, MAC and physical layer protocol [3-5].

Before using the appropriate architectures and radio technologies to reach the green targets, a key thing that matters is to evaluate and measure how green a network can be. On the other hand, green metrics also would offer the direction for the future research on the green communications. However, a widely accepted definition of green wireless communications or green metrics remains an open problem. All researchers agree that a green wireless communication system is highly efficient. However, a widely accepted choice of a criterion characterizing the overall efficiency of a wireless network is also an open problem. In many cases, researchers use the b/s/Hz spectrum efficiency [6], the b/TENU power efficiency [7] and the b/s/Hz/W energy efficiency [8] to evaluate the communications systems. By no means should the efficiency metrics above be classified as less efficient, since in the appropriate circumstances they are capable of considerably improving the overall performance of the entire network. However, the appropriate choice of the network optimization criteria based on different efficiency metrics can have a profound effect on the overall network performance.

After presenting briefly a novel concept of green efficiency [9], we aim to develop green efficiency from a point-to-point (PtP) link to a cellular network in this paper. Firstly, we recast green efficiency in a more comprehensive form, e.g., its mathematical analysis. Secondly, after comparing with the existing efficiency metrics, we introduce green efficiency into network planning.

The rest of this paper is organized as follows. In section II, we present the definition of green efficiency of wireless links. As much investment in wireless systems aims at improving spectrum efficiency, b/TENU power efficiency and energy efficiency, we carry out a comprehensive analysis of green efficiency after comparing with them. In section III, based on the green efficiency criteria, we present a heuristic approach for radio access networks to achieve the maximum overall green efficiency by means of searching for an optimal cell radius. In section IV, simulation studies are carried out to evaluate the performance of the proposed approach. The concluding remarks and the subjects for further study are given in Section V.

II. GREEN EFFICIENCY OF WIRELESS LINKS

In this section, after introducing the existing efficiency metrics of wireless links, we shall present our new green efficiency metrics.

The system capacity is defined as the maximum possible transmission rate such that the probability of error is arbitrarily small, which is quantified by the Shannon-Hartley theorem as:

$$C = B\log_2(1+\gamma_s) \tag{1}$$

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where B is the channel bandwidth in Hz. $\gamma_s = S/N_0$ is the average signal-to-noise ratio (SNR) at the receiver, where S denotes the signal power, and N_0 denotes the noise power.

For simplicity, we assume that the propagation loss $PL = gd^{-a}$ [10], where the value of factor g is determined by the heights and gains of antennas of the transmitter and receiver terminal, carrier frequency and so on; d is the distance between the transmitter and receiver; a is the path loss factor which typically ranges from 2 to 5. So we can arrive at

$$\gamma_s = \frac{S}{N_o} = \frac{P_t \cdot PL}{N_o} = \frac{P_t \cdot gd^{-a}}{N_o}, \tag{2}$$

where P_t denotes the transmission power.

Subsequently, spectrum efficiency is defined as:

$$\eta_s = \frac{C}{B} = \log_2(1+\gamma_s) = \log_2(1+\frac{P_t g}{d^a N_o})\ (b/s/Hz). \tag{3}$$

Energy consumption of the transmitter is mostly dominated by the transceivers. However, it is not the overall power usage, which also includes power amplifiers, antenna, and cooling fans in base stations (BSs), besides P_t. So the total power consumption is expressed as:

$$P_{sum} = b^{-1} \cdot P_t + P_{OM}\ , \tag{4}$$

where b is the efficiency factor of amplifiers and normally $1/b = 0.125$ in BSs. P_{OM} is the operation and maintain consumption including all the other components besides the transceiver power, e.g., cooling fans, which is independent on P_t [11].

Then energy efficiency is defined as:

$$\eta_e = \frac{\eta_S}{P_{sum}} = \frac{\log_2(1+\gamma_s)}{b^{-1}P_t + P_{OM}}\ (b/s/Hz/W). \tag{5}$$

The b/TENU power efficiency is defined as:

$$\eta_{TENU} = \frac{\eta_s}{\gamma_s} = \frac{\log_2(1+\gamma_s)}{\gamma_s}\ (b/TENU), \tag{6}$$

where TNEU refers to the amount of signal energy identical to the variance of the complex-valued AWGN samples recorded at the receiver.

In wireless communications, a user aims at transmitting its packets successfully under its quality-of-service (QoS) requirements and available resources constraints over a certain distance to its receivers. Hence, the radio utility metrics should include the successfully transmitted packets in bit, QoS metrics (such as bandwidth in b/s, delay and jitter in second, and packet loss rate) and the transmission distance in meter. And then a novel concept of green efficiency is obtained as [9]:

$$\eta_G = \eta_e \cdot d = \frac{d \cdot \log_2(1+\gamma_s)}{b^{-1}P_t + P_{OM}}\ ((b \cdot m)/s/Hz/W), \tag{7}$$

where both the successfully transmitted packets and the transmission distance are considered as a utility.

All the four efficiency metrics are dependent on two arguments, P_t and d.

Substituting (2) into (7), we can arrive at

$$\eta_G = \frac{d \cdot \log_2[1 + P_t \cdot g/(d^a \cdot N_o)]}{b^{-1}P_t + P_{OM}}. \tag{8}$$

After taking the partial derivatives of η_G with respect to d, we can get:

$$\frac{\partial \eta_G}{\partial d} = \frac{1}{(b^{-1}P_t + P_{OM})}\left[\log_2(1+\frac{P_t \cdot g}{d^a \cdot N_o}) - \frac{1}{\ln 2}\cdot\frac{a}{1+(d^a \cdot N_o)/(P_t \cdot g)}\right]. \tag{9}$$

Then taking the second order partial derivative of η_G with respect to d, we can arrive at

$$\frac{\partial^2 \eta_G}{\partial^2 d} = \frac{-aP_t gd^{(-a-1)}}{\ln 2 \cdot (b^{-1}P_t + P_{OM})}\left[\frac{P_t \cdot g/(N_o \cdot d^a) + 1 - a}{(P_t \cdot g/(N_o \cdot d^a) + 1)^2)}\right]. \tag{10}$$

Obviously, $\gamma_s = P_t \cdot g \big/ \left(N_0 \cdot d^a\right) > a$ in practical communication systems, so we can conclude that $\frac{\partial^2 \eta_G}{\partial^2 d} < 0$ and $\frac{\partial \eta_G}{\partial d}$ is a monotone decreasing function. Moreover, although we needn't calculate the distance making $\frac{\partial \eta_G}{\partial d} = 0$, as

$$\begin{cases} \left.\frac{\partial \eta_G}{\partial d}\right|_{d=\sqrt[a]{P_t \cdot g/(63N_o)}} > 0 \\ \left.\frac{\partial \eta_G}{\partial d}\right|_{d=\sqrt[a]{P_t \cdot g/(N_o)}} < 0 \end{cases}, \tag{11}$$

there exists only one zero point between $\sqrt[a]{P_t \cdot g/(63N_o)}$ and $\sqrt[a]{P_t \cdot g/N_o}$ for $\frac{\partial \eta_G}{\partial d}$, and the only one maximum for η_G at this point.

From the above analysis, we see that green efficiency η_G is a concave function over the transmission distance d with only one maximal value.

Using the same approach, we can evaluate η_G over P_t. After taking the partial derivatives of η_G with respect to P_t, we can get:

$$\frac{\partial \eta_G}{\partial P_t} = \frac{1}{c}\cdot\left[\frac{cP_t + A - (cP_t+1)\ln(cP_t+1)}{(cP_t + A)^2(cP_t+1)}\right], \tag{12}$$

where $c = g/(N_o \cdot d^a)$ and $A = P_{OM} \cdot g \cdot b/(d^{(a+1)} \cdot N_o) >> 1$.

Then after taking the second order partial derivative of η_G about P_t, we can arrive at:

$$\frac{\partial^2\eta_G}{\partial^2 P_t} = -\Big[(cP_t+1)\ln(cP_t+1)+3(cP_t+1) + 4(A-1)-3\ln(cP_t+1)+(A-1)^2\Big] \Big/ \Big\{c^2\big[(cP_t+A)^2(cP_t+1)\big]\big[(cP_t+A)(cP_t+1)\big]\Big\} \quad (13)$$

Moreover,

$$\begin{aligned}&(cP_t+1)\ln(cP_t+1)+3(cP_t+1)+4(A-1)-3\ln(cP_t+1)+(A-1)^2\\&>3(cP_t+1)-3\ln(cP_t+1)\\&>0\end{aligned} \quad (14)$$

Obviously, $\frac{\partial^2\eta_G}{\partial^2 P_t}<0$. So we know $\frac{\partial\eta_G}{\partial P_t}$ is a monotone decreasing function. Moreover, as

$$\lim_{P_t\to 0^+}\left(\frac{\partial\eta_G}{\partial P_t}\right)>0 \quad and \quad \lim_{P_t\to +\infty}\left(\frac{\partial\eta_G}{\partial P_t}\right)<0\,, \quad (15)$$

there is only one zero point for $\frac{\partial\eta_G}{\partial P_t}$, and only one maximum for η_G.

From the above analysis, we see that green efficiency η_G is also a concave function over the transmission power P_t with only one maximal value.

In the similar manner, we can get:

$$\frac{\partial^2\eta_G}{\partial^2 d}\cdot\frac{\partial^2\eta_G}{\partial^2 P_t}<\frac{\partial^2\eta_G}{\partial P_t\partial d}\cdot\frac{\partial^2\eta_G}{\partial d\partial P_t}\,. \quad (16)$$

Theorem: Supposed that $\boldsymbol{D}$ is a non-empty open convex set, and *f(x)* is twice differentiable in $\boldsymbol{D}$,

$$\nabla^2 f(x)<0,\ \forall x\in D \Rightarrow f(x)\ is\ a\ \ concave\ \ \ function\,,$$

where $\nabla^2 f(x)=\left[\frac{\partial^2 f}{\partial x_i\partial x_j}\right]_{n\times n}=\begin{bmatrix}\frac{\partial^2 f}{\partial x_1\partial x_1} & \cdots & \frac{\partial^2}{\partial x_1\partial x_n}\\ \vdots & \cdots & \vdots\\ \frac{\partial^2 f}{\partial x_n\partial x_1} & \cdots & \frac{\partial^2 f}{\partial x_n\partial x_n}\end{bmatrix}$.

Obviously by using the method of order principal minor determinant and the inequality (16), we can conclude $\nabla^2\eta_G=\begin{bmatrix}\frac{\partial^2\eta_G}{\partial^2 d} & \frac{\partial^2\eta_G}{\partial d\partial P_t}\\ \frac{\partial^2\eta_G}{\partial P_t\partial d} & \frac{\partial^2\eta_G}{\partial^2 P_t}\end{bmatrix}<0$, so green efficiency η_G is a two dimensional concave function.

III. GREEN EFFICIENCY-BASED NETWORK PLANNING

Based on the above analysis, we have discussed green efficiency of a PtP wireless link, and in this section we shall introduce green efficiency into a cellular network, and present a green efficiency-based network planning method, i.e., achieving the maximum green efficiency by tuning the cell size upon an assumption that the transmission power of its BS is fixed.

To provide a certain QoS for all the user equipments (UEs), especially at the cell edge, their SNRs should be greater than the threshold value $\gamma_{\min}$. So we can get the transmission power of the BS,

$$P_t=\frac{S}{PL}=\frac{\gamma_{\min}\cdot N_0}{PL}=\frac{\gamma_{\min}\cdot N_0\cdot R^a}{g}\,, \quad (17)$$

where we consider a circle area with the BS at the center and a radius of *R*.

Without loss of generality, assume that UEs are distributed in the cell randomly with the probability density function *f*(*r*, *θ*), where *r* is the distance between the BS and a given UE, and *θ* is the azimuth angle ranging from 0 to 2π. So the expectation of green efficiency can be expressed as:

$$E[\eta_G]=\int_0^{2\pi}\int_0^{R}\frac{r\cdot\log_2(1+\frac{P_t g}{r^a N_o})}{b^{-1}P_t+P_{OM}}f(r,\theta)rdrd\theta\,. \quad (18)$$

After substituting (17) into (18), and considering the normalization condition, i.e.,

$$\int_0^{2\pi}\int_0^{R}f(r,\theta)rdrd\theta=R^2\int_0^{2\pi}\int_0^{1}f(yR,\theta)ydyd\theta=1\,, \quad (19)$$

Eq. (18) could be expressed as:

$$\begin{aligned}E[\eta_G]&=\frac{R^3}{b^{-1}P_t+P_{OM}}\int_0^{2\pi}\int_0^{1}y\log_2(1+\frac{\gamma_{\min}}{y^a})f(yR,\theta)ydyd\theta\\&\le\frac{R^3}{b^{-1}P_t+P_{OM}}\int_0^{2\pi}\int_0^{1}\max_{y\in(0-1]}\left[y\log_2\left(1+\frac{\gamma_{\min}}{y^a}\right)\right]f(yR,\theta)ydyd\theta\\&\le\frac{R^3}{b^{-1}P_t+P_{OM}}\max_{y\in(0-1]}\left[y\log_2\left(1+\frac{\gamma_{\min}}{y^a}\right)\right]\int_0^{2\pi}\int_0^{1}f(yR,\theta)ydyd\theta\\&\le\frac{R\cdot\max_{y\in(0\text{-}1]}\left[y\log_2\left(1+\frac{\gamma_{\min}}{y^a}\right)\right]}{b^{-1}P_t+P_{OM}}\end{aligned}\,, \quad (20)$$

where $\max_{y\in(0-1]}[y\log_2(1+\frac{\gamma_{\min}}{y^a})]$ is independent of *y*. The upper bound almost varies in accordance with the accurate value and has a simple form.

The above equation shows that the upper bound of green efficiency is dependent on the cell radius *R*, the power transfer efficiency *b*, the transmission power P_t, and the operation & maintain power P_{OM}. As the values of *b* and P_{OM} are not directly controlled variables, the only way to achieve optimal performance is to tune the cell radius for a given transmission power.

Similarly, we can get the upper bound of spectrum efficiency, b/TENU power efficiency and energy efficiency as given in (21) , (22) and (23):

$$E[\eta_s] \le \max_{y\in(0-1]} [\log_2(1+\frac{\gamma_{\min}}{y^a})] \tag{21}$$

$$E[\eta_{TENU}] \le \max_{y\in(0-1]} \left\{ \frac{\log_2\left(1+\frac{\gamma_{\min}}{y^a}\right)}{\frac{\gamma_{\min}}{y^a}} \right\} \tag{22}$$

$$E[\eta_e] \le \frac{\max_{y\in(0-1]} [\log_2(1+\frac{\gamma_{\min}}{y^a})]}{b^{-1}P_t + P_{OM}} \tag{23}$$

IV. SIMULATION RESULTS

To evaluate the proposed green efficiency, the following simulations are carried out for a PtP wireless link. For comparison, we simulate the spectrum efficiency, b/TENU power efficiency and energy efficiency under the same scenarios. The main system parameters are given in Table I [11].

Table I Simulation parameters

P_{OM}	N_0	a	b^{-1}	g
400W	8e-14W/Hz	3.76	0.125	1e-1.53

Fig. 1 shows that spectrum efficiency is a monotone function over both arguments, i.e., transmission power and transmission distance. Meanwhile the larger the transmission power and the shorter the transmission distance, the higher spectrum efficiency is.

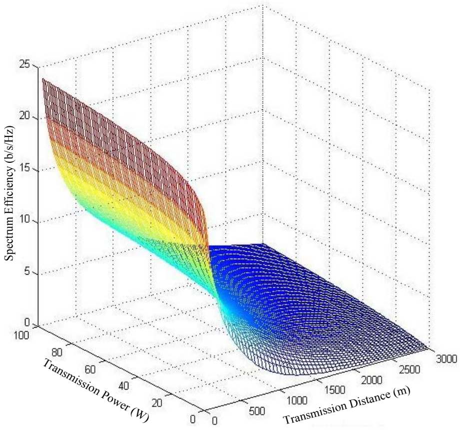

Fig. 1 Spectrum efficiency of a PtP link

Fig. 2 shows that energy efficiency is a monotone decreasing function over distance; however it is a convex function with only one peak value over the argument P_t. And the shorter distance is, the more easily we can observe the peak value.

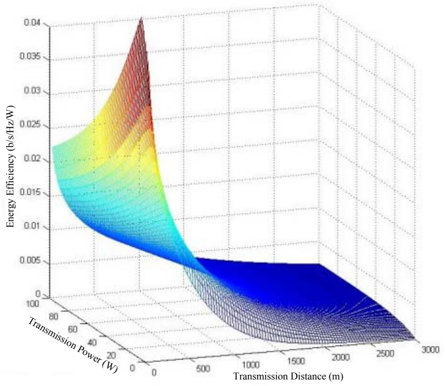

Fig. 2 Energy efficiency of a PtP link

Fig. 3 shows that b/TENU power efficiency is a monotone increasing function over distance; however it is a monotone decreasing function over the transmission power. There is no peak value in this figure.

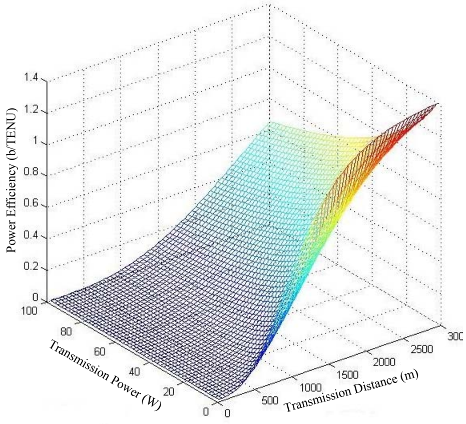

Fig. 3 b/TENU Power efficiency of a PtP link

We can observe the green function is a two dimensional convex function with only one maximum in Fig. 4.

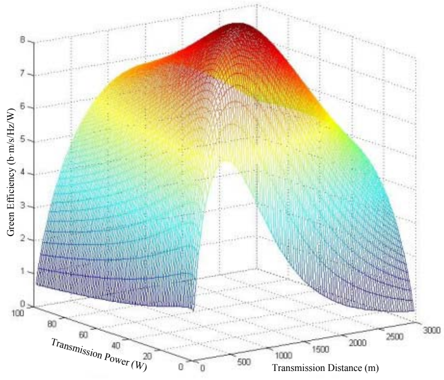

Fig. 4 Green efficiency of a PtP link

To evaluate the proposed green efficiency-based network planning, the following simulations are carried out for a cellular network.

The spectrum efficiency of a cellular cell is almost a constant value versus the radius of the cell as same as the b/TENU power efficiency, as shown in Fig. 5. And the energy efficiency is a monotone decreasing function, as shown in Fig. 6. Green efficiency of a cellular cell is still a convex function as shown in Fig. 7, so we can achieve the maximum green efficiency by tuning the cell radius. Moreover, the upper bound is a reference we can't reach.

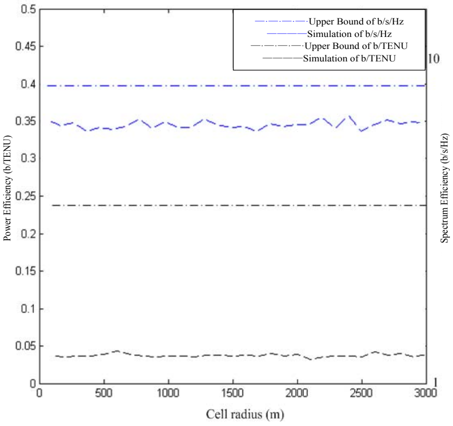

Fig. 5 Spectrum efficiency and b/TENU Power efficiency of a cellular cell

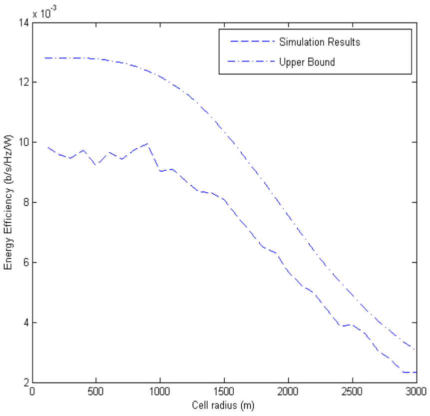

Fig. 6 Energy efficiency of a cellular cell

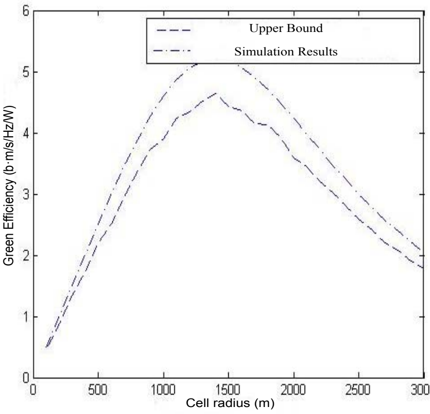

Fig. 7 Green efficiency of a cellular cell

By no means should the first three efficiency metrics be classified as less efficient, since in the appropriate circumstances they are capable of considerably improving the overall performance of the entire network. However, the appropriate choice of the network optimization criteria based on different efficiency metrics can have a profound effect on the overall network performance.

V. CONCLUSIONS

In this paper, firstly, after discussing spectrum efficiency, b/TENU power efficiency and energy efficiency, we introduce a novel efficiency metric of wireless communications, green efficiency. Secondly, we carry out a deep analysis of green efficiency, e.g., its mathematical features. Green efficiency is a two dimensional concave function over the transmission power and transmission distance. Thirdly, we introduce green efficiency from a PtP link into a cellular cell for network planning. Simulation results show that we can achieve the maximal green efficiency of a cellular cell by tuning its cell size.

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