Analysis of electromagnetic scattering by uniaxial anisotropic bispheres

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Based on the generalized multiparticle Mie theory and the Fourier transformation approach, electromagnetic (EM) scattering of two interacting homogeneous uniaxial anisotropic spheres with parallel primary optical axes is investigated. By introducing the Fourier transformation, the EM fields in the uniaxial anisotropic spheres are expanded in terms of the spherical vector wave functions. The interactive scattering coefficients and the expansion coefficients of the internal fields are derived through the continuous boundary conditions on which the interaction of the bispheres is considered. Some selected calculations on the effects of the size parameter, the uniaxial anisotropic absorbing dielectric, and the sphere separation distance are described. The backward radar cross section of two uniaxial anisotropic spheres with a complex permittivity tensor changing with the sphere separation distance is numerically studied. The authors are hopeful that the work in this paper will help provide an effective calibration for further research on the scattering characteristic of an aggregate of anisotropic spheres or other shaped anisotropic particles. © 2011 Optical Society of America

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1. INTRODUCTION

Because of recent advances in material science and technology and increasing applications of anisotropic materials for microstrip circuits, microwave engineering, and enhancement and reduction of radar cross section (RCS) of various scatterers, and so on, interest in the interaction between electromagnetic (EM) wave and anisotropic media is growing. Many scholars have investigated the scattering of a single anisotropic object. Papadakis et al. [1] researched the scattering of a general anisotropic dielectric ellipsoid from a plane wave. Using the Fourier transformation method, Geng et al. [2,3] studied the scattering of a uniaxial and plasma anisotropic sphere from a plane wave. Wu et al. [4,5] extended the scattering to and from a Gaussian beam. Moreover, using the differential theory, Stout et al. [6,7] gave the solution of the scattering of an arbitrary-shaped body made of arbitrary anisotropic medium, but did not give any numerical results. Using the expansion in terms of the scalar eigenfunctions method, Wong and Chen [8] and Qiu et al. [9] also devoted their endeavors to the scattering of a uniaxial sphere. Some numerical methods are also used to study the scattering problem, such as the finite-difference time domain method [10–12], the moment method [13], and the coupled dipole approximation [14]. However, the scatterer in all these studies is limited to a single anisotropic object. The published works on the scattering from multiple anisotropic objects are exiguous.

It is well known that the multiparticle scattering theory has developed very quickly in past decades. Brunding and Lo [15,16] made the first significant contribution to multiparticle scattering by giving a comprehensive solution for a two-sphere chain. Then, Fuller and Kattawar [17,18] introduced the order-of-scattering technique to obtain the consummate solution of EM scattering by a cluster and ensemble spheres. Subsequently, two approaches were developed to study the multiparticle scattering based on the Mie theory [19]: the TM matrix approach developed by many contributors [20–24] and the generalized multiparticle Mie (GMM) theory developed by Xu [25–29] during 1995 to 2003 in order to solve the scattering problem of multiple spheres or particles that have different shapes, such as sphere, spheroid, or cylinder. It is not the intent of this paper to include a general analysis of the difference between the two approaches (refer to the review by Xu [29]). In this paper, the GMM theory as an analytical method will be applied to study the scattering of the uniaxial anisotropic bispheres.

For most papers to which we have referred, the investigations focus on the scattering by multiple isotropic spheres. However, the interaction of an EM wave with multiple anisotropic spheres is still a new problem, but one of considerable merit. Because of the increasing applications of anisotropic medium, it is urgent to exploit the multiple anisotropic spheres scattering problem. An arbitrary anisotropic dielectric can be transformed into a uniaxial anisotropic dielectric by means of coordinate rotation.

In this paper, a rigorous analytical solution on the scattering of uniaxial anisotropic bispheres with parallel primary optical axes based on the Fourier transformation method and the GMM theory are described. In order to verify the proposed solution, we initially make a comparison between the results computed by our codes and those simulated by the Computer Simulation Technology (CST) Microwave Studio. It is found that the values are in good agreement. CST Microwave Studio is a three-dimensional EM simulation software based on the numerical method of finite integration and developed by the German company CST in 1992, and our laboratory is the Sino–Germany Joint CST training center in northwestern...
China. The theoretical and numerical development aim to extend the GMM solution for isotropic spheres to that for anisotropic spheres and provide useful insights into fields such as particle sizing, biomedicine, and Raman scattering diagnosis. In the subsequent depiction, a time dependence of the form $\exp(-\omega t)$ is assumed and suppressed, where $\omega$ is the circular frequency.

2. THEORETICAL FORMULATIONS

A. Expansion of Incident Fields and Scattered Fields

Considering two uniaxial anisotropic spheres with radius $a_j (j = 1, 2)$ and the primary optical axes coincident with the $z$ axis in a global coordinate system $Oxyz$, as shown in Fig. 1, the centers of the spheres are located at $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$, respectively. The particles are illuminated by a $z$-propagating and $x$-polarized plane wave, written in spherical polar coordinates as

$$E^j = E_0 e^{i k_0 \cdot r} \hat{e}_x,$$

where $E_0$ is the amplitude, $k_0$ is the wave vector of the incident wave, $r$ is the position vector, and $\hat{e}_x$ is one of the unit vectors of the Cartesian coordinate system.

In terms of spherical vector wave functions (SVWFs), the incident fields can be expanded in the global coordinate system $Oxyz$ as [19,25,30]

$$E^j = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} E_{mn} a_{mn}^{j}(r, k_0) + b_{mn}^{j} N_{mn}^{(1)}(r, k_0),$$

$$H^j = \frac{k_0}{i \omega \mu_0} \sum_{n=1}^{\infty} \sum_{m=-n}^{n} E_{mn} a_{mn}^{j} N_{mn}^{(1)}(r, k_0) + b_{mn}^{j} M_{mn}^{(1)}(r, k_0),$$

where $k_0 = 2\pi/\lambda$, $\lambda$ is a free-space wavelength, $\mu_0$ is the permeability of the surrounding medium, and the normalization factor $E_{mn}$ is defined as

$$E_{mn} = E_0 \sqrt{\frac{(2n+1)(n-m)!}{n(n+1)(n+m)!}}.$$  

According to the orthogonal relations of SVWFs, the expansion coefficients $a_{mn}^j$ and $b_{mn}^j$ can be derived [30]

![Fig. 1. (Color online) Geometry for light scattering of a plane wave by uniaxial anisotropic bispheres.](image)

$$a_{mn}^j = i \frac{\sqrt{2n+1}}{2} (\delta_{m1} + \delta_{m,-1}),$$

$$b_{mn}^j = i \frac{\sqrt{2n+1}}{2} (\delta_{m1} - \delta_{m,-1}),$$

where $\delta_{m1}$ and $\delta_{m,-1}$ are the Kronecker delta.

Build rectangular coordinate systems $O_1 x_1 y_1 z_1$ and $O_2 x_2 y_2 z_2$ parallel to the global coordinate system $Oxyz$ with sphere centers $O_1$ and $O_2$, respectively. Similarly, the incident fields can be expanded in terms of SVWFs in the $j$th sphere coordinate system $O_j x_j y_j z_j$. The expansion coefficients $a_{mn}^j$ and $b_{mn}^j$ have the following relations with the coefficients $a_{mn}^i$ and $b_{mn}^i$:

$$a_{mn}^j = e^{i k_j r_j} a_{mn}^i, \quad b_{mn}^j = e^{i k_j r_j} b_{mn}^i,$$

where $j = 1, 2$ denotes the correlative parameter with the $j$th sphere and $r_j$ originates at the center of the $j$th sphere.

The scattered fields of the $j$th ($j = 1, 2$) uniaxial anisotropic sphere can also be expanded in terms of SVWFs in the $j$th sphere coordinate system $O_j x_j y_j z_j$

$$E_j^s = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} E_{mn} [a_{mn}^{j} M_{mn}^{(3)}(r_j, k_0) + b_{mn}^{j} N_{mn}^{(3)}(r_j, k_0)],$$

$$H_j^s = \frac{k_0}{i \omega \mu_0} \sum_{n=1}^{\infty} \sum_{m=-n}^{n} E_{mn} [a_{mn}^{j} N_{mn}^{(3)}(r_j, k_0) + b_{mn}^{j} M_{mn}^{(3)}(r_j, k_0)].$$

B. Expansion of Internal Fields

The uniaxial anisotropic medium is characterized by a permittivity and permeability with tensor $\varepsilon_j$ and $\mu_j (j = 1, 2)$, respectively, expressed as

$$\varepsilon_j = \varepsilon_0 \begin{bmatrix} \varepsilon_{ij} & 0 & 0 \\ 0 & \varepsilon_{ij} & 0 \\ 0 & 0 & \varepsilon_{jj} \end{bmatrix}, \quad \mu_j = \mu_0 \begin{bmatrix} \mu_{ij} & 0 & 0 \\ 0 & \mu_{ij} & 0 \\ 0 & 0 & \mu_{jj} \end{bmatrix}. $$

The $E$ field vector wave equation in a uniaxial anisotropic sphere may be written as

$$\nabla \times (\mu^{-1} \cdot \nabla \times E) - \alpha^2 \varepsilon \cdot E = 0.$$  

Then, the internal fields of the uniaxial anisotropic medium cannot be expanded in terms of SVWFs simply. The Fourier transformation method, due to its briefness for a uniaxial anisotropic medium, is introduced to derive the internal fields. The detailed process can be found in [2,3]. Here, we only give the final results on the internal fields of the $j$th ($j = 1, 2$) uniaxial anisotropic sphere,
\[ E_j^I (r_j) = \sum_{q=1}^{n} \sum_{n=-m}^{m} \sum_{l=-n}^{n} 2\pi G_{jmnq} \int_0^{\pi} [A_{jmnq}^{(1)} M_{jmn}^{(1)} (r_j, k_{jq}) + B_{jmnq}^{(1)} N_{jmn}^{(1)} (r_j, k_{jq})] \theta \sin \theta d\theta , \]

\[ H_j^I (r_j) = \sum_{q=1}^{n} \sum_{n=-m}^{m} \sum_{l=-n}^{n} 2\pi G_{jmnq} \int_0^{\pi} [A_{jmnq}^{(1)} M_{jmn}^{(1)} (r_j, k_{jq}) + B_{jmnq}^{(1)} N_{jmn}^{(1)} (r_j, k_{jq})] \sin \theta \cos \theta d\theta , \]

where \( A_{jmnq}^{(1)}, B_{jmnq}^{(1)}, C_{jmn}^{(1)} = 1 \) \( M_{jmn}^{(1)} \) and \( N_{jmn}^{(1)} \) are the expansion coefficients, and their expressions can be found in [3], but the expansions should be coincident with the corresponding permittivity tensor \( \varepsilon \) and permeability \( \mu \) for each uniaxial anisotropic sphere. \( G_{jmnq} \) is the unknown expansion coefficient relative to the internal fields and is determined by the boundary conditions of the \( j \)-th sphere.

### 3. Interactive Scattering Coefficients

On the spherical boundary at \( r_j = a_j \), the tangential components (designated by the subscript \( t \)) of the EM fields continue as

\[ E_j^It = E_j^It + E_j^It, \quad H_j^It = H_j^It + H_j^It \quad (r_j = a_j), \]

where \( E_j^It \) and \( H_j^It \) represent the total EM fields incident on the \( j \)-th \((j = 1, 2)\) anisotropic sphere. They generally consist of the initial incident fields and scattered fields from the other sphere,

\[ E_j^It = E_j^I + E_j^It, \quad H_j^It = H_j^I + H_j^It, \]

where \( I \) is an integer, \( j = 1, 2 \), \( l \neq j \), and \( E_j^I \) and \( H_j^I \) are the initial EM fields. \( E_j^It \) and \( H_j^It \) are the EM fields from the other sphere.

Applying the addition theorem of SVWFs [15,22,26] and Eqs. (2), (6), and (11), the total EM fields can be derived:

\[ E_j^I = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} E_{mn}^{(1)} f_{jmn}^{(1)} M_{jmn}^{(1)} (r_j, k_0) + g_{jmn}^{(1)} N_{jmn}^{(1)} (r_j, k_0), \]

\[ H_j^I = \frac{k_0}{io\mu_0} \sum_{n=1}^{\infty} \sum_{m=-n}^{n} E_{mn}^{(1)} f_{jmn}^{(1)} M_{jmn}^{(1)} (r_j, k_0) + g_{jmn}^{(1)} N_{jmn}^{(1)} (r_j, k_0). \]

The expansion coefficients are

\[ f_{jmn}^{(1)} = a_{jmn}^{(1)} + \sum_{(j_1, j_2, \ldots, j_l, k_0) = 1}^{n} \sum_{m=-n}^{n} a_{jmn}^{(1)} A_{jmn}^{(1)} (l, j) + b_{jmn}^{(1)} B_{jmn}^{(1)} (l, j) \quad (l \neq j), \]

\[ g_{jmn}^{(1)} = b_{jmn}^{(1)} + \sum_{(j_1, j_2, \ldots, j_l, k_0) = 1}^{n} \sum_{m=-n}^{n} a_{jmn}^{(1)} B_{jmn}^{(1)} (l, j) + b_{jmn}^{(1)} A_{jmn}^{(1)} (l, j) \quad (l \neq j), \]

where \( A_{jmn}^{(1)} \) and \( B_{jmn}^{(1)} \) are the so-called addition theorem coefficients [15,22,26].

Substituting Eqs. (6), (9), and (13) into Eq. (11) and utilizing the orthogonality of the associated Legendre function, we gain

\[ f_{jmn}^{(1)} (k_0 r_j) + g_{jmn}^{(1)} (k_0 r_j) = \frac{1}{2} \sum_{q=1}^{n} \sum_{n=-n}^{n} 2\pi G_{jmnq} \int_0^{\pi} [A_{jmnq}^{(1)} M_{jmnq}^{(1)} (r_j, k_{jq}) + B_{jmnq}^{(1)} N_{jmnq}^{(1)} (r_j, k_{jq})] \theta \sin \theta d\theta , \]

\[ g_{jmn}^{(1)} (k_0 r_j) + k_0 \frac{\partial}{\partial k_0} g_{jmn}^{(1)} (k_0 r_j) = \frac{1}{2} \sum_{q=1}^{n} \sum_{n=-n}^{n} 2\pi G_{jmnq} \int_0^{\pi} [A_{jmnq}^{(1)} M_{jmnq}^{(1)} (r_j, k_{jq}) + B_{jmnq}^{(1)} N_{jmnq}^{(1)} (r_j, k_{jq})] \sin \theta \cos \theta d\theta , \]

\[ \frac{k_0}{io\mu_0} f_{jmn}^{(1)} (k_0 r_j) + \frac{k_0}{io\mu_0} h_{jmn}^{(1)} (k_0 r_j) = \frac{1}{2} \sum_{q=1}^{n} \sum_{n=-n}^{n} 2\pi G_{jmnq} \int_0^{\pi} [A_{jmnq}^{(1)} M_{jmnq}^{(1)} (r_j, k_{jq}) + B_{jmnq}^{(1)} N_{jmnq}^{(1)} (r_j, k_{jq})] \sin \theta \cos \theta d\theta . \]

Combining the Eqs. (14)–(17) and eliminating \( a_{jmn}^{(1)} \) and \( b_{jmn}^{(1)} \), we have

\[ f_{jmn}^{(1)} \left[ j_n (k_0 r_j) \frac{d (r_j h_{jmn}^{(1)} (k_0 r_j))}{dr_j} - h_{jmn}^{(1)} (k_0 r_j) \frac{d (r_j j_n (k_0 r_j))}{dr_j} \right] = \frac{1}{2} \sum_{q=1}^{n} \sum_{n=-n}^{n} 2\pi G_{jmnq} \int_0^{\pi} V_{jmnq}^{(1)} (\cos \theta) k_{jq}^2 \sin \theta d\theta . \]

where
\[ V_{jmnq} = \left\{ A_{jmnq}^{\varepsilon} \frac{1}{h_{b}^{(1)}(k_{0}a_{j})} \int_{0}^{\infty} \frac{1}{r} d(rj_{b}^{(1)}(kr_{j})) \right\} + \left\{ B_{jmnq}^{\varepsilon} \frac{1}{h_{b}^{(1)}(k_{0}a_{j})} \int_{0}^{\infty} \frac{1}{r} d(rj_{b}^{(1)}(kr_{j})) \right\} + C_{jmnq}^{\varepsilon} \left\{ \frac{j_{b}^{(1)}(kr_{j})}{r_{j}} \right\} h_{b}^{(1)}(kr_{j}) \right\}. \]  

From Eqs. (18) and (19), the expansion coefficient \( G_{jmnq}^{\varepsilon} \) can be analytically derived. Then, substituting \( G_{jmnq}^{\varepsilon} \) back into Eqs. (14) and (17), we can obtain the following expressions of the interactive scattering coefficients \( a_{jmn}^{\varepsilon} \) and \( b_{jmn}^{\varepsilon} \):

\[ a_{jmn}^{\varepsilon} = \frac{1}{h_{b}^{(1)}(k_{0}a_{j})} \left\{ \frac{1}{E_{mn}} \sum_{q=1}^{\infty} \sum_{w=1}^{\infty} 2\pi G_{jmnq}^{\varepsilon} \right\} \left\{ \int_{0}^{\infty} A_{jmnq}^{\varepsilon} j_{b}^{(1)}(kr_{j}) \right\} \sin \theta_{jk} d\theta_{jk} \right\}. \]  

\[ b_{jmn}^{\varepsilon} = \frac{1}{h_{b}^{(1)}(k_{0}a_{j})} \left\{ \frac{1}{E_{mn}} \sum_{q=1}^{\infty} \sum_{w=1}^{\infty} 2\pi G_{jmnq}^{\varepsilon} \right\} \left\{ \int_{0}^{\infty} A_{jmnq}^{\varepsilon} j_{b}^{(1)}(kr_{j}) \right\} \sin \theta_{jk} d\theta_{jk} \right\}. \]  

It is worth noticing that the expressions of the interactive scattering coefficients in Eqs. (22) and (23) are more complicated than those of scattering by an aggregate of isotropic spheres given by Xu [23], since the internal fields of a uniaxial anisotropic sphere are completely different from those of an isotropic sphere.

4. TOTAL SCATTERED FIELDS

In order to study the scattering characteristic of multiple uniaxial anisotropic spheres, we need to know the total scattered fields that can be obtained by the superposition of individual scattered fields of each sphere,

\[ E^{st} = E_{1}^{*} + E_{2}^{*}, \quad H^{st} = H_{1}^{*} + H_{2}^{*}. \]  

Applying the addition theorem of SVWFs denoted by the first kind of the spherical Hankel function \( h_{b}^{(1)}(kr_{j}) \), but not the spherical Bessel function \( j_{b}^{(1)} \), the total scattered fields in the global coordinate system \( Oxyz \) can be derived:

\[ E^{st} = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} E_{mn}[a_{jmn}^{st} M_{mn}^{(3)}(r_{0}, k_{0}) + b_{jmn}^{st} M_{mn}^{(3)}(r_{0}, k_{0})], \]

\[ H^{st} = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} E_{mn}[a_{jmn}^{st} N_{mn}^{(3)}(r_{0}, k_{0}) + b_{jmn}^{st} N_{mn}^{(3)}(r_{0}, k_{0})]. \]  

where \( a_{jmn}^{st} \) and \( b_{jmn}^{st} \) are the total scattering coefficients. Utilizing the approximate forms of SVWFs when \( r \to \infty \), the total scattering coefficients can be written simply as [27]

\[ a_{jmn}^{st} = \sum_{j=1}^{3} a_{jmn}^{*} \exp(-ik_{0}\Delta_{j}), \quad b_{jmn}^{st} = \sum_{j=1}^{3} b_{jmn}^{*} \exp(-ik_{0}\Delta_{j}). \]  

where \( \Delta_{j} = x_{j} \sin \theta \cos \phi + y_{j} \sin \theta \sin \phi + z_{j} \cos \theta \).

With these solved coefficients, the field components of the total scattered, transmitted, and incident fields can be obtained by corresponding substitutions. Because of great use in radar detection and military purposes, we can calculate the RCS:

\[ \sigma = \lim_{r \to \infty} (4\pi r^{2})|E^{st}|^{2}/|E^{i}|^{2}. \]  

where \( E^{i} \) indicates the initial incident electric field.

5. NUMERICAL RESULTS AND DISCUSSION

A. Validation of Theory and Codes

To validate the theory, some results are selected to compare with numerical results provided by the CST Microwave Studio software. As shown in Fig. 2, the angular distribution of the RCS of two uniaxial anisotropic spheres along the \( z \) axis with both electric and magnetic anisotropy are in good agreement with those generated in the CST Microwave Studio simulation, which can confirm the accuracy of our theory. Note that \( E \) plane and \( H \) plane correspond to the \( xoz \) plane and the \( yoz \) plane, respectively. The remaining figures also have similar expressions.
B. Effects of Size Parameter

Figures 3 and 4 show the angular distributions of the RCS of two close-packed uniaxial anisotropic spheres along the $z$ and $x$ axes with different size parameters, respectively. The size parameter ratio $Ar$ is defined as $\frac{x_2}{x_1} = \frac{k_2a_2}{k_1a_1}$. It can be observed that the larger the Ar, the more visible the interaction of the uniaxial anisotropic bispheres; then, the sharper the oscillation of the RCS. It should be pointed out here that the size parameter of the second sphere is less than that of the first one. While $Ar$ is more than 1, it will produce the same results if defining $Ar = \frac{x_1}{x_2}$. The angular distributions are all symmetric about scattering angle $180^\circ$ except that in the $E$ plane while the bispheres are arrayed along the $x$ axis due to the destructive symmetrical configuration on this case.

C. Effects of Anisotropic Absorbing Dielectric

The effects of the complex electric anisotropic medium on RCS for uniaxial anisotropic bispheres along the $z$ and $x$ axes are shown in Figs. 5 and 6, respectively. Case 1 indicates that both of the two spheres are lossless; case 2 indicates that the first sphere is lossless, but the second sphere is lossy; case 3 indicates that the first sphere is lossy, but the second sphere is lossless; and case 4 indicates that both of the two spheres are lossy. The corresponding permittivity and permeability tensor are $\varepsilon_{1t} = \varepsilon_{2t} = 2$, $\varepsilon_{1z} = \varepsilon_{2z} = 2.4$ for case 1; $\varepsilon_{1t} = 2$, $\varepsilon_{1z} = 2.4$, $\varepsilon_{2t} = 2 + i$, $\varepsilon_{2z} = 2.4 + 0.5i$ for case 2; $\varepsilon_{1t} = 2 + i$, $\varepsilon_{1z} = 2.4 + 0.5i$, $\varepsilon_{2t} = 2$, $\varepsilon_{2z} = 2.4$ for case 3; $\varepsilon_{1t} = \varepsilon_{2t} = 2 + i$, $\varepsilon_{1z} = \varepsilon_{2z} = 2.4 + 0.5i$ for case 4; and $\mu_{1t} = \mu_{2t} = \mu_{1z} = \mu_{2z} = 1.0$ for all cases. It can be observed that the RCS decreases if one of the anisotropic spheres is lossless, and the phenomenon will
be more visible if both of the two anisotropic spheres are lossless. The angular distributions of the RCS are distinct from that of one uniaxial anisotropic lossy sphere described by Wu et al. [4], where the RCS rapidly decays to a constant for a large-sized uniaxial anisotropic absorbing sphere with the increase of the scattering angle. In Figs. 5 and 6, the RCS still has high oscillation with the increase of the scattering angle as a result of the interaction of the bispheres.

By comparing Fig. 6(a) with Fig. 4(a), it can be found that the symmetric configuration of the bispheres for some plane is determined not only by the size parameter but also by the permittivity and permeability tensor. The RCSs of the bispheres along the $x$ axis for cases 2 and 3 in the $H$ plane are absolutely the same because the configurations of these cases for the $H$ plane are the same.

After careful examination and simulation, it is found that the RCS of uniaxial anisotropic bispheres along the $y$ axis exhibits a similar scattering characteristic with those of uniaxial anisotropic bispheres along the $x$ axis. Hence, the figures of normalized RCS results for uniaxial anisotropic bispheres along the $y$ axis are not given in detail due to length restrictions.

D. Effects of Sphere Separation Distance

TiO$_2$, characterized by $\varepsilon_r = 5.913$ and $\varepsilon_z = 7.197$, is a typical uniaxial anisotropic medium. In Fig. 7, the RCSs of two TiO$_2$ spheres placed along the $z$ axis with different sphere separation distances $k_0d = 2\pi$, $4\pi$, $6\pi$, and $10\pi$ are given. Two interesting phenomena can be found: the angular distribution of the RCS of bispheres scattering oscillates sharper than that of a single sphere scattering and the angular distribution of the RCS of bispheres with larger sphere separation distance oscillates sharper than that of bispheres with the smaller sphere separation distance. They can be explained as follows: both the interacting scattering and the interference scattering are considered, which can enhance the total amplitude and the complexity of the scattering characteristic. With the increase of the sphere separation distance, the interacting effect of the anisotropic bispheres will be decreased while the

![Fig. 6](image1.png)  
**Fig. 6.** (Color online) Effects of complex uniaxial electric anisotropy on RCS for uniaxial anisotropic bispheres along the $x$ axis.

![Fig. 7](image2.png)  
**Fig. 7.** (Color online) Normalized RCS values versus the scattering angle for TiO$_2$ bispheres along the $z$ axis with different sphere separation distances.

![Fig. 8](image3.png)  
**Fig. 8.** (Color online) Backward RCS values versus the sphere separation distance for isotropic bispheres.
interference effect will be more visible; thus, the angular distribution of the RCS will oscillate sharper and sharper until it arrives at the resonance zoom.

To verify the accuracy of our theory further, we make another comparison. As Fig. 8 shows, the backward RCS changing with the sphere separation distance obtained from our codes reduced to isotropic bispheres are coincident with those generated in [16].

The backward RCS changing with the sphere separation distance for uniaxial anisotropic bispheres along the x, y, or z axes are shown in Fig. 9. Figure 9(a) indicates that both of the two spheres are lossless; Fig. 9(b) indicates that the first sphere is lossless, but the second sphere is lossy; Fig. 9(c) indicates that the first sphere is lossy, but the second sphere is lossless; and Fig. 9(d) indicates that both of the two spheres are lossy. It can be found that the backward RCS for uniaxial anisotropic bispheres along the z axis oscillates much higher than that for uniaxial anisotropic bispheres along the x or y axes due to the stronger interaction of uniaxial anisotropic bispheres along the z axis when the incident wave propagates along the z axis. It can be noticed that the limits of oscillations of the backward RCS for lossy uniaxial anisotropic bispheres are smaller than that for lossless uniaxial anisotropic bispheres, but the periods are the same as a result of the same real of the permittivity tensor. By comparing the curves denoted by “z-axis” in Fig. 9 and the curve in Fig. 8, it can be found that backward RCS at the wave hollow decreases with the increase of the sphere separation distance until the value is invariable. The process is much quicker if one of the uniaxial anisotropic bispheres is lossy, and the backward RCS have a minimum at a certain sphere separation distance while both of the uniaxial anisotropic bispheres are lossy.

In the above study, only the electric anisotropic bispheres are considered. In fact, the magnetic anisotropic bispheres exhibit the similar scattering characteristics after careful calculations. Hence, the figures on the RCS values for magnetic anisotropic bispheres are not given in detail due to length restrictions.

6. CONCLUSION

In summary, the analytic solution of the EM scattering of two interacting homogeneous uniaxial anisotropic spheres with parallel primary optical axes is studied. By introducing the Fourier transformation, the internal fields of each uniaxial anisotropic sphere are expanded in terms of SVWFs. Matching the continuous boundary conditions at each surface among the bispheres and considering their interaction by transforming the scattered fields of one sphere into the incident fields of the other sphere, the interactive scattering coefficients and the expansion coefficients of the internal fields are derived. The influences of the size parameter, the uniaxial anisotropic absorbing dielectric, and the sphere separation distance on the scattering properties are numerically analyzed. The symmetrical configuration for some plane of uniaxial anisotropic bispheres is determined not only by the size parameter, but
also by the permittivity and permeability tensor. Because of the interaction of uniaxial anisotropic bispheres, the angular distributions of the RCS of uniaxial anisotropic absorbing bispheres are distinct from that of a uniaxial anisotropic absorbing sphere. The performances on the backward RCS values of uniaxial anisotropic bispheres with a complex permittivity tensor versus the sphere separation distance are considered. In this paper, only the TM polarization wave is considered, while the scattering characteristic of uniaxial anisotropic bispheres for TE polarization wave can be analyzed similarly. The theoretical method and numerical results presented in this paper aim to extend GMM solution for isotropic spheres to that for anisotropic spheres and provide an effective calibration for further research on the scattering of an aggregate of uniaxial anisotropic spheres or arbitrary anisotropic spheres. Scattering of multiple uniaxial anisotropic spheres with unparallel primary optical axes will be discussed in our future research.

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