# Optimized Iterative Clipping and Filtering for PAPR Reduction of OFDM Signals

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**Abstract**

Iterative clipping and filtering (ICF) is a widely used technique to reduce the peak-to-average power ratio (PAPR) of orthogonal frequency division multiplexing (OFDM) signals. However, the ICF technique, when implemented with a fixed rectangular window in the frequency-domain, requires many iterations to approach specified PAPR threshold in the complementary cumulative distribution function (CCDF). In this paper, we develop an optimized ICF method which determines an optimal frequency response filter for each ICF iteration using convex optimization techniques. The design of optimal filter is to minimize signal distortion such that the OFDM symbol's PAPR is below a specified value. Simulation results show that our proposed method can achieve a sharp drop of CCDF curve and reduce PAPR to an acceptable level after only 1 or 2 iterations, whereas the classical ICF method would require 8 to 16 iterations to achieve a similar PAPR reduction. Moreover, the clipped OFDM symbols obtained by our optimized ICF method have less distortion and lower out-of-band radiation than the existing method.

**Index Terms**

orthogonal frequency division multiplexing, peak-to-average power ratio, iterative clipping and filtering, convex optimization.

## I. Introduction

ORTHOGONAL frequency division multiplexing (OFDM) is widely used in modern wireless communication systems because of its high spectrum efficiency and low susceptibility to multi-path effects [1]. However, one major drawback of OFDM is its high peak-to-average power ratio (PAPR), which makes it sensitive to nonlinear effects of power amplifiers.

Many techniques have been proposed to reduce the PAPR of OFDM symbols [2]. Among all these existing techniques, the iterative clipping and filtering (ICF) procedure maybe the simplest to approach a specified PAPR threshold in the processed OFDM symbols [3], [4]. However, clipping time-domain signals causes out-of-band spectral regrowth and in-band distortion. The latter can also degrade bit error performance of the OFDM system. In addition, frequency-domain filtering can reduce the spectral regrowth, but may still generate large time-domain peaks. For this reason, the ICF technique requires many iterations to approach a desired PAPR reduction.

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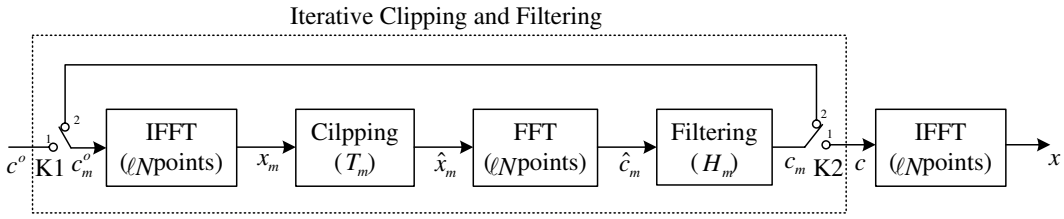


Fig. 1. Cliping and filtering scheme, where $c^o \in \mathbb{C}^N$, $c_m^o \in \mathbb{C}^{\ell N}$, $x_m \in \mathbb{C}^{\ell N}$, $\hat{x}_m \in \mathbb{C}^{\ell N}$, $\hat{c}_m \in \mathbb{C}^{\ell N}$, $c_m \in \mathbb{C}^{\ell N}$, and $c \in \mathbb{C}^N$. $m = 1, ..., M$ denotes the iteration number and $M$ is a preset maximum number of iterations to perform.

In recent years, convex optimization has been widely used in communication signal processing applications, including PAPR reduction of OFDM signals [5]–[10]. The greatest advantage of convex optimization techniques is that a global optimal solution can be guaranteed and computed efficiently, i.e., in polynomial time. In this paper, we propose a new ICF procedure which modifies the classic ICF by replacing the rectangular-window filter with the one designed by convex optimization techniques. When compared to existing ICF techniques, the new optimized ICF procedure dramatically decreases the number of required iterations to reach a given PAPR level while simultaneously achieving a sharp drop of CCDF, a smaller symbol distortion and a lower out-of-band radiation. Simulation results demonstrate the effectiveness of our proposed approach.

## II. The Classic ICF Method

### A. OFDM Symbol

Let $c \in \mathbb{C}^N$ be the frequency-domain OFDM symbol and $\{c(i), i = 1, ..., N\}$ be the symbol value carried by the $i$-th sub-carrier. Then, the time-domain OFDM symbol, $x \in \mathbb{C}^{\ell N}$, corresponding to $c$ with $\ell$ times over-sampling is expressed as

$$x(k) = \frac{1}{\sqrt{\ell N}} \sum_{i=1}^{N} c(i) e^{j\frac{2\pi}{\ell N}ki}, \tag{1}$$

where $k = 1, ..., \ell N$ is time index. The frequency-domain OFDM symbol $c$ is computed using

$$c(i) = \frac{1}{\sqrt{\ell N}} \sum_{k=1}^{\ell N} x(k) e^{-j\frac{2\pi}{\ell N}ki}, \tag{2}$$

where $i = 1, ..., N$. In practice, OFDM modulation and demodulation can be implemented via IFFT and FFT, respectively.

### B. OFDM System Parameters

Let us define two parameters related to OFDM constellation reshaping: PAPR and error vector magnitude (EVM). For convenience, we will use $c^o$ and $x^o$ to represent the original (undistorted) frequency-domain and time-domain OFDM symbols, respectively, and $c$ and $x$ for the processed frequency-domain and time-domain OFDM symbols.

*1) PAPR:* The PAPR of the original time-domain OFDM symbol, $x^o$, can be defined as

$$\text{PAPR} = \frac{\max\limits_{k=1,...,\ell N} |x^o(k)|^2}{\frac{1}{\ell N}\sum\limits_{k=1}^{\ell N} |x^o(k)|^2} = \frac{\|x^o\|_\infty^2}{P_{x^o}} = \frac{\|x^o\|_\infty^2}{\frac{1}{\ell N}\|x^o\|_2^2}, \tag{3}$$

where $\|\cdot\|_2$ is the 2-norm and $\|\cdot\|_\infty$ stands for the $\infty$-norm. Making appropriate substitutions, (3) also defines the PAPR of the processed time-domain OFDM symbol $x$.

*2) EVM and RMS-EVM:* EVM is used to describe the distortion of the processed OFDM symbol. A single OFDM symbol's EVM is defined as

$$\mathrm{EVM} = \sqrt{\frac{\sum_{i=1}^{N} |c^o(i) - c(i)|^2}{\sum_{i=1}^{N} |c^o(i)|^2}} = \frac{\|c^o - c\|_2}{\|c^o\|_2}. \tag{4}$$

Taking the root mean square of $K$ single OFDM symbols' EVM, we obtain RMS-EVM (or long-term EVM, denoted by $\Lambda$) as follows:

$$\Lambda = \sqrt{\frac{1}{K}\sum_{i=1}^{K} \mathrm{EVM}_i{}^2}. \tag{5}$$

Clearly, a larger RMS-EVM value corresponds to larger distortion of the OFDM signal and reduced bit error rate (BER) performance.

### *C. The Classic ICF Technique*

Figure 1 shows the basic block diagram of the classic ICF PAPR reduction scheme as described in [3]. It modifies each OFDM symbol one at a time. In the first iteration ($m = 1$), switch K1 is set to 1 and the new OFDM symbol enters the ICF block. Then, both K1 and K2 are set to 2 and clipping and filtering is iteratively performed. In the $M$-th (final) iteration, the switches are returned to position 1 and thus the output $c$ is produced. More specifically, the clipping procedure is performed by

$$\hat{x}_m(k) = \begin{cases} T_m e^{j\theta_m(k)}, & |x_m(k)| > T_m \\ x_m(k), & |x_m(k)| \le T_m \end{cases}, \tag{6}$$

where $1 \le k \le \ell N$, $\theta_m(k)$ represents the phase of $x_m(k)$, and $T_m$ is the clipping level in the $m$-th iteration. The clipping level is recalculated in each iteration according to a constant value called the clipping ratio (CR), which is related to the desired PAPR (denoted $\mathrm{PAPR}_{\max}$) as follows:

$$\mathrm{CR} = \sqrt{\mathrm{PAPR}_{\max}} = \frac{T_m}{\frac{1}{\sqrt{\ell N}}\|x_m\|_2}. \tag{7}$$

In the classic ICF method, the filtering step is based on a rectangular window with frequency response defined by

$$H_m(i) = \begin{cases} 1, & 1 \le i \le N \\ 0, & N+1 \le i \le \ell N \end{cases}. \tag{8}$$

The above filtering step simply removes the out-of-band spectral regrowth without considering the effect on the time domain peak after the IFFT operation. As a result, it tends to cause sizable time-domain peaks, requiring clipping and filtering to be repeated many iterations before achieving the desired PAPR.

## III. An Optimized ICF Method

### A. Optimal Filter for Each Iteration in ICF

For each filtering iteration of ICF, we propose to choose a filter that minimizes the current OFDM symbol's EVM subject to the desired $\mathrm{PAPR}_{\max}$. This designed filter will replace the rectangular filter in the classic ICF method. Specifically, let $\hat{c}_m$ denote the frequency domain OFDM symbol at the $m$-th iteration (See figure 1). Then the filter design problem can be formulated as follows:

$$\min_{H_m \in \mathbb{C}^N} \quad \mathrm{EVM} = \frac{\|c^o - c_m^{'}\|_2}{\|c^o\|_2} \tag{9a}$$

$$\text{subject to} \quad c_m^{'} = \hat{c}_m^{'} \bullet H_m \tag{9b}$$

$$c_m^{''} = 0 \tag{9c}$$

$$x_{m+1} = \mathrm{IFFT}(c_m) \tag{9d}$$

$$\frac{\|x_{m+1}\|_\infty}{\frac{1}{\sqrt{\ell N}}\|x_{m+1}\|_2} \leq \sqrt{\mathrm{PAPR}_{\max}} = \mathrm{CR}. \tag{9e}$$

Here, $c_m^{'}$ and $\hat{c}_m^{'}$ are in-band component, $c_m^{''}$ is out-of-band components, i.e., $c_m^{'} = [c_m(1) \cdots c_m(N)]^T$, $\hat{c}_m^{'} = [\hat{c}_m(1) \cdots \hat{c}_m(N)]^T$, $c_m^{''} = [c_m(N+1) \cdots c_m(\ell N)]^T$, and $c_m = [c_m^{'}; c_m^{''}]$. The operator '$\bullet$' denotes element-by-element product. (9b) and (9c) perform the filtering procedure, which weights the in-band components $\hat{c}_m^{'}$ using $H_m \in \mathbb{C}^N$ (the optimized filter at iteration $m$) and sets the out-of-band components $c_m^{''}$ to zero.

The optimization model (9) is non-convex because the constraint function in (9e) is non-convex. This makes numerical solution of (9) difficult.

### B. Convex Reformulation

To modify the optimization problem to a convex formulation, firstly we approximate $\|x_{m+1}\|_2$ by $\|\hat{x}_m\|_2$. In particular, in light of (9e) and (7), the non-convex constraint (9e) can be changed to a convex constraint

$$\|x_{m+1}\|_\infty \leq T'_{m+1}, \tag{10}$$

where approximated clipping level, according to (7), can be calculated through

$$T'_{m+1} = \sqrt{\frac{1}{\ell N}}\|\hat{x}_m\|_2 \mathrm{CR}. \tag{11}$$

Secondly we define a matrix $A$, which consists of the first $N$ columns of $\ell N - \mathrm{IFFT}$ twiddle factor matrix. Finally, we simplify the objective (9a) by defining an auxiliary variable, $t$, and letting $t = \frac{\|c^o - c_m^{'}\|_2}{\|c^o\|_2}$. With these modifications, the filter

design problem (9) can now be formulated as a second-order cone program (SOCP):

$$\min_{H_m \in \mathbb{C}^N, t \in \mathbb{R}} \quad t \tag{12a}$$

$$\text{subject to} \quad \|c^o - \hat{c}_m^{'} \bullet H_m\|_2 \leq \|c^o\|_2 t \tag{12b}$$

$$\|A(\hat{c}_m^{'} \bullet H_m)\|_\infty \leq T_{m+1}^{'}. \tag{12c}$$

### C. An Optimized Procedure

The new ICF algorithm, summarized below, combines the classic clipping procedure with the optimized filtering step described above. Step 1 is performed once for initialization, and then steps 2 through 7 are performed $M$ times for each OFDM symbol.

**Step 1**.**Initialization** : Set CR value and maximum iteration number, $M$.

**Step 2**. Set switch K1 to 1. A new OFDM symbol, $c^o$, enters the ICF loop, and then both K1 and K2 are set to 2.

**Step 3**. If $m = 1$, set $c^o = [c_1^o(1) \cdots c_1^o(N)]^T$ and $[c_1^o(N+1) \cdots c_1^o(\ell N)]^T = 0$. Otherwise, let $c_m^o = c_{m-1}$. Then convert $c_m^o$ to time domain signal, $x_m$, using IFFT.

**Step 4**. Calculate clipping level, $T_m$, using (7); generate $\hat{x}_m$ by clipping $x_m$ using (6).

**Step 5**. Convert $\hat{x}_m$ into frequency domain signal, $\hat{c}_m$, using FFT.

**Step 6**. Perform optimal filtering (12), i.e., weight $\hat{c}_m^{'}$ using optimized $H_m$ and set $\hat{c}_m^{''} = 0$.

**Step 7**. Let $m = m + 1$. If $m > M$, reset $m = 1$ and go to Step 2 to process the next OFDM symbol. Otherwise, go to Step 3 to proceed with the next iteration for the current OFDM symbol.

### D. Algorithm Complexity

SOCP model (12) can be solved by standard interior-point methods and the cost of solving the optimization model is $O(N^3)$ in the worst-case [11]. In the first iteration, the computation complexity is $O(N^3 + 2\ell N \log_2(\ell N))$. In the subsequent iterations, the computation complexity can be reduced to $O(N^3 + \ell N \log_2(\ell N))$ because OFDM time domain symbol $x_{m+1}$ can be achieved through (12c). Let the maximum iteration number be $M$. Then the computation complexity of the proposed procedure in Section III-C is $O(MN^3 + (M+1)\ell N \log_2(\ell N)))$.

When compared to that of the classic ICF method shown in Section II-C, the computation complexity of the proposed method is higher because $N \gg \ell$ in most practical applications. However, it should be noted that the said $O(N^3)$ is a complexity upper bound for solving the model (12). Designing a customized interior-point method for SOCP model can significantly reduce the computation complexity, as was shown in [7].

## IV. Simulation and Analysis

We now present simulation results for a 128 sub-carrier OFDM system with QPSK modulation and over-sampling factor $\ell = 4$. The convex optimization problem presented in (12) is solved using the public software CVX [12].

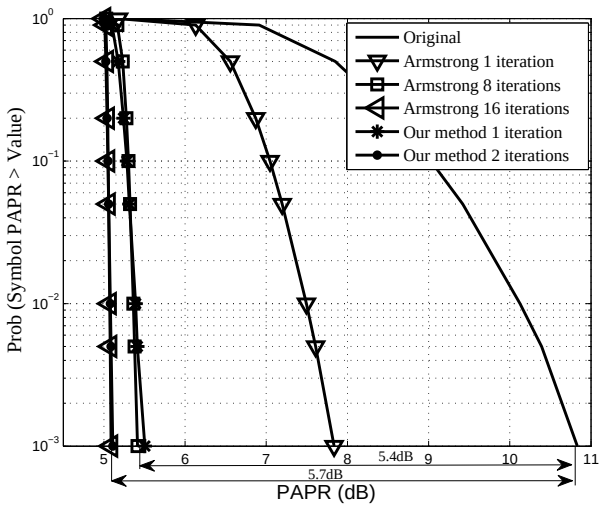


Fig. 2. PAPR statistics for an OFDM system (128 sub-carriers, QPSK modulation, and over-sampling factor $\ell = 4$).

Figure 2 plots the CCDFs of the PAPR for original symbols, modified symbols using Armstrong's method (for 1, 8, and 16 iterations) [3], and modified symbols using our proposed method (for 1 and 2 iterations) involving filter optimization. For both ICF methods, the desired PAPR is set to 5dB (or CR=1.7783). From the figure, it can be seen that both the Armstrong method and our optimization method can significantly reduce the PAPR while simultaneously achieving a comparable sharp drop in their CCDFs. However, for our ICF method, the PAPR is reduced by about 5.4dB at a probability of $10^{-3}$ after only one iteration and by about 5.7dB after two iterations. On the other hand, Armstrong's method requires 8 and 16 iterations, respectively, to achieve these same levels of PAPR reduction. Thus, the optimized ICF method requires far fewer iterations to approach the desired PAPR.

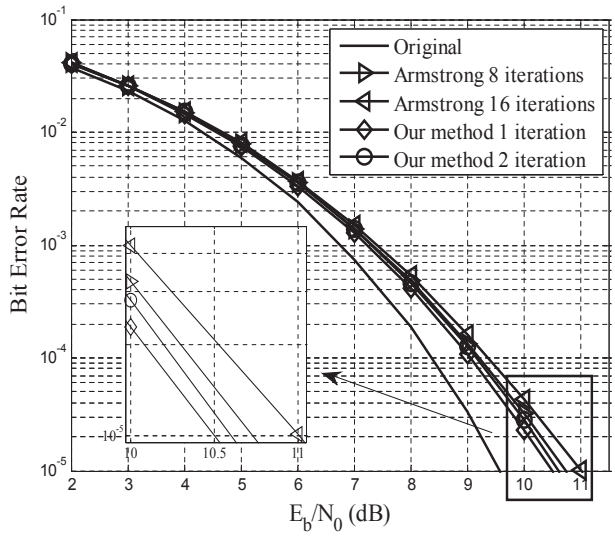


Fig. 3. BER comparison for original OFDM symbols, clipped OFDM symbols using Armstrong method, and clipped OFDM symbols using our proposed ICF method. Inset clarifies which line corresponds to which method.

Figure 3 plots the bit error rate curves of the original signal, the modified signal using Armstrong's method (for 8 and 16 iterations), and modified signal using our method (for 1 and 2 iterations) through an AWGN channel. All four BER curves

corresponding to modified symbols are located to the right of the original signal's curve because the clipping procedure causes signal distortion. Comparing the two BER curves of our method with those of the Armstrong method, it is observed that the signal to noise ratio (SNR) of our clipped OFDM symbols are better than Armstrong's for a given BER. For example, at a BER level of $10^{-5}$, 8 iterations of Armstrong's method yields an SNR about 0.42dB worse than that of the optimized ICF method after 1 iteration, and 16 Armstrong iterations results in an SNR about 0.20dB worse than 2 iterations of the optimized ICF method. The RMS-EVM values for the clipped OFDM symbols using Armstrong (8 and 16 iterations) and our method (1 and 2 iterations) can be calculated using (5), yielding -18.88dB, -18.43dB, -19.70dB and -18.96dB, respectively. This demonstrates that our clipped OFDM symbols have less distortion than that of Armstrong's clipped symbols.

The frequency responses (magnitude and phase) and time impulse response of Armstrong's filter ($H$) and our optimized filter ($H_1$, generated in the first iteration) for a random OFDM symbol are plotted in figure 4. It is observed that while the amplitude and phase frequency response of Armstrong's filter are constant across the entire in-band region, our optimized filter fluctuates with frequency. Comparing time impulse responses of $H$ and $H_1$, the latter can generate time dispersion effects which introduce some difference between the clipped symbols ($\hat{c}_m$) and the filtered symbols ($c_m$). From figure 3, it can be seen that this difference compensates in part the distortion caused by clipping procedure and optimizes the constellations of clipped OFDM symbols.

Finally, we consider passing the clipped signals through a solid-state power amplifier (SSPA), which is modeled by

$$s_o(t) = \frac{|s_i(t)|}{[1 + (\frac{|s_i(t)|}{C})^{2p}]^{\frac{1}{2p}}} e^{j\phi(t)}, \tag{13}$$

where $s_i(t) = Gxe^{j\phi(t)}$ is the input signal, $s_o(t)$ is the output of SSPA, and $G$ is gain of SSPA. The out-of-band radiation comparison is shown in figure 6. In the simulation, $G = 27dB$ and SSPA parameters $p$ and $C$ are set 3 and 1dB, respectively. We can see that Armstrong method leads to about 5dB lower out-of-band radiation than without using any PAPR reduction technique, and our approach leads to about 5dB lower out-of-band radiation than ICF at normalized frequency 0.4.

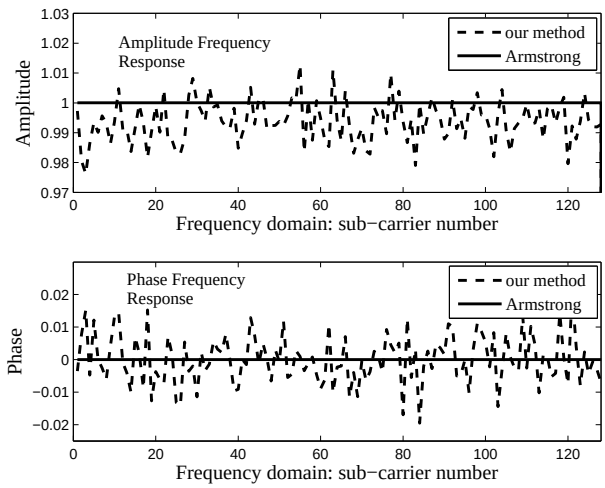


Fig. 4. Frequency response comparison between our proposed filter and Armstrong's filter.

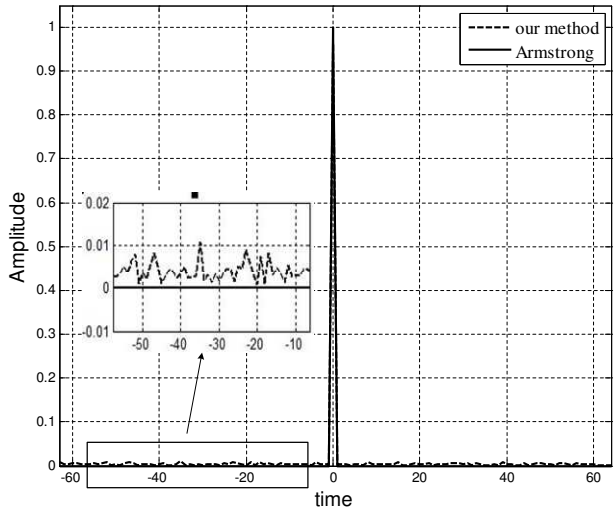


Fig. 5. Impulse response comparison between our proposed filter and Armstrong's filter. Inset clarifies which line corresponds to which method.

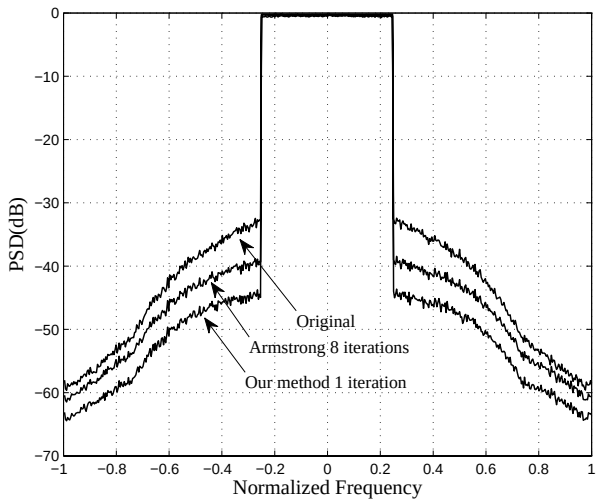


Fig. 6. Out-of-band radiation comparison of OFDM signals through SSPA- optimized method, Armstrong method, and none-processed . SSPA gain $G = 27dB$, $p = 3$, and $C = 1dB$.

## V. Conclusion

This paper proposes a convex optimization technique to dynamically modify the filter response in an ICF procedure. The resulting optimized ICF scheme can be used to reduce the PAPR of OFDM symbols. Simulations comparing this new strategy to that of the classic ICF method demonstrate three key advantages of the proposed method: the number of iterations required to reach a given PAPR level is greatly decreased, the processed OFDM symbols have less distortion and better out-of-band radiation.

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