Cooperative User Scheduling in Massive MIMO Systems

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ABSTRACT Taking advantage of distributed computation and the characteristic of Device-to-Device (D2D) communication among users, a cooperative user scheduling (CUS) scheme is firstly proposed. Through reducing the number of users which feed back their channel state information (CSI) to the base station (BS), the problem of huge feedback in massive multiple-input multiple-output (MIMO) systems can be, to a large extent, solved. The property of D2D allows users to exchange information, so the users can calculate their correlation coefficients with the selected best user locally. Then, these coefficients are used to filter out these strongly correlated users without greatly affecting the system sum rate. Specially, to decrease the computational complexity and reduce the number of feedback users, the cell users are divided into several groups randomly. The users in the same group can determine by themselves which need to feed back their CSIs, so the global optimal selection problem is decomposed into several local optimization problems. Through theoretical analysis on the CUS scheme, we obtain the lower bound expression of system sum rate. Simulation results indicate that, the uplink feedback resources can be greatly saved with the CUS scheme, whilst the proposed scheme’s influence on the system sum rate is negligible.

INDEX TERMS Massive MIMO, cooperative user scheduling, D2D, user filtering, huge feedback.

I. INTRODUCTION

RECENTLY, the next-generation wireless communication becomes the focus of attention, worldwide researchers have achieved numerous achievements on the fifth-generation (5G) wireless communication. Since proposed, massive antenna array is regarded as one of the key techniques in the 5G communication. In massive multiple-input multiple-output (MIMO) systems, the base station (BS) is equipped with dozens or even hundreds of antennas to serve multiple users. Through deeply developing the wireless resources in the spatial dimension, the large-scale antenna array has the potential to significantly improve the spectral efficiency and power efficiency. However, the technique also faces many challenges, such as pilot contamination, the channel modelling and the huge feedback in user scheduling [1], [2].

In many user scheduling algorithms [3], [4], the BS needs the channel state information (CSI) of all the users in cell to select the optimal user set to maximize the system sum rate. For example, in the traditional MIMO systems, the authors in [5] propose a low-complexity semi-orthogonal user selection (SUS) algorithm based on the exact CSI. The SUS algorithm iteratively selects the user with the greater channel norm and the correlation coefficient. The SUS algorithm can effectively decrease the computational complexity, while it needs the exact CSIs of all the users. The limitation makes the algorithm difficult to be applied to massive MIMO systems. Since, in the frequency division duplexing (FDD) mode, the feedback load is positively correlated with the numbers of the BS antennas and users, the huge amount of feedback would occupy a great part of spectral resources, which, in turn, decreases the system spectral efficiency when the numbers of the BS antennas and users increase sharply. In addition to the huge feedback overhead, it is also extremely difficult for the BS to select the optimal user set from a large number of users. Nowadays, many researchers have come up with a great variety of solutions to ease the problems of high computational complexity and huge feedback [3], [6]–[10].
authors in [3] propose a greedy user scheduling algorithm based on the rate allocation in vector perturbation precoding systems, which reduces the computational complexity through removing the insignificant users from the candidate user set. Considering that the principle of user scheduling is similar with that of antenna selection, the authors in [6] propose a joint strategy which simultaneously performs antenna selection and schedules the optimal users with the objective of maximizing the system sum rate and reducing the computational complexity of user scheduling and antenna selection. Owing to the limited space of the BS antenna array, the adjacent antennas are unavoidably correlated with each other when the number of antennas increases sharply. The authors in [7], [8] utilize the correlations among the adjacent antennas to group the correlated antennas, thus the dimensionality of the antennas at the BS and the feedback overhead are reduced. Similarly, taking advantage of the correlations among the users to group the users with the similar covariance eigenvectors [9], [10], the joint spatial division and multiplexing (JSDM) algorithm can reduce the dimensionality of the effective channels, simplify the system operations, and reduce the system feedback through randomly selecting part of users to feedback their CSIs.

For the articles above-mentioned, the user scheduling, the compression and recovery of CSI and the user grouping are accomplished at the BS, which would increase the feedback overhead of the uplink channel and the computational complexity at the BS. If part of the task of user scheduling is allocated to users, the computational complexity at the BS and the feedback load would be effectively decreased. Motived by the above viewpoints, we propose a cooperative user scheduling scheme for Device-to-Device (D2D) communication systems [11]. The key of the proposed scheduling scheme is to utilize the D2D communication among users to filter out those non-significant users and decrease computational complexity at the BS via the distributed computation. The proposed cooperative scheduling scheme consists of two phases: user filtering and user scheduling. During the user filtering phase, the cell users are divided into several groups firstly. Through a timer, the user with the best channel condition in each group is found, then the optimal user broadcasts its CSI to the other users in the same group. Thus, the residual users can calculate their correlation coefficients with the best user locally. Only the user with the best channel condition and the users whose correlation coefficients are greater than the filtering threshold can feed back their CSIs to the BS. During the second phase, the BS selects the user set through the popular user scheduling algorithms. Considering the similarity that both SUS algorithm [5] and the proposed user filtering algorithm simplify the candidate user set based on the correlation coefficients, we combine SUS algorithm with user filtering for maximizing the system sum rate.

In this paper, we analyze the system sum rate and feedback overhead of the proposed scheme in massive MIMO systems. According to the performance analysis, we conclude that the cooperative user scheduling scheme can reduce the computational complexity at the BS and the system feedback overhead without greatly decreasing the sum rate performance. The main contributions of this paper include:

- To ensure the low feedback and efficient transmission in massive MIMO systems, we propose a cooperative user scheduling scheme based on the cooperation among users. Through filtering out the non-significant users and simplifying the candidate user set before CSI feedback, the cooperative user scheduling (CUS) scheme can greatly reduce the number of feedback users. As far as we know, it is the first time to apply the concept of “user filtering” in user scheduling. Besides the advantage of reducing the system feedback, the user filtering is accomplished at users, so the proposed scheme can reduce the search complexity at the BS. Furthermore, through combining SUS algorithm with the user filtering, the CUS scheme can achieve excellent sum rate performance through selecting the appropriate user set from the permitted feedback users.

- To avoid occupying huge system resources during the feedback phase, we design an efficient CSI feedback strategy to improve the system spectral efficiency. By utilizing the cooperation among users, each user can obtain the correlation coefficient with the strongest user, which is found through a timer. Through comparing these coefficients with a filtering threshold, each user can locally determine which users can feed back their CSIs.

- Through analyzing the probability that each user is selected and the probabilities under different numbers of selected users, we obtain the lower bound of the system sum rate of the proposed CUS scheme. According to the simulation results, we verify the validity of the lower bound of sum rate under different antenna numbers, and conclude that the lower bound would get tighter when the number of BS antennas increases.

The remaining parts of this paper are organized as follows. In Section II, a multi-user MIMO system is considered and the section also quantifies the effect of zero-forcing (ZF) beamforming on effective signal to interference plus noise ratio (SINR). In Section III, the proposed cooperative user scheduling scheme is discussed in detail. Section IV gives an elaborate theoretical analysis on the sum rate of the scheduling algorithm. The simulation results are shown in Section V and the final concluding remarks are provided in Section VI.

As for notations, the uppercase boldface and lowercase boldface are used to denote matrices and vectors. $(\cdot)^H$ denotes the conjugate transpose of a matrix or vector. $|\cdot|$ denotes the Euclidean vector norm. $\text{card}(\cdot)$ is the number of the elements in set. $\mathbb{E}\{\cdot\}$ denotes the mathematical expectation operator. $A_{k,j}$ denotes the element in the $k$-th row and $j$-th column of matrix $A$. 

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II. SYSTEM AND CHANNEL MODELS

A single cell massive MIMO broadcast system is considered in this paper, where a BS transmits space information to \( N \) users, as shown in Fig. 1. In the system, the BS is equipped with \( N_t \) transmitting antennas, and each user is equipped with one antenna. We consider frequency division duplexing (FDD) in this paper, where the downlink and uplink channels occupy different frequency bandwidths, and the BS obtains the downlink CSI via the uplink feedback channel. It is assumed that the downlink (DL) channel is quasi-static Rayleigh block fading channel, which means that the channel condition changes from one frame until the next frame but remains constant within one frame. The uplink (UL) channel is assumed to be perfect and free from the channel noise.

As shown in Fig. 1, the BS transmits the data \( x \) to these users belonging to the user set \( S \). All the scheduled users’ received data \( y \) can be expressed as

\[
y = Hx + n = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_k \\ \vdots \\ h_K \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_k \\ \vdots \\ n_K \end{bmatrix}, \quad k \in S, \tag{1}
\]

where \( y \in \mathbb{C}^{K \times 1} \) is the received signal, in which \( K \) is the number of the selected users in the user set \( S \), that is, \( K = \text{card}(S) \leq N \). \( x \in \mathbb{C}^{N_t \times 1} \) denotes the transmitted signal from the BS. \( H = [h_1^T, h_2^T, \ldots, h_k^T, \ldots, h_K^T]^T \) denotes the channel gain matrix, in which \( h_k \in \mathbb{C}^{1 \times N_t} \) is the channel condition vector of the \( k \)-th user. The \( j \)-th entry in \( h_k \) is the channel fading coefficient between the \( j \)-th BS antenna and the \( k \)-th user, and it obeys Rayleigh distribution. As [12] considered, the antennas of BS are independent, \( n \in \mathbb{C}^{K \times 1} \) is the additive white noise vector, where \( n_k \) is the noise interfering the \( k \)-th user’s receiving signal, and \( n \sim \mathcal{CN}(0, I) \). \( I \in \mathbb{C}^{K \times K} \) denotes the unitary matrix. The power constraint for the transmitted signal is \( \mathbb{E}\{x^H x\} = P_T \). Since the noise variance is unit, \( P_T \) also is the transmitting signal-to-noise ratio (SNR).

In order to effectively eliminate the interference between users, the original signal \( s \) needs to be preprocessed. We adopt the zero-forcing beamforming (ZFBF) transmitter [13] for data preprocessing, so the post-processed signal \( x \) can be expressed as

\[
x = \sum_{k=1}^{K} \sqrt{P_k} w_k s_k, \quad k \in S, \tag{2}
\]

where \( s_k \) is the original transmitting signal of the \( k \)-th user, and \( s = [s_1, s_2, \ldots, s_k, \ldots, s_K]^T \). \( w_k \in \mathbb{C}^{N_t \times 1} \) is the zero-forcing beamforming vector of the \( k \)-th user. \( P_k \) is the power allocated to the \( k \)-th user, and \( \sum_{k=1}^{K} P_k = P_T \). Let \( \hat{W} = H^H(\hat{H}^H)^{-1} = [\hat{w}_1, \hat{w}_2, \ldots, \hat{w}_K] \) [14] be the beamforming matrix. For satisfying the power constraint, the beamforming vector of each user has to be normalized, i.e., \( w_i = \frac{w_i}{\|w_i\|} \), so \( \hat{W} = [\hat{w}_1, \hat{w}_2, \ldots, \hat{w}_K] = \left[ \frac{\hat{w}_1}{\|\hat{w}_1\|}, \frac{\hat{w}_2}{\|\hat{w}_2\|}, \ldots, \frac{\hat{w}_K}{\|\hat{w}_K\|} \right] \). Thus, the \( k \)-th user’s received signal \( y_k \) can be denoted as

\[
y_k = \sqrt{P_k} h_k w_k x_k + \sum_{j=1}^{K} \sqrt{P_j} \hat{w}_j x_j + n_k, \quad k, j \in S, \tag{3}
\]

where \( n_k \) is the noise received by the \( k \)-th user. According to (3), the effective receiving SINR of the \( k \)-th user is

\[
\gamma_k = \frac{P_k \|h_k w_k\|^2}{1 + \sum_{j \neq k} P_j \|h_j w_j\|^2}. \tag{4}
\]

Without loss of generality, equal power allocation is assumed, i.e., \( \bar{P}_k = \frac{P_T}{K} \).

III. COOPERATIVE USER SCHEDULING (CUS) SCHEME

In this section, we would introduce the proposed cooperative user scheduling scheme in detail. The CUS scheme is divided into two parts: user filtering based on D2D communication and user scheduling.

A. USER FILTERING BASED ON THE D2D COMMUNICATION

The first step of the proposed CUS scheme is to filter out some users via the cooperation among users, which means that not all users in cell can feed back their CSIs. Thus the scheme can decrease the system feedback overhead.

For reducing the computational complexity and saving the hardware cost, we need to assign the ultra-high dimension matrix computation of BS to multiple user groups. Considering that most of the existing grouping algorithms have strong dependency on CSI or channel correlation information [15]–[17], and need extra function evaluations, we adopt random user grouping algorithm [15] to ease the
computational complexity and reduce the time delay at BS. Furthermore, in the proposed scheme, we cannot adopt the grouping algorithms which depend on the instant channel information, since the grouping is accomplished at the users before the CSI feedback. The cell users are randomly divided into \( M \) groups. For simplicity, it is assumed that the number of users in each group is same, namely, \( N_G = \left\lceil \frac{N}{M} \right\rceil \). The users in different groups work in the different time slots, so these groups are independent with each other, and there is no inter-group interference.

Focusing on the optimal path selection in relay networks, the authors in [18] propose a cooperative protocol to find the best relay, and its idea of using a timer can be extended to our scheme. Before feedback, each user is set a timer and the timer’s start point is directly correlated with the user’s CSI [18]. The timer of the user having the best end-to-end channel condition will expire firstly. The start point of the timer can be set as

\[
T_s = \frac{\lambda}{\|h_s\|}, \quad s = 1, 2, \ldots, N_G, \tag{5}
\]

where \( T_s \) is an initial value of the \( s \)-th user’s timer. \( \lambda \) is a time constant. The start point \( T_s \) is inversely proportional to the channel state vector norm \( \|h_s\| \), so the user with the strongest channel would expire firstly. Here, We assume that all the timers in the same group start at the same time.

Based on the assumption that the users can estimate their CSIs exactly through channel estimation techniques, each user calculates its timer start point according to (5) independently. As shown in Fig. 2, the timer of the user with the strongest channel will expire first. Then the best user transmits a short flag packet [18] to signal its presence. The flag packet also contains the best user’s channel state vector. All the users in the same group, while waiting for their own timers to reach zero, are in listening mode (each user cannot overhear the flag packet of the users in other groups). As soon as they hear one user in their own group flagging its presence (the best user), they back off immediately. In this way, we can easily find the user with the strongest channel in each group (the user with the greatest channel norm).

The other users in each group, obtain the best user’s channel vector \( h_b \). In the following parts, we assume that the index of the best user is \( b \). Then they calculate the correlation coefficients between their own channel vectors and the \( b \)-th user’s channel vector, that is

\[
\alpha_{sb} = \frac{\|h_s h_b^H\|}{\|h_s\| \|h_b\|}, \quad 1 \leq s, b \in \mathbb{Z}, \quad s \neq b, \tag{6}
\]

where \( s \) and \( b \) are the indices of the users which are in the same group. \( \mathbb{Z} \) denotes the integer set. Only the user with the best channel condition and the users satisfying the filtering threshold can feed back theirs CSIs to the BS. That is, if \( \alpha_{sb} \leq \alpha \), user \( s \) can feed back its CSI to the BS, otherwise, user \( s \) is filtered out and will not communicate with the BS in this frame. \( \alpha \) is a given filtering threshold, which is same for all the groups. Thus, after the user filtering, the user set can be denoted as

\[
S_f = \bigcup_{m=1}^{M} S_{f,m} = \bigcup_{m=1}^{M} \left\{ s,b \in \mathbb{G}_m \bigg| \max_{b \in \mathbb{G}_m} \|h_s\|, \alpha_{sb} = \frac{\|h_s h_b^H\|}{\|h_s\| \|h_b\|} \leq \alpha, b \neq s \right\}, \tag{7}
\]

where \( S_{f,m} \) denotes the permitted user set of the \( m \)-th group, which can feed back its CSI to the BS. \( \mathbb{G}_m \) is the initial user set containing all the users of the \( m \)-th group before the user filtering, and \( \text{card}(\mathbb{G}_m) = N_G \).

**B. USER SCHEDULING**

After the user filtering phase, the BS can get the most wanted users’ CSIs from all the groups. Through combining with the semi-orthogonal user scheduling algorithm [19], the appropriate user set can be selected from all the feedback users for data transmission.

As shown in Tab. 1, \( n_f \) denotes the total of the feedback users, \( M \leq n_f \leq N \). \( S_{c,i} \) and \( S_i \) respectively denote the candidate user set and the selected user set at the \( i \)-th iteration. \( \pi(i) \), \( i = 1, \ldots, N_i \), denotes the index of the selected user at the \( i \)-th iteration. Step 2 is carried out to select the user with the strongest channel as the first scheduled user, and Step 3 is used for obtaining the projection vectors of the candidate users on the orthogonal complement space of the selected user set. In Step 4, the candidate user with the strongest projection vector joins in the selected user set. Step 5 is used to simplify the candidate user set, and in this step the correlation coefficients between the selected users and the candidate users are obtained through the projected vector \( g_n \) and \( g_{i(i)} \). After the SUS scheduling phase, the BS obtains the final user set \( S \) for data transmission.
In this section, we analyze the achievable sum rate of the proposed scheme with ZFBF transmitters. The sum rate of the massive MIMO system can be denoted as

\[
C_{CUS} = \frac{N_t}{K-1} \sum_{K=1}^{N_t} P_S(K) \sum_{k=1}^{K} \mathbb{E}\{\log_2 (1 + SINR_k)\} = \sum_{K=1}^{N_t} P_S(K) \int_{0}^{\infty} \log_2 (1 + \gamma_k) P_k(\gamma_k) d\gamma_k
\]

(8)

(9)

(10)

where \(P_S(K)\) is the probability that the number of the selected users is \(K\). \(P_k(\gamma_k)\) is the probability that the \(k\)-th user’s SINR is \(\gamma_k\). As ZFBF can effectively suppress the interference, \(SINR_k = SNR_k\), so according to (4), we have

\[\gamma_k = \frac{P_k|h_k|^2}{1 + \sum_{i \neq k} P_i|h_i|^2}\]

(11)

(12)

(13)

where \(P_{filt}\) denotes the probability that the correlation coefficient between user \(s\) and the best user is less than \(\alpha\). \(P\{N_m = n\}\), \(m = 1, 2, \ldots, M\), denotes the probability that the remaining user number of the \(m\)-th group is \(n\), which can be calculated as

\[
P\{N_m = n\} = \frac{(N_G - 1)}{n_m - 1} P_{filt}^{n_m - 1}(1 - P_{filt})^{N_G - n_m}
\]

(12)

(13)

After the user filtering, g-th group’s user set can be expressed as

\[W_g = S_{f,g}/b = \left\{ s \left| \frac{|h_s|}{h_0} \leq \alpha, s, b \in \mathbb{G}_g \right. \right\} \]

(14)

The probability that user \(s\) belongs to the set \(W_g\) can be described as [5]

\[
P\{s \in W_g\} \triangleq P_{filt} = F_{2j,2(N_t-j)} \left( \frac{N_t - j}{j} \frac{\alpha^2}{1 - \alpha^2} \right) = I_2(j, N_t - j)
\]

(15)

where \(s_g\) denotes the user \(s\) in the \(g\)-th group. \(F_{n,m}(x)\) is the cumulative density function (CDF) of the \(F\) distribution. \(I_2(a, b) = B_2(a, b)/B(a, b)\) is the regularized incomplete beta function, in which \(B_2(a, b)\) is the incomplete beta function, and \(B(a, b)\) is the complete beta function. Note that \(j = 1\) in our case, since we only need to calculate the correlation coefficient with one vector. Due to the fact that each group is i.i.d., we have, \(P\{s_1 \in W_1\} = P\{s_2 \in W_2\} = \cdots = P\{s_M \in W_M\} = P_{filt}\).

Let the number of all the remaining users, after the user filtering, be \(n_f\), that is, \(\sum_{m=1}^{M} n_m = n_f\). (12) can be further denoted as

\[
P_{n_f} = \prod_{m=1}^{M} \left( \frac{N_G - 1}{n_m - 1} \right) P_{filt}^{n_m - 1}(1 - P_{filt})^{N_G - n_m}
\]

(16)

(17)

2) Semi-orthogonal user scheduling (SUS)

As Step 5 shown in Tab. 1, the probability that the absolute value of the correlation coefficient between two vectors is less than \(\beta\) is defined as the “survival probability”. The “survival probability” at the \(i\)-th iteration of SUS can be expressed as [19]

\[
P_i(\beta) = 1 - (1 - \sqrt{\beta})^{N_t-i}
\]

(17)

### IV. SUM RATE ANALYSIS OF THE COOPERATIVE USER SCHEDULING

In this section, we analyze the achievable sum rate of the proposed scheme with ZFBF transmitters. The sum rate of the massive MIMO system can be denoted as
The final probability $P_s$ that each cell user is selected for data transmission can be calculated as

$$P_s = \sum_{i=1}^{N_1} P_{s,i},$$

(18)

where $P_{s,i}$, $1 \leq i \leq N_1$, denotes the probability that user $s$ is selected at the $i$-th iteration.

The probability $P_{s,1}$ that user $s$ is selected at the first iteration of the SUS algorithm is

$$P_{s,1} = \frac{1}{M} \sum_{n_f=1}^{N} P_{f,1}, \sum_{m=1}^{M} n_m = n_f, 1 \leq n_m \leq n_f,$$

(19)

where $P_{f,1}$ is the probability that user $s$ is the best user of the $g$-th group and the remaining user number of the $g$-th group is $n_g$. That is, $P_{f,1} = P(N_g = n_g, s \in S_{f,g}, s \notin W_g)$, its details will be given in the Appendix A. The probability $P_{s,2}$ that user $s$ is selected at the second iteration of the SUS algorithm is expressed as

$$P_{s,2} = P_{s,2,1} + P_{s,2,2},$$

(20)

where $P_{s,2,1}$ denotes the probability that user $s$ is selected at the second iteration and it is the best user of the $g$-th group. $P_{s,2,2}$ denotes the probability that user $s$ is selected at the second iteration and it is not the best user of the $g$-th group. The probability $P_{s,3}$ that user $s$ is selected at the third iteration of the SUS algorithm is

$$P_{s,3} \geq P\{\text{card}(S_{c,1}) = n_{s,1}, s \in S_{c,1}\} \sum_{n_{s,1}=1}^{n_{s,1}-1} \left(\begin{array}{c} n_{s,1} - 2 \\ n_{s,1} - 1 \end{array}\right) P_{s,2}^{n_{s,2}} (1 - P_{s,2}^{n_{s,2}}) P_{s,1}^{n_{s,1}-n_{s,2}-1},$$

(21)

where $P\{\text{card}(S_{c,1}) = n_{s,1}, s \in S_{c,1}\}$ denotes the probability that, at the first iteration of SUS algorithm, the candidate user number is $n_{s,1}$ and user $s$ is in the candidate user set $S_{c,1}$. $n_{s,2}$ is the candidate user number at the second iteration of SUS algorithm.

The probability $P_{s,k}, 3 \leq k \leq N_1$ that user $s$ is selected at the $k$-th iteration of the SUS algorithm is

$$P_{s,k} \geq \frac{1}{n_{s,k-1}} P\{\text{card}(S_{c,k-1}) = n_{s,k-1}, s \in S_{c,k-1}\},$$

(22)

where, similarly, $P\{\text{card}(S_{c,k-1}) = n_{s,k-1}, s \in S_{c,k-1}\}$ denotes the probability that, at the $k-1$-th iteration, the candidate users number is $n_{s,k-1}$ and user $s$ is in the candidate user set $S_{c,k-1}$. The details of the final probability of each user being selected are given in Appendix A.

As (19)-(22) shown, we can obtain the exact probability of each user being selected at the first iteration of SUS algorithm, and the lower bound of the probability that user $s$ is selected at the $k$-th ($k \geq 2$) iteration. Thus, according to (18), we obtain the lower bound of the probability of each user being selected.

Fig. 3 depicts the probability density function (PDF) that each user is selected. In this simulation, the BS is equipped with 4 antennas, and there are 20 users which are divided into 2 groups. The channels between the BS and the users obey Rayleigh fading. The filtering threshold $\alpha$ is 0.5, and the correlation coefficient $\beta$ is in the range of $(0, 0.45)$. In reality, for achieving a better scheduling user set, the parameters $\alpha$ and $\beta$ should be smaller than the values given in the simulation (it will be proved in Section V), which means that the selected users would be less correlated. Here, we just consider the conventional antenna array instead of massive MIMO systems due to its huge computational complexity of Fig. 3 and Fig. 4.

Just as Fig. 3 shown, we obtain a lower bound of the probability that each user is selected, which coincides with the analysis of (18)-(22). The theoretical lower bound has an excellent performance. Especially, when the coefficient efficient $\beta$ is in the range of $(0, 0.33)$, the result of theoretical analysis is almost equivalent to the simulation result. In reality, when there are thousands of users in cell, the optimal value of $\beta$ is just in this interval $(0, 0.33)$ [5], thus the theoretical lower bound (18) that each user is selected can be regard as the actual probability.

Based on the probability of each user being selected, we give a probability analysis about the different selected user numbers after the SUS algorithm. According to (16) and (42), the probability $P_S(1)$ of the selected user number being one can be calculated as

$$P_S(1) = \frac{1}{M} \sum_{n_f=1}^{N} P_{f,1} \sum_{i=1}^{M} (1 - P_{s,i}^{n_{s,i}}) \prod_{m=1}^{M} \left(1 - P_{s,i}^{n_{s,i}}\right)^{n_{s,i}}$$

(23)

Following this, the probability $P_S(k)$ of the selected user number being $k, 2 \leq k \leq N_1$ can be expressed as

$$P_S(k) \approx P\{\text{card}(S_{c,k-1}) = n_{s,k-1}\} \left(1 - P_{s,i}^{n_{s,i}}\right)^{n_{s,i}-1},$$

(24)

where $P\{\text{card}(S_{c,k-1}) = n_{s,k-1}\}, n_{s,k-1} \geq 1$ represents the probability that the candidate user number at the $k-1$-
the iteration is $n_{s,k-1}$, in which $P\{\text{card}(S_{c,1}) = n_{s,1}\}$ can be calculated as

$$P\{\text{card}(S_{c,1}) = n_{s,1}\} = \frac{1}{M} \sum_{m=1}^{M} P_{n_{f}} \sum_{m=1}^{N} P\{\text{card}(S_{c,1m}) = n_{s,1}\} \cdot P\{\text{card}(\tilde{S}_{c,1m}) = n_{s,12}\},$$

(25)

where $P\{\text{card}(S_{c,1m}) = n_{s,1}\}$ denotes the probability that the $m$-th group’s remaining user number is $n_{s,1}$, and $P\{\text{card}(S_{c,1m}) = n_{s,11}\}$ denotes that the other groups’ (except the $m$-th group) remaining user number is $n_{s,12}$, and the probability can be expressed as $P\{\text{card}(S_{c,1m}) = n_{s,12}\} = \binom{n_{m} - 1}{n_{s,12}} (1 - P_{c})^{n_{m} - n_{s,11} - 1}$. The probability $P\{\text{card}(S_{c,1m}) = n_{s,11}\}$ denotes the number of the candidate user number being $n_{s,k}$ can be expressed as

$$P\{\text{card}(S_{c,k}) = n_{s,k}\} \approx P\{\text{card}(S_{c,k-1}) = n_{s,k-1}\} \cdot \binom{n_{s,k-1} - 1}{n_{s,k}} P_{n_{s,k}} (\beta) (1 - P_{c})^{n_{s,k-1} - n_{s,k}-1}.$$  

(26)

Here, we obtain an approximated result based on the assumption that each user is i.i.d. Since the relationships would become weak and complicated after the second iteration due to the projection in Step 3 of Tab. 1. According to (24), (25) and (26), we can obtain the probability $P_{S}(k)$ of the selected user number being $n_{s,k}$, $2 \leq k \leq N_{t}$.

**FIGURE 4.** The probability analysis of different selected user numbers

Fig. 4 depicts the PDF that the numbers of the final selected users are one, two, three, and four. The simulation conditions are same as those of the Fig. 3. As seen from Fig. 4, an exact result is obtained when the selected user number is one. When the user numbers are two, three or four, we only obtain the approximated results, while the theoretical results are very close to the simulation results. Concluding from Fig. 3 and Fig. 4, we can verify the correctness of the theoretical analysis about the probability of different selected numbers, which accomplishes the first part of (10).

**B. SUM RATE ANALYSIS**

**Theorem 1** : The lower bound of the sum rate of the proposed CUS algorithm can be denoted as

$$C_{CUS} = \sum_{k=1}^{N_{t}} P_{S}(K) C_{zf,SUS}(K) \geq \log(e) \sum_{k=1}^{N_{t}} P_{S}(K) \frac{e^{K/P_{T}N_{t}} - K + 2}{P_{T}N_{t}}$$

(27)

$$\sum_{n=1}^{N_{t} - K + 1} \left( \frac{P_{T}}{K} \right)^{n} \Gamma \left( n - N_{t} + K - 1, \frac{K}{P_{T}} \right),$$

(28)

where $\Gamma(a, x) = \int_{0}^{x} t^{a-1} e^{-t} dt$ is the upper incomplete gamma function [13].

**Proof** : The effective SINR of user $k, 1 \leq k \leq K$ with ZFBF can be expressed as

$$\gamma_{k} = \frac{\bar{P}_{k}}{\left( HH^{H} \right)^{-1}} = \frac{\bar{P}_{T}}{K \left( HH^{H} \right)^{-1}},$$

(29)

where $K$ is the total number of the effective users. Set

$$Z = HH^{H},$$

(30)

where $Z$ is termed as a complex Wishart distribution [13], and $Z \sim W_{K}(N_{t}, \Lambda), \Lambda = I$.

According to [20], $\gamma_{k}$ in (29) is a Chi-squared distribution random variable with degrees of freedom $2(N_{t} - K + 1)$, so the PDF of $\gamma_{k}$ can be calculated as

$$f_{\gamma_{k}}(\gamma_{k}) = \frac{K e^{K/\bar{P}_{T}}}{\bar{P}_{T}^{N_{t} - K} \Gamma(N_{t} - K, \frac{K}{\bar{P}_{T}})^{N_{t} - K}} \gamma_{k}^{N_{t} - K - 1}, \gamma_{k} \geq 0.$$  

(31)

Note that the constraint $N_{t} \geq K$ (the constraint of Wishart distribution) is satisfied because of the SUS algorithm.

Given the user number $K$, the sum rate [21–23] with the ZFBF transmitters when the cell users are independent is

$$C_{zf}(K) = \sum_{k=1}^{K} E\{ \log(1 + \gamma_{k}) \} = \sum_{k=1}^{\infty} \int_{0}^{\infty} f_{\gamma_{k}}(\gamma_{k}) \log(1 + \gamma_{k}) d\gamma_{k}$$

$$= K \int_{0}^{\infty} f_{\gamma_{k}}(\gamma_{k}) \log(1 + \gamma_{k}) d\gamma_{k}$$

$$= \log(e) \frac{e^{K/P_{T}N_{t} - K + 2}}{P_{T}N_{t} + K + 1}$$

$$\sum_{n=1}^{N_{t} - K + 1} \left( \frac{P_{T}}{K} \right)^{n} \Gamma \left( n - N_{t} + K - 1, \frac{K}{P_{T}} \right),$$

(32)
where (32) is obtained from the integral identity [13]

\[
\int_0^\infty \log(1 + t) e^{-\mu t} t^{n-1} dt = (n - 1)! e^{\mu} \sum_{i=1}^n \frac{\Gamma(i - n, \mu)}{\mu^i}, n = 1, 2, \ldots.
\]

(33)

Then, we will consider the effect of correlation on the sum rate. The sum rate \( C_{zf}(K) \) in (32) is obtained based on the assumption of independent users. However, in the proposed CUS scheme, users are selected based on the correlation coefficients. The selected users are semi-orthogonal with each other instead of being randomly selected. So, the elements off the diagonal of \( Z \) in our scheme are smaller compared with those of independent users.

Given \( Z_{k,k} = Z_{k,k} \) and \( Z_{k,j} > Z_{k,j}, k \neq j \), we have \( (Z^{-1})_{k,k} > (Z^{-1})_{k,k} \). Thus, according to (29), the effective \( \gamma_k \) would be greater after the SUS procedure. So,

\[
C_{zf,SUS}(K) \geq C_{zf}(K).
\]

(34)

Thus, we obtain the lower bound of the sum rate with the SUS algorithm and the ZFBF transmitters, which accomplishes the second part of (10). According to the known channel-hardening [24] property in massive MIMO systems, as the number of the BS antennas increases sharply, the different users’ channels are nearly orthogonal. Thus the non-diagonal elements in \( Z \) would get smaller further, and the lower bound will get tighter. Combining the sum rate \( C_{zf,SUS} \) and the probability of different selected user numbers, we obtain the lower bound of the sum rate of the CUS scheme according to (10), (24) and (34), which can be calculated as

\[
C_{CUS} = \sum_{K=1}^{N_t} P_S(K) C_{zf,SUS}(K) \geq \log(e) \sum_{K=1}^{N_t} P_S(K) e^{K/P_T} K^{N_t - K + 2} \left( \frac{P_T}{K} \right)^{N_t - K + 1} \cdot \sum_{n=1}^{N_t - K + 1} \left( \frac{P_T}{K} \right)^n \Gamma(n - N_t + K - 1, K/P_T).
\]

(35)

So we obtain the sum rate expression in Theorem 1. Here, we give a detailed performance analysis of the proposed algorithm step by step. In [5], the authors just give an asymptotic sum rate expression when the users number approaches infinity, and do not give an exact probability analysis about the selected user number. Except obtaining the probabilities of different selected user numbers, our formulation obtains an elaborate lower bound of the sum rate. And the theoretical lower bound would get closer to the simulation results as the BS antenna number increases.

![Fig. 5. The sum rate comparisons of the simulation results and the theoretical results](image)

Fig. 5 depicts the sum rate comparisons of the simulation results and theoretical results. In the simulation, there are 32 users, they are randomly divided into two groups. For analytical tractability, the numbers of the BS transmit antennas are set equal to 4, 8, and 16. The filtering threshold \( \alpha \) and correlation coefficient \( \beta \) are 0.3 and 0.25. The simulation results of the sum rate are obtained over \( 10^5 \) independent channel realizations. Concluding from Fig. 5, we obtain the lower bound of the system sum rate, which coincides with the analysis in (35).

As shown in Tab. 2, given the receiving SNR, the normalized relative differences between the simulation results and the theoretical results decrease obviously with the increase of antenna number, in which the normalized relative difference is calculated as \( \left| \frac{C_{sim} - C_{theory}}{C_{sim}} \right| \), where \( C_{sim} \) and \( C_{theory} \) respectively denote the simulation sum rate and the theoretical sum rate. Concluding from Tab. 2, the lower bound would get tighter when the antenna number increases, which coincides with our previous analysis. So it is reasonable to believe that the theoretical analysis would obtain a closer approximation, when the theoretical results are applied in massive MIMO systems. According to Fig. 3, Fig. 4, and Fig. 5, we can verify the correctness of the theoretical lower bound of sum rate (35).

![TABLE 2. The normalized relative differences between simulation results and theoretical results](image)

<table>
<thead>
<tr>
<th>SNR</th>
<th>0 dB</th>
<th>2 dB</th>
<th>4 dB</th>
<th>6 dB</th>
<th>8 dB</th>
<th>10 dB</th>
<th>12 dB</th>
<th>14 dB</th>
<th>16 dB</th>
<th>18 dB</th>
<th>20 dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_t = 4 )</td>
<td>0.450</td>
<td>0.416</td>
<td>0.385</td>
<td>0.356</td>
<td>0.319</td>
<td>0.289</td>
<td>0.264</td>
<td>0.242</td>
<td>0.226</td>
<td>0.207</td>
<td>0.199</td>
</tr>
<tr>
<td>( N_t = 8 )</td>
<td>0.318</td>
<td>0.290</td>
<td>0.264</td>
<td>0.237</td>
<td>0.214</td>
<td>0.198</td>
<td>0.178</td>
<td>0.164</td>
<td>0.149</td>
<td>0.137</td>
<td>0.129</td>
</tr>
<tr>
<td>( N_t = 16 )</td>
<td>0.220</td>
<td>0.198</td>
<td>0.174</td>
<td>0.157</td>
<td>0.147</td>
<td>0.133</td>
<td>0.113</td>
<td>0.110</td>
<td>0.097</td>
<td>0.096</td>
<td>0.077</td>
</tr>
</tbody>
</table>

In the performance analysis, we give the figures about the theoretical results and simulation results in the conventional MIMO systems. Since the computation complexity of the theoretical analysis would have an exponential growth when the number of antennas increase sharply due to the step-by-step theoretical analysis. While, as stated in our paper,
we analyze the results in the massive MIMO systems, and conclude that, in the massive MIMO scenario, the theoretical results would get closer to the simulation results. That is, in the massive MIMO, the theoretical results would obtain a better performance. Except that, in the Section V, we give the figures of performance comparison in the massive MIMO systems.

V. SIMULATION AND DISCUSSION

In this part, we mainly focus on the sum rate performance and the system feedback overhead of the proposed CUS scheme. The noise variance of each user is one. The sum rate is obtained over $10^3$ channel realizations. The channel in the paper obeys the Rayleigh distribution with mean 0 and variance 1.

According to Tab. 3, through setting $\alpha = \beta$, the system feedback load can be greatly saved. Especially, with $\alpha = \beta = 0.15$, only about half of the cell users need to feed back their CSIs, and the sum rate only has a slight degradation as depicted in Fig. 6.

![Fig. 6](image-url)  
**Fig. 6.** The system sum rate with different filtering thresholds $\alpha$ and correlation coefficients $\beta$

![Fig. 7](image-url)  
**Fig. 7.** The relationship between the system sum rate and filtering threshold $\alpha$

Fig. 7 depicts the effect of filtering threshold $\alpha$ on the system sum rate. The simulation conditions are same as those of Fig. 6. As seen in Fig. 7, the sum rate of the CUS scheme increases with the filtering threshold $\alpha$ given the correlation coefficients $\beta$. But if the threshold $\alpha$ is too large, it cannot effectively reduce the feedback users, and if $\alpha$ is too small, it would greatly degrade the system sum rate. Considering this factor, we can set $\beta = 0.04$ according to curves in Fig. 7. Since, in this interval, the scheme would not greatly decrease system sum rate (less than 15%), and it can effectively save the system resources. Thus the system can get a good trade-off between the sum rate and the system feedback load. As for the realization in practice, the system can determine the values of $\alpha$ and $\beta$ beforehand according to the system configuration [5], then BS informs all the users in the way similar to the realization of SINR threshold in limited feedback [25].

![Table 3](table-url)  
**Table 3.** The average numbers of transmitting users and filtering users with different filtering thresholds $\alpha$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Average number of transmitting users</th>
<th>Average number of filtering users</th>
<th>Percentage of reduction of feedback users</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>34</td>
<td>66</td>
<td>66%</td>
</tr>
<tr>
<td>0.15</td>
<td>55</td>
<td>45</td>
<td>45%</td>
</tr>
<tr>
<td>0.22</td>
<td>80</td>
<td>20</td>
<td>70%</td>
</tr>
</tbody>
</table>

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slightly decrease the number of the groups \( M \) to reduce the system feedback overhead. As shown in Tab. 4, \( M \) has little influence on the system sum rate (the simulation conditions are same as those of Fig. 6).

**TABLE 4.** The relationship between the sum rate and user group number \( M \)

<table>
<thead>
<tr>
<th>SNR</th>
<th>( P=0 ) dB</th>
<th>( P=5 ) dB</th>
<th>( P=10 ) dB</th>
<th>( P=15 ) dB</th>
<th>( P=20 ) dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M = 5 )</td>
<td>25.3</td>
<td>25.1</td>
<td>34.6</td>
<td>34.6</td>
<td>34.6</td>
</tr>
<tr>
<td>( M = 10 )</td>
<td>25.3</td>
<td>25.1</td>
<td>34.6</td>
<td>34.6</td>
<td>34.6</td>
</tr>
<tr>
<td>( M = 20 )</td>
<td>25.3</td>
<td>25.1</td>
<td>34.6</td>
<td>34.6</td>
<td>34.6</td>
</tr>
</tbody>
</table>

In this paper, through making use of the cooperation among users to filter out the users whose correlation coefficients are greater than the filtering threshold, a new cooperative user scheduling scheme was proposed based on the D2D communication among users. The cooperative user scheduling scheme could effectively ease the problem of huge feedback in massive MIMO systems and decrease the computational complexity of user scheduling at the BS, which can be applied to the maritime communication systems. Meanwhile, we analyzed the influence of filtering threshold, correlation coefficient, and the number of groups on the system sum rate performance and feedback load. According to the simulations, the sum rate and feedback overhead could achieve a good balance, while the corresponding values of these parameters were changeable according to specific needs of different systems. Through exhaustive analysis on the CUS scheme, we obtained the probability of each user being selected, the probabilities of different selected user numbers and the lower bound of the sum rate. For analytical tractability, we just plotted the curves of the theoretical results in conventional MIMO systems. Whilst, as analyzed, when the number of BS antennas increased, the theoretical analysis could obtain better performance, and the lower bound would get tighter.

**APPENDIX A PROOF OF EQUATIONS (18)-(22)**

1) user filtering

After the user filtering phase, the probability that the remaining user number is \( n_f \) and user \( s \) belongs to the set \( S_f \) can be calculated as

\[
P(N_{filt} = n_f, s \in S_f) = \prod_{m=1}^{M} P_{f,m}
\]

\[
= P(N_g = n_g, s \in S_{f,g}) \prod_{m=1}^{M} P(N_m = n_m)
\]

\[
= (P_{f,g1} + P_{f,g2}) \prod_{m=1}^{M} \left( \frac{N_G - 1}{n_m - 1} \right) P_{f,m}^{n_m - 1} (1 - P_{f,m})^{N_G - n_m}
\]

\[
= P_{f1} + P_{f2},
\]

where \( P_{f,m} = P(N_m = n_m), m = 1, 2, \ldots, M, m \neq g \) is the probability that the \( m \)th group’s remaining user number is \( n_m \), that is, \( P_{f,m} = P(\text{card}(S_{f,m}) = n_m) = \)
\[ P_{s,2} \geq \frac{1}{n_{s,1}} P \{ \text{card}(S_{c,1}) = n_{s,1}, s \in S_{c,1} \} \]
\[ = \frac{M-1}{M} \sum_{n_f=1}^{N} P_{f_{g}} \sum_{n_s=1}^{n_f-1} P \{ N_{i,s} = n_{i,s} \} P \{ N_{g,s} = n_{f,g,s} \} \prod_{m=1}^{M} P \{ N_{m,s} = n_{m,s} \} \]  
\[ + \frac{1}{M} \sum_{n_f=1}^{N} P_{f_{g}} \sum_{n_s=1}^{n_f-1} P \{ N_{g,s} = n_{f,g,s} \} \prod_{m=1}^{M} P \{ N_{m,s} = n_{m,s} \} . \]

**case 1**

\[ \left( \frac{N_G - 1}{n_f - 1} \right) P_{f_{g}}^{n_f-1} \left( 1 - P_{f_{g}} \right)^{N_G - n_f}, \text{ and } \sum_{m=1}^{M} n_m = n_f. \]  

**case 2**

\[ \frac{1}{n_f - 1} P_{f_{g}}^{n_f-1} \left( 1 - P_{f_{g}} \right)^{N_G - n_f}. \]

\[ P_{f_{g}} \text{ is the probability that the } g\text{-th group’s remaining user number is } n_f \text{ and user } s \text{ is not the best one.} \]

\[ P_{f_{g}} = P \{ N_{g} = n_{g}, s \in S_{f,g}, s \notin W_{g} \}, \]

\[ P_{f_{g}} = \left( \frac{N_G - 1}{n_f - 2} \right) P_{f_{g}}^{n_f-1} \left( 1 - P_{f_{g}} \right)^{N_G - n_f}. \]

\[ P_{f_{g}} = P \{ N_{g} = n_{g}, s \in W_{g} \}, \]

\[ P_{f_{g}} = \left( \frac{N_G - 1}{n_f - 1} \right) P_{f_{g}}^{n_f-1} \left( 1 - P_{f_{g}} \right)^{N_G - n_f}. \]

2) Semi-orthogonal user scheduling (SUS)

The probability that user \( s \) is selected for data transmission can be calculated as

\[ P_s = \sum_{i=1}^{N_s} P_{s,i} , \quad (37) \]

where \( P_{s,i} \) denotes the probability that user \( s \) is selected at the \( i \)-th iteration of the SUS algorithm. \( P_{s,1} \) can be expressed as

\[ P_{s,1} = \frac{1}{M} \sum_{n_f=1}^{N} P_{f_{g}} \sum_{n_m=1}^{n_f} P_{f_{m}} \prod_{m=1}^{M} P_{f_{m}}. \]

Since the selected user at the first iteration must be one of the best users of \( M \) groups, so we do not need to consider the case that user \( s \) is not the best user of the \( g \)-th group. The probability \( P_{s,2} \), that user \( s \) is selected at the second iteration and is the best user of the \( g \)-th group, can be expressed as

\[ P_{s,2} \geq \frac{1}{n_{s,1}} P \{ \text{card}(S_{c,1}) = n_{s,1}, s \in S_{c,1} \} \]
\[ = \frac{M-1}{M} \sum_{n_f=1}^{N} P_{f_{g}} \sum_{n_s=1}^{n_f-1} P \{ N_{i,s} = n_{i,s} \} P \{ N_{g,s} = n_{f,g,s} \} \prod_{m=1}^{M} P \{ N_{m,s} = n_{m,s} \} \]  
\[ + \frac{1}{M} \sum_{n_f=1}^{N} P_{f_{g}} \sum_{n_s=1}^{n_f-1} P \{ N_{g,s} = n_{f,g,s} \} \prod_{m=1}^{M} P \{ N_{m,s} = n_{m,s} \} . \]

where \( P \{ N_{m,s} = n_{m,s} \}, m = 1, 2, \cdots, M, \) denotes the probability that, after the first iteration of the SUS algorithm, the remaining user number of the \( m \)-th group is \( n_{m,s} \). \( \sum_{m=1}^{M} n_{m,s} = n_{s,1} \), and \( \text{card}(S_{c,1}) = n_{s,1} \). Here, it is assumed that the selected user at the first iteration is coming from the \( i \)-th group, \( i \neq g \).

\[ P \{ N_{m,s} = n_{m,s} \} =\]
\[ \left\{ \begin{array}{ll}
\frac{n_i-1}{n_i} P_{f_{1,c}}^{n_i-1} (1 - P_{f_{1,c}})^{\beta}, & m = i \\
\frac{n_g-1}{n_{g,s_1}} P_{f_{1,g}}^{n_g-1} (1 - P_{f_{1,g}})^{\beta}, & m = g \\
\frac{n_{m,s}}{n_{m,s}} P_{f_{1,m}}^{n_{m,s}} (1 - P_{f_{1,m}})^{\beta}, & \text{others}
\end{array} \right. \]  
\[ (41) \]

when \( m = i \), the “survival probability” \( P_{1,c} \) of the candidate users in the \( i \)-th group, at the first iteration, is a conditional probability instead of \( P_{1}^{\beta} \). Since the candidate users from the \( i \)-th group are already satisfied with the coefficient constraint (14). Thus the “survival probability” can be expressed as

\[ P_{1,c} = \frac{P_{f_{g}}^{\beta}}{P_{f_{g}}} \leq \beta \]
\[ = \frac{P_{f_{g}}^{\beta}}{P_{f_{g}}} \leq \beta \]
\[ = P_{f_{g}}^{\beta} \leq \beta . \]  
\[ (42) \]

Here, \( \alpha \geq \beta \) (when \( \alpha \leq \beta, P_{1,c} = 1 \)). The inequality (39) holds since the candidate users which are in the same group
with the first selected user satisfy (14), which would lead to a high selection probability. While, in this analysis, we assume that all the remaining users’ selection probability are $P_1(\beta)$, which causes that the theoretical probability are lower than the actual one.

The probability $P_{s,2}$ that user $s$ is selected at the second iteration is not the best user of the $g$-th group is calculated in (43), in which the case 1 represents the selected user and user $s$ are in the different groups. The case 2 represents the first selected user is in the same group with the user $s$, in which

$$P\{\text{card}(S_{c,k}) = n_{s,k}, s \in S_{c,k}\} = P\{\text{card}(S_{c,k-1}) = n_{s,k-1}, s \in S_{c,k-1}\} \frac{n_{s,k-1} - 1}{n_{s,k-1}} \left(\frac{n_{s,k-1} - 2}{n_{s,k-1}}\right) P_k^{n_{s,k}}(\beta)(1 - P_k(\beta))^{n_{s,k} - n_{s,k} - 1}.$$  (51)

The probability $P_{s,1}$ that user $s$ is selected at the second iteration of the SUS algorithm is

$$P_{s,2} = P_{s,2_1} + P_{s,2_2}.$$  (47)

According to (39), (43), and (47), we obtain the lower bound of the probability that user $s$ is selected at the second iteration.

Similarly, the probability that user $s$ is selected at the $k$-th iteration of the SUS algorithm ($3 \leq k \leq N_i$) can be described as

$$P_{s,k} \geq \frac{1}{n_{s,k-1}} P\{\text{card}(S_{c,k-1}) = n_{s,k-1}, s \in S_{c,k-1}\}$$  (48) 
$$\geq P\{\text{card}(S_{c,k-2}) = n_{s,k-2}, s \in S_{c,k-2}\} \frac{n_{s,k-2} - 1}{n_{s,k-2} n_{s,k-1}}$$  (49) 
$$\frac{n_{s,k-2} - 2}{n_{s,k-1} - 1} P_k^{n_{s,k-1}}(\beta)(1 - P_k(\beta))^{n_{s,k-2} - n_{s,k-1} - 1}$$

where $n_{s,k-1}$ and $n_{s,k-2}$ respectively represent the remaining users number at the $k-1$-th and $k-2$-th iteration, that is, $\text{card}(S_{c,k-1}) = n_{s,k-1}$ and $\text{card}(S_{c,k-2}) = n_{s,k-2}$. According to (40) and (44), the probability $P\{\text{card}(S_{c,1}) = n_{s,1}, s \in S_{c,1}\}$ that the total survival user number, at the first iteration, is $n_{s,1}$ and user $s$ is in the candidate user set $S_{c,1}$ can be obtained as (50), in which the equality $\sum$ is used for simplifying the equation.
The probability $P\{\text{card}(S_{c,k}) = n_{s,k}\}, 2 \leq k \leq N_t$ that, at the $k$-th iteration of the SUS algorithm, the remaining users number is $n_{s,k}$ and user $s$ is in the set $S_{c,k}$ can be calculated as (51). So, according to (38), (47), and (48), we can obtain the lower bound of the probability of each user being selected, which also proves (18).

REFERENCES


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