Synthesis of Cross-Coupled Triple-Passband Filters Based on Frequency Transformation

Hao Di, Bian Wu, Xin Lai, and Chang-Hong Liang, *Senior Member, IEEE*

***Abstract*—A synthesis method is presented for triple-passband filters. The proposed method is based on a frequency transformation from the normalized frequency domain to the actual frequency domain. A novel triple-passband filter topology with cross coupling is obtained, which consists of admittance inverters and parallel resonators. The coupling coefficients and the external quality factors can be calculated analytically. To validate the theory, a triple-passband filter is designed and fabricated using microstrip structure. The theoretical response meets the specifications very well, and the measured result shows a good agreement with the simulated one.**

***Index Terms*—Cross-coupled filter, frequency transformation, synthesis method, triple-passband filters.**

I. INTRODUCTION

WITH the development of modern wireless communication, high-performance microwave filters in communication systems are largely required. In multichannel communication systems, dual-band and multiband microwave filters can simplify the system and reduce the volume and mass of the circuit. Recently, synthesis techniques for dual- and triple-passband microwave filters are widely studied in several literatures. The most direct method for dual-passband filters is cascading a wideband bandpass filter with a narrowband stopband filter [1]. In [2], the dual-and triple-passband filters are designed by placing transmission zeros within the passband of a wideband bandpass filter. Synthesis method using frequency transformation to locate the poles and zeros of the dual- and triple-passband filters is proposed in [3] and [4], respectively. The transmission zeros used to split single passband can also be produced by bandstop resonators that are coupled to the bandpass resonators [5]–[7], and these methods are also based on frequency transformations.

In this letter, we present a synthesis technique for the triple-passband filters. This design method is based on a frequency transformation from the normalized frequency domain to the actual frequency domain. The frequency transformation is applied to a cross-coupled low-pass prototype filter (with any filtering characteristics, symmetric or asymmetric), and a novel cross-coupled triple-passband filter is obtained with several transmission zeros located at different frequencies inside the stopbands and providing the equiripple response in the stopbands. The coupling coefficients and the external quality factors for the triple-passband filters can be calculated directly using the parameters in the frequency transformation proposed in this method without any need of optimization; however, in [3] and [4], the locations of poles and zeros need to be determined, and then the coupling matrix is solved according to the transfer function obtained from poles and zeros. With this method, the triple-passband filters can be designed with prescribed passbands regardless of whether the bandwidths of the three passbands and the spacings between the passbands are equal or not. A triple-passband generalized Chebyshev filter is designed and fabricated using microstrip structure to validate the proposed method.

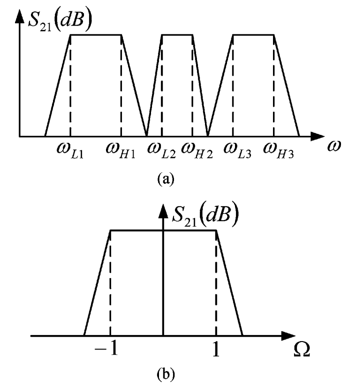

Fig. 1. Schematic frequency response of (a) the triple-passband filter in ω domain and (b) the low-pass prototype filter in the normalized Ω domain.

II. SYNTHESIS METHOD

The triple-passband filter that operates at the ω domain has the prescribed passbands at $(\omega_{\mathrm{L1}}, \omega_{\mathrm{H1}})$, $(\omega_{\mathrm{L2}}, \omega_{\mathrm{H2}})$, and $(\omega_{\mathrm{L3}}, \omega_{\mathrm{H3}})$, its frequency response is shown in Fig. 1(a). The low-pass prototype filter operates at the normalized frequency Ω domain, and its frequency response is depicted in Fig. 1(b). As the synthesis method of low-pass prototype filter is well established, the triple-band filter can be designed using a frequency transformation and a related low-pass prototype filter. The frequency transformation from low-pass prototype to triple-passband used in this synthesis method can be written as

$$\Omega = T(\omega) = b_1\left(\frac{\omega}{\omega_{01}} - \frac{\omega_{01}}{\omega}\right) - \frac{1}{b_2\left(\frac{\omega}{\omega_{02}} - \frac{\omega_{02}}{\omega}\right) - \frac{1}{b_3\left(\frac{\omega}{\omega_{03}} - \frac{\omega_{03}}{\omega}\right)}} \tag{1}$$

where b_1, b_2, b_3, ω_{01}, ω_{02}, ω_{03} are the parameters that define the frequency transformation. To determine these parameters, the same technique is used as in [3] for dual-passband filters.

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The authors are with the National Key Laboratory of Antennas and Microwave Technology, Xidian University, Xi'an 710071, China (e-mail: dihaoxdu@gmail.com).

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Actually, through the frequency transformation, the three lower limits $(\omega_{L1}, \omega_{L2}, \omega_{L3})$ of the passbands are mapped to -1 in the normalized Ω domain, and the three upper limits $(\omega_{H1}, \omega_{H2}, \omega_{H3})$ are mapped to 1. Taking into account that the transformation $T(\omega)$ in (1) is an odd function, these mapping relationships can be expressed as

$$T(\omega_{H1}) = T(\omega_{H2}) = T(\omega_{H3}) = 1 \quad (2a)$$
$$T(-\omega_{L1}) = T(-\omega_{L2}) = T(-\omega_{L3}) = -(-1) = 1. \quad (2b)$$

Here, let us introduce a new function $U(\omega) = T(\omega) - 1$. It is noted that the frequencies $(-\omega_{L1}, \omega_{H1}, -\omega_{L2}, \omega_{H2}, -\omega_{L3}, \omega_{H3})$ are just six zeros of function $U(\omega)$. Substituting (1) for $T(\omega)$, $U(\omega)$ can be expressed as the ratio of two polynomials

$$U(\omega) = \frac{N(\omega)}{D(\omega)} = \frac{\omega^6 + n_5\omega^5 + n_4\omega^4 + n_3\omega^3 + n_2\omega^2 + n_1\omega + n_0}{-n_5\omega^5 - n_3\omega^3 - n_1\omega}. \quad (3)$$

It is obvious that $(-\omega_{L1}, \omega_{H1}, -\omega_{L2}, \omega_{H2}, -\omega_{L3}, \omega_{H3})$ are also the zeros of the numerator $N(\omega)$ in (3). The unknown parameters n_0-n_5 can be solved. The parameters defining the transformation $T(\omega)$ can be expressed as functions of n_0-n_5. They are shown as

$$\omega_{01} = \sqrt{-\frac{n_0 n_5}{n_1}} \quad (4)$$
$$b_1 = \sqrt{-\frac{n_0}{n_1 n_5}} \quad (5)$$
$$\omega_{02} = \sqrt{\frac{n_1^2 n_4 n_5 - n_0 n_1 n_5^2 - n_1^2 n_3}{n_0 n_3 n_5^2 + n_1^2 n_5 - n_1 n_2 n_5^2}} \quad (6)$$
$$b_2 = \sqrt{\frac{n_1 n_5^2}{\left(\frac{n_0 n_3}{n_1} + \frac{n_1}{n_5} - n_2\right)\left(n_4 n_5 - \frac{n_0}{n_1} n_5^2 - n_3\right)}} \quad (7)$$
$$\omega_{03} = \sqrt{\frac{n_1}{n_5 \omega_{02}^2}} \quad (8)$$
$$b_3 = \frac{\omega_{01}\omega_{02}\omega_{03}}{b_1 b_2 \left[n_3 + n_5\left(\omega_{02}^2 + \omega_{03}^2\right)\right]}. \quad (9)$$

Thus, b_1, b_2, b_3, ω_{01}, ω_{02}, ω_{03} can be calculated using (4)–(9), respectively, and the transformation $T(\omega)$ is determined finally.

Employing the frequency transformation determined above and network analysis, a unit capacitance can be transformed to an inverter coupled resonator section shown in Fig. 2. Fig. 2 also indicates the physical meanings of the parameters in the frequency transformation. ω_{01}, ω_{02}, and ω_{03} represent the resonant angular frequencies, and b_1, b_2, and b_3 represent the susceptance slope parameters of the parallel resonators when characteristic admittances are unity. It can be derived that the characteristic admittance J of the inverter can be designated arbitrarily according to the practical situation.

The two inverter-coupled resonator sections can be used as basic building blocks for the construction of triple-passband filters. Based on a cross-coupled low-pass prototype, a cross-coupled triple-passband filter can be obtained. The topology structure of such a filter is schematically depicted in Fig. 3. Each black node represents a resonator, and the lines between resonators represent the couplings.

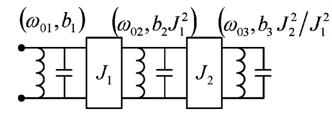

Fig. 2. Inverter-coupled resonator section obtained using the frequency transformation.

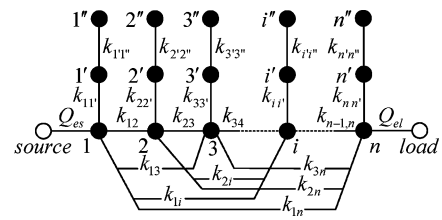

Fig. 3. Topology structure of the cross-coupled triple-passband filter.

It is noted that the first resonators in the inverter-coupled resonator sections are cross-coupled in the triple-passband filter, and the coupling coefficients of these resonators can be expressed in the form of matrix. This coupling matrix is calculated as

$$k = \frac{M}{b_1} \quad (10)$$

where M is the coupling matrix of the cross-coupled low-pass prototype in [8].

The coupling coefficients of the resonators in the inverter coupled resonator section is expressed as

$$k_{i,i'} = \frac{J_1}{\sqrt{b_1 b_2 J_1^2}} = \frac{1}{\sqrt{b_1 b_2}} \quad (11)$$
$$k_{i',i''} = \frac{J_2}{\sqrt{b_2 J_1^2 b_3 \frac{J_2^2}{J_1^2}}} = \frac{1}{\sqrt{b_2 b_3}}. \quad (12)$$

The external quality factors can be calculated using

$$Q_{es} = \frac{b_1}{R_1} \quad (13)$$
$$Q_{el} = \frac{b_1}{R_N} \quad (14)$$

where R_1 and R_N is the source impedance and load impedance of the low-pass prototype in [8], respectively.

III. Triple-Passband Filter Design Example

A triple-passband filter is designed and fabricated using microstripe structure to validate the presented synthesis theory. The three passbands are designated to be 3.3–3.4, 3.5–3.6, and 3.7–3.8 GHz, and each passband has a maximum return loss of 20 dB. The parameters in the frequency transformation are calculated as

$$[b_1, b_2, b_3] = [11.795, 13.301, 19.006] \quad (15)$$
$$[\omega_{01}, \omega_{02}, \omega_{03}] = 2\pi \times [3.538, 3.544, 3.555]. \quad (16)$$

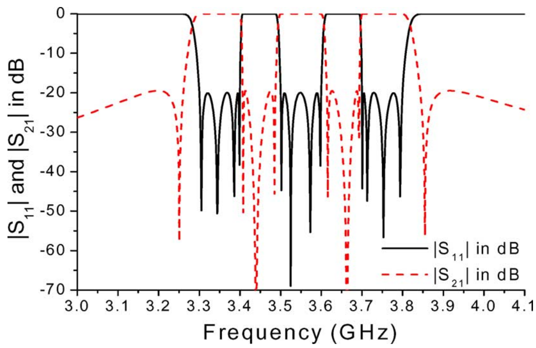

Fig. 4. Theoretical filter characteristics in frequency domain.

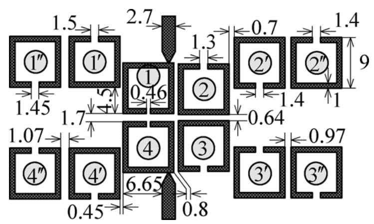

Fig. 5. Layout of the triple-passband filter. All dimensions are in millimeters.

The filter is designed based on a cascaded quadruplet (CQ) generalized Chebyshev low-pass prototype that has two finite transmission zeros at $\pm j1.5$ and 20 dB return loss in the passband. The circuit parameters are $k_{12} = k_{34} = 0.0687$, $k_{23} = 0.0707$, $k_{14} = -0.0298$, $k_{i,i'} = 0.0798$, $k_{i',i''} = 0.0629$, $Q_{\mathrm{e}} = 11.456$. The theoretical filter characteristics of the filter can be obtained by substituting the frequency transformation into the transfer function as shown in Fig. 4, and it meets the prescribed specification very well. It can be observed easily that all the passbands have the same filtering characteristics as the CQ generalized Chebyshev low-pass prototype.

Finally, microstrip square open-loop resonators are applied to realize the parallel resonators. The relative dielectric constant and the thickness of the substrate is 2.65 and 1 mm, respectively. The parameters are extracted by Zeland IE3D software using the method proposed in [9]. Fig. 5 illustrates the layout and dimensions of the filter, and the structure is symmetrical about the horizontal axis. Since the resonant characteristics of any resonator will be influenced by the resonators aside, the lengths of the resonators are not exactly half-wavelength and are not proportional to their resonant frequencies.

The filter is also fabricated and measured. Fig. 6 compares the simulated and measured frequency responses using Zeland IE3D software and Agilent's 8719ES network analyzer, respectively. Both simulated and measured results lose reflection zeros and some transmission zeros due to the parasitical couplings. The measured result agrees well with the simulated result and shows the validity of the synthesis method.

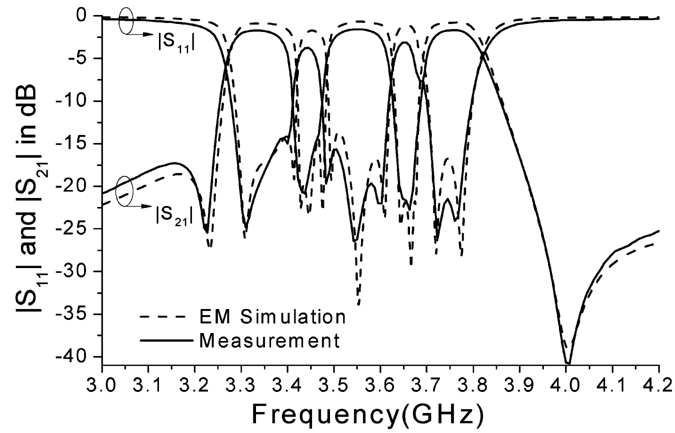

Fig. 6. Comparison of simulated and measured frequency response.

IV. Conclusion

An analytical synthesis method has been presented in this letter. A frequency transformation has been developed. Applying this transformation, the triple-passband filter circuits with cross-coupled configuration have been constructed. The expressions of the coupling coefficients and the external quality factors are given. To validate the synthesis technique, a triple-passband filter is designed, fabricated, and measured. The theoretical frequency response of this filter meets the prescribed specifications very well, and the measured result shows a good agreement with the simulated one.

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