Seven-parameter statistical model for BRDF in the UV band

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Abstract: A new semi-empirical seven-parameter BRDF model is developed in the UV band using experimentally measured data. The model is based on the five-parameter model of Wu and the fourteen-parameter model of Renhorn and Boreman. Surface scatter, bulk scatter and retro-reflection scatter are considered. An optimizing modeling method, the artificial immune network genetic algorithm, is used to fit the BRDF measurement data over a wide range of incident angles. The calculation time and accuracy of the five- and seven-parameter models are compared. After fixing the seven parameters, the model can well describe scattering data in the UV band.

OCIS codes: (290.1483) BSDF, BRDF, and BTDF; (290.5820) Scattering measurements; (290.5880) Scattering, rough surfaces.

References and links
1. Introduction

Interest in the study of scattering and reflective properties in the UV band (200 nm to 400 nm) has recently increased. The applications of such properties include UV space object detection and subsurface defect short wavelength monitoring in the semiconductor industry, among others [1,2].

The bidirectional reflectance distribution function (BRDF) is often used to describe the directional dependence of the scattering properties of a surface. The BRDF has been extensively studied and surveyed in various fields [3–10]. Many BRDF models have been developed, and they can be classified into two categories, namely, purely empirical and analytical models.

Both models have their advantages and disadvantages. The purely empirical BRDF models are simple and useful but have no physical basis, such as the Minnaert BRDF model [3], Walthall model [5], etc. The analytical BRDF models are derived from more complex physical theory by simplifying some assumptions and approximations. They are needed in many applications, especially in simulation software, but they usually require the determination of many parameters. Examples include the Roujean [7], LiSparse-Dense BRDF [8], and Renhorn and Boreman 14-parameter [9] models.

Most BRDF models are developed over the visible and near-infrared spectrum. There are numerous papers that have discussed the measurement or model method in the visible to infrared band [7–12]. Although there are some related papers reporting UV scatter in rough surfaces [13–17], a suitable BRDF model in the UV band is still lacking. In the UV band, the refraction light spectrum is often not easy to be measured, and we should consider more complex model to describe the measurement data. Bulk or volume scattering are usually caused by the in-homogeneities of materials. Relative studies about volume scattering provide a powerful tool to investigate the scattering properties of roughness surfaces [18–22].

The present paper aims to provide a model of reflection by a rough surface that can successfully predict the experimental findings in the UV band. In section one, the schematic diagram of the instrument used to performing scatter measurements is presented. In section two, a novel seven-parameter model is developed. There are three terms in this model: surface scatter, bulk scatter, and retro-reflection. An optimizing modeling method is used to model the BRDF measurement data of typical samples in the UV band. In the last section, the calculation results using the new seven-parameter model are compared with the five-parameter model.

2. Measurement of BRDF in the UV band

A BRDF is actually an angle-resolved energy distribution. As shown in Fig. 1, an element of surface dA is illuminated by a source of incident wave with wave vector \( \hat{k}_i \). Letting \( \hat{k}_r \) be the reflected direction wave vector, the symbol \( \hat{\varepsilon} \) denotes the normal of the mean surface of the z axis and \( \hat{n} \) denotes the normal direction of the micro-facet dA. Letting \( \alpha \) be the angle between \( \hat{n} \) and \( \hat{\varepsilon} \), where \( \gamma \) is the angle between \( \hat{k}_i \) and \( \hat{n} \), angles \( \alpha \) and \( \gamma \) satisfy the following relationships

\[
\cos \alpha = \left( \cos \theta_i + \cos \theta_r \right) / (2 \cos \gamma)
\]  

(1)
\[
\cos^2 \gamma = \frac{1}{2} \left( \cos \theta_i \cos \theta_r + \sin \theta_i \sin \theta_r \cos \phi_r + 1 \right)
\]  \hspace{1cm} (2)

where \( \theta \) and \( \phi \) are the zenith and azimuthal angles, respectively. The subscripts \( i \) and \( r \) denote the incident and reflected angles, respectively. The definition of BRDF, \( f_r \), can be expressed as the differential radiance \( dL_r \) scattered by a uniformly illuminated, homogeneous material per unit differential incident irradiance \( dE_i \):

\[
f_r(\theta, \phi, \theta_i, \phi_i) = \frac{dL_r(\theta, \phi, \theta_i, \phi_i)}{dE_i(\theta, \phi_i)}
\]  \hspace{1cm} (3)

Another equivalent definition of the BRDF is

\[
f_r = \frac{P_r}{(P_i \cos \theta_i \cdot \Omega_i)}
\]  \hspace{1cm} (4)

where \( \Omega_i \) is the solid angle through which the scattered power \( P_r \) is collected. It is then normalized with respect to the total incident power \( P_i \). On the other hand, \( \cos \theta_r \) is the factor that can be thought of as a correction to adjust the illuminated area to its apparent area when viewed from the direction of the scatter.

Fig. 1. Angle definition of BRDF.

Fig. 2. A schematic diagram of the instrument.
Figure 2 shows the schematic diagram of the instrument used to perform angle-resolved optical scatter measurements. This BRDF measurement installation system was designed and constructed by the Anhui Institute of Optics and Fine Mechanics, Chinese Academy of Sciences. This measurement system has three parts, i.e., the source, sample holder, and receiver. The design details of each part are described elsewhere [23].

The UV light source used in these investigations was a 150 mW-power deuterium lamp, which can generate continuous emission spectra. After emerging from the exit slit, the beam is quasi-collimated by passing through a convex lens. After passing through two mirrors, the beam is reflected to the slit. By passing through the slit, the beam enters the sample and is then be collected by the detector. The system can measure three-dimensional BRDF data by rotating the sample and light source using electromotors A, B, and C in different three-dimensional directions. The detector used was a spectrometer made by Ocean Optic Co. in USA. The sample can be rotated about a vertical axis to allow different angles of incidence, and the detector can be rotated independently about the same axis to allow the measurement of angular scatter distribution. The step of the zenith angle is 1°, and the step of the azimuth angle is 5°. Near the specular direction, the sample angle is 1° to guarantee sufficient precision. The accuracy of the zenith and azimuth angles is 0.1°, and the relative error of the measured BRDF of this system is less than 5%. The incident UV wavelength varies from 250.45288 nm to 368.89652 nm. The total 1024 output voltages $V_i$ for the sample are measured within this wavelength range. The voltages of reflectance standard plate $V_{ref}$ are also measured within the corresponding wavelengths. Then, the ratio is as follows

$$\frac{f_r(\theta_i, \varphi; \theta_r, \varphi_r, \lambda)}{f_r(\theta_i, \varphi; \theta_r, \varphi_r, \lambda)} = \frac{V_i}{V_{ref}}$$

where $f_r(\theta_i, \varphi; \theta_r, \varphi_r, \lambda)$ and $f_r(\theta_i, \varphi; \theta_r, \varphi_r, \lambda)$ are the BRDFs of the sample and reflectance standard plate at a given incident wavelengths $\lambda$, respectively.

Notably, $V_{ref}$ of the reference standard plate must be determined under the same measurement conditions used to detect the samples, and all measurements must be performed correspondingly. In our experiments, the reference standard plate is the pressed polytetrafluoroethylene plate, which can be considered as a perfect Lambertian plate.

3. Development of a seven-parameter BRDF model

Torrance and Sparrow [10] have deduced a useful light-scattering model, which assumes that the surface consists of small, randomly disposed mirror-like facets. An implied criterion of this model is that the root-mean-square (RMS) surface roughness is greater than the wavelength of the incident radiant energy. There are three parameters in the Torrance-Sparrow model, based on which Wu [11] has proposed a five-parameter BRDF model, whose expression can be expressed as

$$BRDF_{surface} = f_r(\theta_i, \theta_r, \varphi, \lambda)$$

$$= k_\alpha \frac{k^2 \cos \alpha}{1 + (k^2 - 1) \cos \alpha} \exp \left[ b \left( 1 - \cos \gamma \right)^\alpha \right] V(\theta_i, \varphi, \lambda) \frac{V(\theta_i, \varphi, \lambda)}{\cos \theta_i \cos \theta_r} + k_d$$

where $\varphi = \varphi_o - \varphi_i$. For isotropic materials at a given wavelength, the BRDF can be safely assumed to be invariant to azimuthal rotations of the incident and $\varphi_i = 0$. There are two components in Eq. (6); one is the specular reflection from mirror-like surface facets, and the other is a diffuse component. In the modified Torrance-Sparrow model of Wu [11], there are five parameters $a, b, k_\alpha, k_d,$ and $k_r$ to be determined. $k_\alpha$ is the mirror-direction component, $k_d$ is related with the diffuse reflection component, and $k_r$ is related to the distribution of the
subsurface \( \text{d}A \) (determined by the slope distribution of the surface). 
\[
k^2 \cos \alpha / [1 + (k^2 - 1) \cos \alpha]
\] is the distribution function of the sub-surface, \( \exp[b \cdot (1 - \cos \gamma)^r] \) is the approximation description of the Fresnel reflectance function, and \( V(\theta, \theta', \phi, \lambda) \) is the masking and shadowing effect. In this model, the exponential function in the Torrance-Sparrow model is substituted by an elliptical function to describe the distribution of the normal of the facets. The exponential function with two parameters is used to substitute the Fresnel reflectance function to avoid the calculation of many trigonometric functions. The model can be used to describe isotropic surfaces with non-polarization incident light. The selection criteria of these five parameters are to minimize the RMS errors between the simulation and experimental data.

UV wavelengths are shorter than visible and infrared bands. Thus, a new model that suited the experimental measurement data in the UV band is developed. Our model partly adopted the five-parameter model of Wu and partly adopted the Renhorn-Boreman fourteen-parameter analytical model. The corresponding bulk scatter and retro-reflection terms are introduced to improve the accuracy of the simulation results of the five-parameter model.

According to the Renhorn-Boreman model [9], a Gaussian surface auto-covariance function \( g(x, y) \) is given by

\[
g(x, y) = \sigma_s^2 \exp\left(-\left(x^2 + y^2\right)\rho^2\right)
\]

where \( \sigma_s \) is the RMS surface roughness and \( \rho \) is the inverse of the surface correlation length. Its Fourier transform corresponding to the surface power spectrum is \( G(\xi, \eta) \). The parameters \( \xi \) and \( \eta \) are direction cosines defined as \( \xi = \sin(\theta) \cos(\phi) \) and \( \eta = -\sin(\theta) \sin(\phi) \). The corresponding Fourier transform of the surface auto-covariance function is

\[
G(\xi, \eta) = \frac{\sigma_s^2}{\sqrt{2\rho}} \exp\left[-\frac{1}{4\rho^2}(\xi^2 + \eta^2)\right]
\]

An exponential auto-covariance function is given by

\[
g(x, y) = \sigma_s^2 \exp\left[-\left(|x| + |y|\right)\rho^2\right]
\]

The Fourier transformation results of Eq. (9) can be obtained as follows using Mathematic under the condition \( 0 \leq \xi \leq 1, 0 \leq \eta \leq 1 \) and a sufficiently large \( \rho \)

\[
G(\xi, \eta) = \frac{2}{\pi} \frac{\sigma_s^2}{\xi^2 + \eta^2 + \rho^2}
\]

In the Renhorn-Boreman model, the exponential statistical distribution is combined with the Gaussian distribution. Taking the angle of incidence into account, the Renhorn-Boreman model yields the two-dimensional Lorentzian BRDF by

\[
BRDF = \frac{\sigma N}{(\xi - \xi_0)^2 + \eta^2 + \rho^2}
\]

where \( \sigma \) is the integrated reflectance. To ensure that the analytical two-dimensional integral of the BRDF is unitary, the parameter \( N \) gives
Considering the geometrical foreshortening effect, the expression 
\[
\rho \left( \sqrt{1 - \xi^2} + \sqrt{1 - \xi^2 - \eta^2} \right) \frac{1}{2}
\]
is used instead of the surface correlation parameter \( \rho \). Equation (11) of the BRDF model of Renhorn-Boreman is finally given as

\[
BRDF = \frac{\sigma N}{(\xi - \xi_0)^2 + \eta^2 + \frac{1}{4} (\sqrt{1 - \xi_0^2} + \sqrt{1 - \xi^2 - \eta^2})^2 \rho^2}
\]  

(13)

The Renhorn-Boreman BRDF model, which considers the surface scattering and includes the bulk and retro-reflection scattering sections, is partly adopted. The total BRDF is given by three separate parts as follows

\[
BRDF_{\text{total}} = BRDF_{\text{surface}} + BRDF_{\text{bulk}} + BRDF_{\text{retro}}
\]  

(14)

The surface scattering BRDF part is described by Eq. (6), whereas the bulk scattering and retro-reflection scattering BRDF are described as follows

\[
BRDF_{\text{bulk}} = \frac{N}{(\xi - \xi_0)^2 + \eta^2 + \frac{1}{4} (\sqrt{1 - \xi_0^2} + \sqrt{1 - \xi^2 - \eta^2})^2 \rho_1^2}
\]  

(15)

\[
BRDF_{\text{retro}} = \frac{N}{(\xi - \xi_0)^2 + \eta^2 + \frac{1}{4} (\sqrt{1 - \xi_0^2} + \sqrt{1 - \xi^2 - \eta^2})^2 \rho_2^2}
\]  

(16)

where \( \rho_1 \) and \( \rho_2 \) are the parameters related to the bulk and retro-reflection scatterings, respectively. Given that the proportions of the bulk and retro-reflection scattering sections are very small, only the masking and shadowing effects are considered in the surface scattering section. For the bulk and retro-reflection scattering sections, only the two-dimensional Lorentzian BRDF is considered.

4. Comparison of the seven- and five-parameter BRDF models

At this point, a new BRDF model has been developed. There are seven parameters to be determined in this model, namely, \( \{k_x, k_y, k_z, a, b, \rho_1, \rho_2 \} \). The artificial immune network algorithm is used to fit the parameters in the model. The optimization method is used to determine the optimum values of the fitting parameters of the BRDF to minimize the squared error between the measured data and the model.

1. Measured BRDF data of sample #1: Fine grinding rough aluminum plate.
Fig. 3. Comparison of the seven- and five-parameter models of sample #1 for various angles of incidence when wavelength of incidence is $\lambda = 266$ nm.

Figures 3 and 4 compare the measured data (shown as discrete points) with the fitted BRDF derived from the seven-parameter (solid curves in blue) and five-parameter (short dash curves in red) models. The sample used in this simulation is a fine grinding rough aluminum plate. The zenith angles of incidence are 20°, 30°, 45°, and 60°. The scattering angle of scatter is over a range of angles from –70° to 70°. In the given wavelengths, six groups of measured data at various angles of incidence are usually used to fit the parameters to be determined (only four groups of measured data are shown in the Figs. 3-6). For convenience, the azimuthal angles of incidence and scatter are both set to 0°. The wavelengths of incidence are $\lambda = 266$ nm and $\lambda = 369$ nm in Figs. 3 and 4, respectively.

Fig. 4. Comparison of the seven- and five-parameter models of sample #1 for various angles of incidence when wavelength of incidence is $\lambda = 369$ nm.
Based on Figs. 3-4, sample #1 can be concluded as sufficiently rough, and the diffuse component dominates this rough surface. Compared with the same sample at different incident wavelengths, Figs. 3 and 4 show that if the wavelength is shorter, the sample is rougher. The mirror reflection at 369 nm is distinct from that at 266 nm. At both wavelengths, the seven-parameter model can meet the measured data better than the five-parameter one.

Table 1 lists the fitting parameters, RMS error, and calculation times of the two different models. Compared with the five-parameter model, our newly developed seven-parameter model has a smaller RMS error. Although the calculation time is longer than that of the five-parameter model, it is still tolerable.

Table 1. Comparison of the Five- and Seven-Parameter Models of Sample #1

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$k_b$</th>
<th>$k_r$</th>
<th>$k_d$</th>
<th>$a$</th>
<th>$\rho_1$</th>
<th>$\rho_2$</th>
<th>Error (%)</th>
<th>Time(s)</th>
<th>$\lambda$(nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>five-parameter model</td>
<td>0.140</td>
<td>10.860</td>
<td>0.156</td>
<td>4.497</td>
<td>10.957</td>
<td>—</td>
<td>2.15%</td>
<td>140</td>
<td>266</td>
</tr>
<tr>
<td>seven-parameter model</td>
<td>0.078</td>
<td>1.503</td>
<td>0.167</td>
<td>14.286</td>
<td>26.911</td>
<td>1.751</td>
<td>11.070</td>
<td>0.70%</td>
<td>171</td>
</tr>
<tr>
<td>five-parameter model</td>
<td>0.114</td>
<td>17.092</td>
<td>0.204</td>
<td>3.700</td>
<td>21.443</td>
<td>—</td>
<td>1.78%</td>
<td>146</td>
<td>369</td>
</tr>
<tr>
<td>seven-parameter model</td>
<td>0.051</td>
<td>1.128</td>
<td>0.252</td>
<td>0.369</td>
<td>24.929</td>
<td>5.788</td>
<td>1.579</td>
<td>0.85%</td>
<td>177</td>
</tr>
</tbody>
</table>

All calculation time simulations shown in Table 1 is developed using a Fortran code running on a standard computer (Intel Core 2 Duo processor, 1.86 GHz, 500GB EMS memory). The parameters are determined by an optimizing artificial immune network genetic algorithm.

According to the Kirchhoff approximation method for electromagnetic wave scattering, the bulk scattering increases with decreased incident wavelength. This model considers bulk and retro-reflection scatterings. Thus, it can effectively reduce the RMS error between the experimental data and model simulation results.

(2) Measured BRDF data of sample #2: white painted surface

Fig. 5. Comparison of the seven- and five-parameter models of sample 2# for various angles of incidence when wavelength of incidence is $\lambda=266$nm.
Another example is shown in Figs. 5 and 6. Table 2 lists the model parameters, RMS error, and computing time of sample #2. The sample used in this case is a white painted surface.

The mirror-reflective section of sample #2 is more obvious than that of sample #1. Thus, sample #2 is relatively smoother than sample #1. The seven-parameter model still has less RMS error for different incidence wavelengths in this case.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$k_b$</th>
<th>$k_r$</th>
<th>$k_d$</th>
<th>$a$</th>
<th>$b$</th>
<th>$\rho_1$</th>
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<th>Error</th>
<th>Time (s)</th>
<th>$\lambda$ (nm)</th>
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<tbody>
<tr>
<td>five-parameter</td>
<td>1.214</td>
<td>5.826</td>
<td>0.990</td>
<td>0.312</td>
<td>25.498</td>
<td></td>
<td></td>
<td></td>
<td>4.51%</td>
<td>130</td>
</tr>
<tr>
<td>seven-parameter</td>
<td>1.036</td>
<td>2.511</td>
<td>0.852</td>
<td>0.328</td>
<td>28.912</td>
<td>1.874</td>
<td>1.438</td>
<td>2.73%</td>
<td>168</td>
<td>266</td>
</tr>
<tr>
<td>model</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>five-parameter</td>
<td>2.582</td>
<td>11.788</td>
<td>0.941</td>
<td>0.314</td>
<td>22.831</td>
<td>1.350</td>
<td>1.438</td>
<td>6.23%</td>
<td>141</td>
<td>369</td>
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<tr>
<td>seven-parameter</td>
<td>2.271</td>
<td>23.309</td>
<td>0.791</td>
<td>0.330</td>
<td>25.350</td>
<td>4.799</td>
<td>1.350</td>
<td>4.42%</td>
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</table>

5. Conclusion

A new seven-parameter BRDF model of rough surfaces in the UV band is established. An angle-resolved BRDF measurement system operates on some typical samples at different incident angles. The measured data are compared with the results of the seven-parameter model. Both for different wavelengths of the same sample or for different samples, the new seven-parameter model meets the experimental data better than the five-parameter one. It can significantly improve the calculation accuracy, although it costs a little more calculation time.

Both seven- and five-parameter model has its advantages and disadvantages. Optimization method will become more difficult when parameter in a model is so many. And the calculation time will be longer. If the precision requirement is not high, for most engineering-oriented application fields, the five-parameter model is a good enough model. But for some...
special use such as in the UV band, it is difficult to detect the scattering signal. If you want to improve the calculation precision, more precision model should be used.

This paper offers a new developed BRDF model of rough surfaces in the UV band. The scattering properties of materials in the UV band are elucidated. And this kind of researches is helpful to understand the scattering properties of materials in the UV band.

Acknowledgments

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